The correlation between transverse momentum and multiplicity of charged particles in a two-component model

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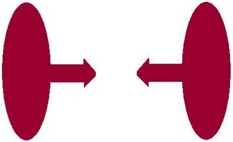
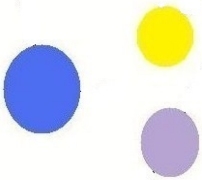
Introduction

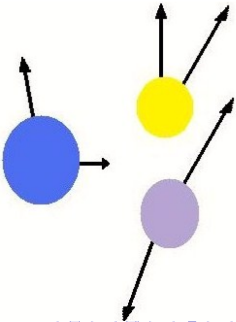
Two-stage scenario of particle production



Creation of emitters

Decomposition of emitters





Introduction

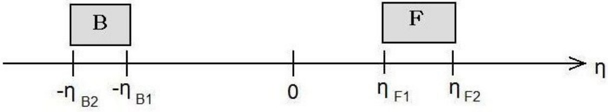


Figure 1: The backward and forward pseudorapidity windows

$$\eta = -ln\left[tg\left(\frac{\theta}{2}\right)\right]$$

LRC observables

- the event multiplicity in the Backward or Forward window n_B, n_F
- the event mean transverse momentum in the Backward or Forward window $p_{tB} \equiv \frac{1}{n_B}\sum_{i=1}^{n_B}|\mathbf{p}_{tBi}|, \quad p_{tF} \equiv \frac{1}{n_F}\sum_{i=1}^{n_F}|\mathbf{p}_{tFi}|$

Introduction

LRC $\sim$ studying $\langle B \rangle_F$ as a function of F

Types of correlations:

- $n_B - n_F$ - the correlation between charged particle multiplicities in B and F windows
- $p_{tB} - n_F$ - the correlation between the event mean transverse momentum in the B window and the charged particle multiplicity in the F window
- $p_{tB} - p_{tF}$ - the correlation between the event mean transverse momenta in B and F windows

Introduction

Correlation coefficients

- $${b^{abs}}_{n-n} \equiv \frac{\langle n_B\, n_F\rangle - \langle n_B\rangle \langle n_F\rangle}{\langle {n_F}^2\rangle - \langle n_F\rangle^2}; \;(\langle n_B\rangle_{n_F} = f\,(n_F)\,,\, b = \frac{d\langle n_B\rangle_{n_F}}{dn_F})$$
  $${b^{rel}}_{n-n} \equiv {b^{abs}}_{n-n} \cdot \frac{\langle n_F\rangle}{\langle n_B\rangle}$$
- $${b^{abs}}_{p_t-n} \equiv \frac{\langle p_{tB}\, n_F\rangle - \langle p_{tB}\rangle \langle n_F\rangle}{\langle {n_F}^2\rangle - \langle n_F\rangle^2}; \;(\langle p_{tB}\rangle_{n_F} = g\,(n_F)\,,\, b = \frac{d\langle p_{tB}\rangle_{n_F}}{dn_F})$$
  $${b^{rel}}_{p_t-n} \equiv {b^{abs}}_{p_t-n} \cdot \frac{\langle n_F\rangle}{\langle p_{tB}\rangle}$$

For more information - see e.g. I.Altsybeev talk "Forward-backward correlations in pp collisions with ALICE detector", QFTHEP 2013, June 24

Model

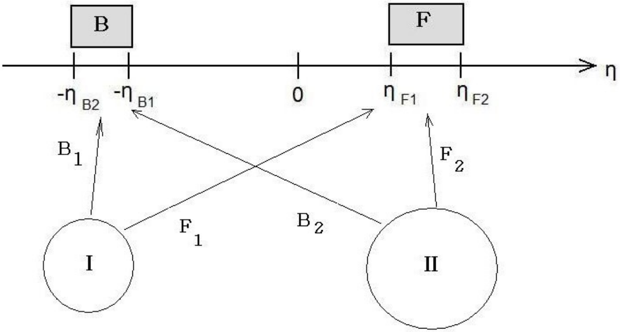


Figure 2: Primary and secondary emitters produce particles in the B and F windows

Calculation of the $n - n$ correlation coefficient

Let the probability to have N_1 primary emitters and N_2 secondary emitters be $\omega(N_1, N_2)$. Let $\{N_1, N_2\} \equiv C$. Then

$$\langle n_F \rangle = \sum_C \omega(C) \langle n_F \rangle_C = \sum_{n_{F1}, n_{F2}} (n_{F1} + n_{F2}) \sum_{N_1, N_2} \omega(N_1, N_2) P_{N_1}(n_{F1}) P_{N_2}(n_{F2});$$

$$\langle n_B \, n_F \rangle = \sum_{n_{F1}, n_{F2}} \sum_{n_{B1}, n_{B2}} (n_{F1} + n_{F2})(n_{B1} + n_{B2}) \sum_{N_1, N_2} P_{N_1}(n_{F1}) P_{N_1}(n_{B1}) P_{N_2}(n_{F2}) P_{N_2}(n_{B2}).$$

Here

$$P_{N_1}(B_1) = \sum_{\{B_{i1}\}} \delta_{B_1 \; \sum_{i1=1}^{N_1} B_{i1}} \prod_{i1=1}^{N_1} p_{B_1}(B_{i1});$$

$$\sum_n p_{B_1}(n) = 1; \qquad \sum_n n \, p_{B_1}(n) = \overline{\mu}_{B1}.$$

Calculation of the $n-n$ correlation coefficient

$$b^{abs}{}_{n-n} \equiv \frac{\langle n_B\, n_F\rangle - \langle n_B\rangle\,\langle n_F\rangle}{\langle {n_F}^2\rangle - \langle n_F\rangle^2};\; b^{rel}{}_{n-n} \equiv b^{abs}{}_{n-n} \cdot \frac{\langle n_F\rangle}{\langle n_B\rangle}$$

$$b^{abs}{}_{n-n} = \frac{D_{N_1}\overline{\mu}_{B1}\,\overline{\mu}_{F1} + cov\,(N_1, N_2)\,(\overline{\mu}_{B1}\,\overline{\mu}_{F2} + \overline{\mu}_{B2}\,\overline{\mu}_{F1}) + D_{N_2}\overline{\mu}_{B2}\,\overline{\mu}_{F2}}{\overline{N_1}D_{\mu_{F1}} + \overline{N_2}D_{\mu_{F2}} + D_{N_1}\overline{\mu}_{F1}{}^2 + D_{N_2}\overline{\mu}_{F2}{}^2 + 2cov\,(N_1, N_2)\,\overline{\mu}_{F1}\,\overline{\mu}_{F2}},$$

$$b^{rel}{}_{n-n} = b^{abs}{}_{n-n} \cdot \frac{\overline{N_1}\overline{\mu}_{F1} + \overline{N_2}\overline{\mu}_{F2}}{\overline{N_1}\overline{\mu}_{B1} + \overline{N_2}\overline{\mu}_{B2}},$$

where

$$\overline{N}_{1,2} = \sum_C \omega\,(C)\,N_{1,2}; \qquad D_{N_{1,2}} = \overline{{N_{1,2}}^2} - \overline{N_{1,2}}^2; \qquad cov\,(N_1, N_2) = \overline{N_1\,N_2} - \overline{N_1}\,\overline{N_2}.$$

If we assume that $\overline{\mu}_{F1} = \overline{\mu}_{F2}$, $\overline{\mu}_{B1} = \overline{\mu}_{B2}$, $D_{\mu_{F1}} = D_{\mu_{F2}}$, $D_{\mu_{B1}} = D_{\mu_{B2}}$, $N_1 + N_2 = N$,:

$$b^{rel}{}_{n-n} \longrightarrow \frac{D_N\,\overline{\mu}_F^2}{\overline{N}\,D_{\mu_F} + D_N\overline{\mu}_F^2}$$

V.V. Vechernin, arxiv:1012.0214v1 (2010); Proc. of Baldin ISHEPP XX, Volume II, 10 (2010)

Model

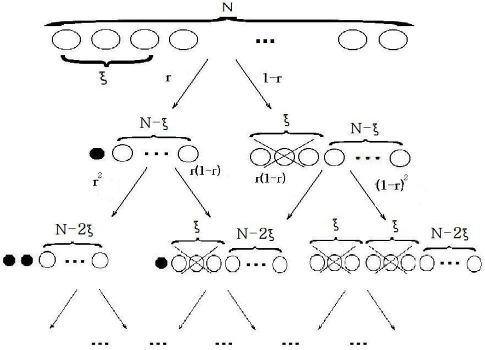
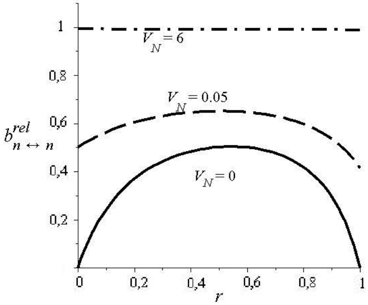


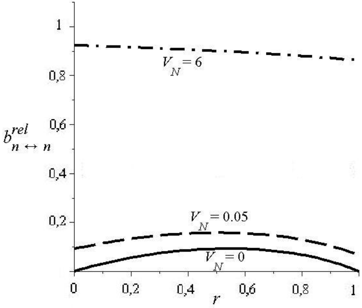
Figure 3: Interaction scheme

$$P_N\left(N_2\right) = C^{N_2}_{\left[\frac{N}{\xi}\right]} \cdot r^{N_2} \cdot (1-r)^{\left[\frac{N}{\xi}\right]-N_2}$$

$n - n$ correlations

$$b^{rel}{}_{n-n} \equiv \frac{\langle n_B\, n_F \rangle - \langle n_B \rangle \langle n_F \rangle}{\langle n_F{}^2 \rangle - \langle n_F \rangle^2} \cdot \frac{\langle n_F \rangle}{\langle n_B \rangle}$$





$W_{\mu_{F1}} = 0.05$ $\qquad$ $W_{\mu_{F1}} = 0.5$

$$V_N = \frac{D_N}{\overline{N}}, W_{\mu_{F1}} = \frac{D_{\mu_{F1}}}{\overline{\mu}_{F1}{}^2}$$

$p_t - n$ correlations

$$p_{tB} \equiv \frac{1}{n_B} \sum_{i=1}^{n_B} |\mathbf{p}_{tBi}|$$

$$\langle p_{tB} \rangle = \sum_C \omega(C) \langle p_{tB} \rangle_C = \sum_{n_{B1}, n_{B2}} \frac{\overline{k_1} n_{B1} + \overline{k_2} n_{B2}}{n_{B1} + n_{B2}} \sum_{N_1, N_2} \omega(N_1, N_2) P_{N_1}(n_{B1}) P_{N_2}(n_{B2})$$

$$\langle p_{tB}\, n_F \rangle = \sum_C \omega(C) \langle p_{tB} \rangle_C \langle n_F \rangle_C = \sum_{n_{B1}, n_{B2}} \frac{\overline{k_1} n_{B1} + \overline{k_2} n_{B2}}{n_{B1} + n_{B2}} \sum_{N_1, N_2} \omega(N_1, N_2) \times$$

$$\times (N_1 \overline{\mu}_{F1} + N_2 \overline{\mu}_{F2}) P_{N_1}(n_{B1}) P_{N_2}(n_{B2}),$$

where

$$P_{N_1}(B_1) = \sum_{\{B_{i1}\}} \delta_{B_1 \; \sum_{i1=1}^{N_1} B_{i1}} \prod_{i1=1}^{N_1} p_{B_1}(B_{i1});$$

$\overline{\mu}_{F1}$-the mean charged particles multiplicity in the F window from a primary emitter
$\overline{k_1}$-the mean transverse momentum of particles in the B window from a primary emitter

$p_t - n$ correlations

$$cov\left(p_{tB}, n_F\right) = \langle p_{tB}\, n_F\rangle - \langle p_{tB}\rangle \langle n_F\rangle = \sum_{N_1, N_2} \omega\left(N_1, N_2\right)\left(\left(N_1 - \overline{N_1}\right)\overline{\mu}_{F1} + \right.$$

$$\left. + \left(N_2 - \overline{N_2}\right)\overline{\mu}_{F2}\right) \sum_{n_{B1}, n_{B2}} \frac{\overline{k_1}\, n_{B1} + \overline{k_2}\, n_{B2}}{n_{B1} + n_{B2}} P_{N_1}\left(n_{B1}\right) P_{N_2}\left(n_{B2}\right).$$

One should make an approximation:

$$n_{B1} + n_{B2} \longrightarrow \langle n_{B1}\rangle_{N_1} + \langle n_{B2}\rangle_{N_2} = N_1\,\overline{\mu}_{B1} + N_2\,\overline{\mu}_{B2}$$

Then

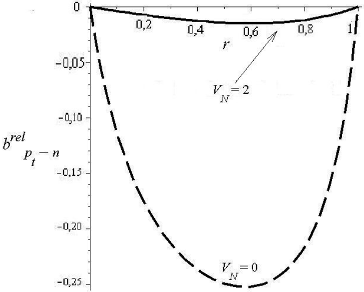
$$cov\left(p_{tB}, n_F\right) = \sum_{N_1, N_2} \omega\left(N_1, N_2\right)\left(\left(N_1 - \overline{N_1}\right)\overline{\mu}_{F1} + \right.$$

$$\left. + \left(N_2 - \overline{N_2}\right)\overline{\mu}_{F2}\right) \frac{\overline{k_1}\, N_1\,\overline{\mu}_{B1} + \overline{k_2}\, N_2\,\overline{\mu}_{B2}}{N_1\,\overline{\mu}_{B1} + N_2\,\overline{\mu}_{B2}}.$$

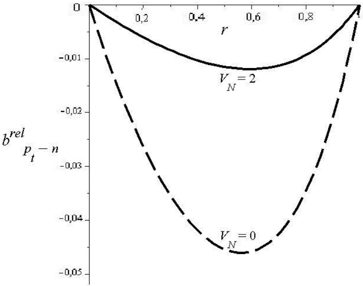
Note: if $\overline{k_1} = \overline{k_2}$, then $cov\left(p_{tB}, n_F\right) = 0$, if $\overline{k_1} \neq \overline{k_2}$, then $cov\left(p_{tB}, n_F\right) \neq 0$

$p_t - n$ correlations: numerical results

$$b^{rel}{}_{p_t - n} \equiv \frac{\langle p_{tB}\, n_F \rangle - \langle p_{tB} \rangle \langle n_F \rangle}{\langle {n_F}^2 \rangle - \langle n_F \rangle^2} \cdot \frac{\langle n_F \rangle}{\langle p_{tB} \rangle}$$



$W_{\mu_{F1}} = 0.05$

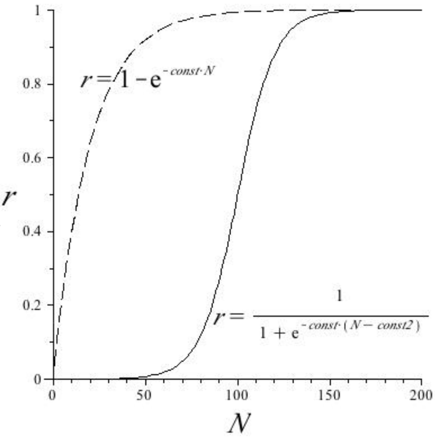


$W_{\mu_{F1}} = 0.5$

Modification

Possible functions

- $r = 1 - e^{-const \cdot N}$
- $r = \dfrac{1}{1 + e^{-const1 \cdot (N - const2)}}$



Monte-Carlo simulation

$$N \Rightarrow N_2 \Rightarrow n_F \Rightarrow n_B \Rightarrow {b^{abs}}_{n-n} \equiv \frac{\langle n_B\, n_F \rangle - \langle n_B \rangle \langle n_F \rangle}{\langle {n_F}^2 \rangle - \langle n_F \rangle^2}$$

Conclusions

- Expressions for the $n - n$ and $p_t - n$ correlation coefficients were obtained in the simple toy model with interaction
- $n - n$ correlation coefficient may increase with the inclusion of the interaction as well as decrease
- Only negative $p_t - n$ correlations take place in this model
- Application of Monte-Carlo simulations allows us to express correlation coefficients as a function of average number of emitters

Plans

- calculation of $n - n$ and $p_t - n$ correlation coefficients via Monte-Carlo simulation
- comparison with the experimental results ("independent seeds" [1], negative $pt - n$ correlations [2], etc.)
- addition of other types of emitters

1 Eva Sicking on behalf of the ALICE Collaboration. CERN LHC Seminar "Multiple Parton Interactions in ALICE" 05-03-2013

2 NA49 Collaboration, G.A. Feofilov, R.S. Kolevatov, V.P. Kondratiev, P.A. Naumenko, V.V. Vechernin Proc. of Baldin ISHEPP XVII, Volume I, 222 (2005)

Thank you!

Backup

Introduction

Comment

Multiplicity distribution depends on probability distribution of emitter configuration and multiplicity distribution from one emitter.

Two-stage scenario

- A. Capella, U.P. Sukhatme, C.–I. Tan, J. Tran Thanh Van
  Phys. Lett. **B81**, 68 (1979); Phys. Rep. **236**, 225 (1994).
- A.B. Kaidalov, Phys. Lett. **B116**, 459 (1982).
- A.B. Kaidalov, K.A. Ter-Martirosyan, Phys. Lett. **B117**, 247 (1982).
- V.A. Abramovsky, O.V. Cancelli, JETP Letters, 31, 566 (1980).

Introduction

Shortcoming of the $n-n$ correlations

The $n-n$ correlations, arised from the interaction between emitters, are suppressed by the correlations, arised from the fluctuations in number of primary emitters[1]. Therefore in [1, 2, 3] it is supposed to use the event mean transverse momentum in the backward and forward window as a dynamical variable. One will consider only $n-n$ and p_t-n correlations.

Referencies

- [1] M.A. Braun, C. Pajares, Eur. Phys. J. **C16**, 349 (2000)
- [2] M.A. Braun, C. Pajares, Phys. Rev. Lett. **85**, 4864 (2000)
- [3] E.V. Shuryak, Nucl. Phys. **A661**, 119 (1999)

$n - n$ correlations

Model

The amount of emitters in collision should be even [1, 2]. So if $\xi = 2$, then $\left[\frac{N}{\xi}\right] = \frac{N}{\xi}$.

Primary and secondary parameters are connected [3, 4, 5, 6]:

$$\overline{\mu}_{F2} = \sqrt{2}\overline{\mu}_{F1}, \quad \overline{\mu}_{B2} = \sqrt{2}\overline{\mu}_{B1}$$

- [1] A. Capella, U.P. Sukhatme, C.–I. Tan, J. Tran Thanh Van Phys. Lett. **B81**, 68 (1979); Phys. Rep. **236**, 225 (1994).
- [2] J. Dias de Deus, R. Ugoccioni, A. Rodrigues, Eur. Phys. J. **C16**, 537 (2000).
- [3] T.S. Biro, H.B. Nielsen, J. Knoll, Nucl. Phys. **B245**, 449 (1984).
- [4] A. Bialas, W. Czyz, Nucl. Phys. **B267**, 242 (1986).
- [5] M.A. Braun, C. Pajares, Eur. Phys. J. **C16**, 349 (2000).
- [6] M.A. Braun, C. Pajares, Phys. Rev. Lett. **85**, 4864 (2000).

$n - n$ correlations

Correlation coefficient

$$
{b^{rel}}_{n-n} = \frac{r^2\left(\sqrt{2}-1\right)^2\left(\frac{V_N}{2}-1\right) + r\left(\sqrt{2}-1\right)\left(\sqrt{2}-1-\sqrt{2}\,V_N\right) + V_N}{r^2\left(\sqrt{2}-1\right)^2\left(\frac{V_N}{2}-1\right) + r\left(\sqrt{2}-1\right)\left(\sqrt{2}-1-\sqrt{2}\,V_N - \frac{W_{\mu_{F1}}}{\sqrt{2}}\right) + V_N + W_{\mu_{F1}}}
$$

$$
V_N = \frac{D_N}{\overline{N}}, \qquad W_{\mu_{F1}} = \frac{D_{\mu_{F1}}}{{\overline{\mu}_{F1}}^2}
$$

$p_t - n$ correlations

$$b^{abs}{}_{p_t - n} \equiv \frac{\langle p_{tB}\, n_F \rangle - \langle p_{tB} \rangle \langle n_F \rangle}{\langle {n_F}^2 \rangle - \langle n_F \rangle^2};\; b^{rel}{}_{p_t - n} \equiv b^{abs}{}_{p_t - n} \cdot \frac{\langle n_F \rangle}{\langle p_{tB} \rangle}$$

-

$$b^{rel}{}_{p_t - n} = \frac{\left(1 - r + \frac{r}{\sqrt{2}}\right)^2}{1 - r + \frac{r}{2^{\frac{1}{4}}}} \times$$

$$\times \frac{\overline{N} \cdot \left(1 - r + \frac{r}{2^{\frac{1}{4}}} - \left(1 - r + \frac{r}{\sqrt{2}}\right) \sum_{N,N_2} P\left(N\right) C_{\frac{N}{2}}^{N_2} \cdot r^{N_2} \cdot \left(1 - r\right)^{\frac{N}{2} - N_2} \cdot \frac{N - 2\,N_2 + 2^{\frac{3}{4}}\,N_2}{N - 2\,N_2 + \sqrt{2}\,N_2}\right)}{r^2\left(\sqrt{2} - 1\right)^2 \left(\frac{V_N}{2} - 1\right) + r\left(\sqrt{2} - 1\right)\left(\sqrt{2} - 1 - \sqrt{2}\,V_N - \frac{W_{\mu_{F1}}}{\sqrt{2}}\right) + V_N + W_{\mu_{F1}}}$$

Formalism

Notations and definitions

Let us consider the set of N_1 emitters of the first type, N_2 emitters of the second type. Let the probability of a configuration be $\omega(N_1, N_2)$.

$$\sum_C \omega(C)\ \ldots \equiv \sum_{N_1,N_2} \omega(N_1, N_2)\ \ldots\ ; \qquad \sum_C \omega(C) = 1;$$

$$\sum_{N_1,N_2} N_1 \omega(N_1, N_2) = \overline{N_1}; \qquad \sum_{N_1,N_2} N_2 \omega(N_1, N_2) = \overline{N_2};$$

$$n_F = F_1 + F_2 = \sum_{i1=1}^{N_1} F_{i1} + \sum_{i2=1}^{N_2} F_{i2}\ ; \qquad n_B = B_1 + B_2 = \sum_{i1=1}^{N_1} B_{i1} + \sum_{i2=1}^{N_2} B_{i2}\ ;$$

$$\{B_{i1}, B_{i2}\} \equiv B_{11}, \ldots, B_{N_1 1}; B_{12}, \ldots, B_{N_2 2}\ , \quad i1 = 1, \ldots, N_1\ , \quad i2 = 1, \ldots, N_2;$$

$$\{F_{i1}, F_{i2}\} \equiv F_{11}, \ldots, F_{N_1 1}; F_{12}, \ldots, F_{N_2 2}\ , \quad i1 = 1, \ldots, N_1\ , \quad i2 = 1, \ldots, N_2$$

Formalism

Basic formulae

Let $k_{i1}^{(j1)} = |\mathbf{k}_{i1}^{(j1)}|$ $(j1 = 1, ..., B_{i1})$ be transverse momenta of particles, produced by the $i1$-th emitter in the backward window, $k_{i2}^{(j2)} = |\mathbf{k}_{i2}^{(j2)}|$ $(j2 = 1, ..., B_{i2})$ be transverse momenta of particles, produced by the $i2$-th emitter in the backward window. Similarly, $q_{i1}^{(j1)} = |\mathbf{q}_{i1}^{(j1)}|$ $(j1 = 1, ..., F_{i1})$ - transverse momenta of particles, produced by the $i1$-th emitter in the forward window, $q_{i2}^{(j2)} = |\mathbf{q}_{i2}^{(j2)}|$ $(j2 = 1, ..., F_{i2})$ - by the $i2$-the in the forward window .

$$P^{C}_{\{B_{i1},B_{i2}\}}(p_{tB}) \equiv \int \delta\left(p_{tB} - \left[\sum_{i1=1}^{N_1}\sum_{j1=1}^{B_{i1}} k_{i1}^{(j1)} + \sum_{i2=1}^{N_2}\sum_{j2=1}^{B_{i2}} k_{i2}^{(j2)}\right]\left[\sum_{i1=1}^{N_1} B_{i1} + \sum_{i2=1}^{N_2} B_{i2}\right]^{-1}\right) \times$$

$$\times \prod_{i1=1}^{N_1}\prod_{j1=1}^{B_{i1}} \rho_1\left(k_{i1}^{(j1)}\right)\, dk_{i1}^{(j1)} \prod_{i2=1}^{N_2}\prod_{j2=1}^{B_{i2}} \rho_2\left(k_{i2}^{(j2)}\right)\, dk_{i2}^{(j2)}$$

Formalism

Basic formulae

$$\int \rho_1(k)\; dk = \int \rho_2(k)\; dk = 1, \quad \int \rho_1(k)\, k\, dk = \overline{k_1},$$

$$\int \rho_2(k)\, k\, dk = \overline{k_2}, \quad \int P^{C}_{\{B_{i1},B_{i2}\}}(p_{tB})\; dp_{tB} = 1\ ,$$

$$P(n_B, n_F) = \sum_C \omega(C)\, P_C(n_B, n_F)\ , \quad P(p_{tB}, n_B, n_F) = \sum_C \omega(C)\, P_C(p_{tB}, n_B, n_F)\,,$$

$$P(n_B, n_F) = \int P(p_{tB}, n_B, n_F)\; dp_{tB}\ , \quad P_C(n_B, n_F) = \int P_C(p_{tB}, n_B, n_F)\; dp_{tB}\ ,$$

$$\sum_{n_B, n_F} \int P(p_{tB}, n_B, n_F)\; dp_{tB} = 1\ , \quad \sum_{n_B, n_F} \int P_C(p_{tB}, n_B, n_F)\; dp_{tB} = 1$$

Formalism

Basic formulae

For independent emitters:

$$P_C(n_B, n_F) = \sum_{\{B_{i1}, B_{i2}\}} \delta_{n_B \; \sum_{i1=1}^{N_1} B_{i1} + \sum_{i2=1}^{N_2} B_{i2}} \sum_{\{F_{i1}, F_{i2}\}} \delta_{n_F \; \sum_{i1=1}^{N_1} F_{i1} + \sum_{i2=1}^{N_2} F_{i2}} \prod_{i1=1}^{N_1} p_1(B_{i1}, F_{i1}) \times$$

$$\times \prod_{i2=1}^{N_2} p_2(B_{i2}, F_{i2}) \; ,$$

In case of the long-range correlations:

$$p_1(B_{i1}, F_{i1}) = p_{B_1}(B_{i1}) \; p_{F_1}(F_{i1}) \; , \qquad p_2(B_{i2}, F_{i2}) = p_{B_2}(B_{i2}) \; p_{F_2}(F_{i2}) \; .$$

Correlations arise only due to fluctuations of the number and types of emitters from event to event

Formalism

Basic formulae

Factorization:

$$P_C(n_B, n_F) = P_C(n_B)\, P_C(n_F)\ ,$$

$$P_C(n_B) = \sum_{\{B_{i1}, B_{i2}\}} \delta_{n_B \ \sum_{i1=1}^{N_1} B_{i1} + \sum_{i2=1}^{N_2} B_{i2}} \prod_{i1=1}^{N_1} p_{B_1}(B_{i1}) \prod_{i2=1}^{N_2} p_{B_2}(B_{i2})\ ,$$

$$P_C(n_F) = \sum_{\{F_{i1}, F_{i2}\}} \delta_{n_F \ \sum_{i1=1}^{N_1} F_{i1} + \sum_{i2=1}^{N_2} F_{i2}} \prod_{i1=1}^{N_1} p_{F_1}(F_{i1}) \prod_{i2=1}^{N_2} p_{F_2}(F_{i2})\ ,$$

$$\sum_n p_{F_{1,2}}(n) = \sum_n p_{B_{1,2}}(n) = 1,$$

$$\sum_n n\, p_{F_{1,2}}(n) = \overline{\mu_{F_{1,2}}}\ , \quad \sum_n n\, p_{B_{1,2}}(n) = \overline{\mu_{B_{1,2}}}.$$

Formalism

Basic formulae

Similarly

$$P_C\left(p_{tB}, n_B, n_F\right) = P_C\left(p_{tB}, n_B\right) P_C\left(n_F\right) \ ,$$

$$P_C\left(p_{tB}, n_B\right) = \sum_{\{B_{i1}, B_{i2}\}} \delta_{n_B \ \sum_{i1=1}^{N_1} B_{i1} + \sum_{i2=1}^{N_2} B_{i2}} P^C_{\{B_{i1}, B_{i2}\}}\left(p_{tB}\right) \prod_{i1=1}^{N_1} p_{B_1}\left(B_{i1}\right) \prod_{i2=1}^{N_2} p_{B_2}\left(B_{i2}\right) \ ,$$

$$P_C\left(p_{tB}\right) = \sum_{n_B} P_C\left(p_{tB}, n_B\right) = \sum_{\{B_{i1}, B_{i2}\}} P^C_{\{B_{i1}, B_{i2}\}}\left(p_{tB}\right) \prod_{i1=1}^{N_1} p_{B_1}\left(B_{i1}\right) \prod_{i2=1}^{N_2} p_{B_2}\left(B_{i2}\right) \ ,$$

$$\int P_C\left(p_{tB}\right) dp_{tB} = 1.$$

Formalism

Basic formulae

Notations:

$$P_{N_1}(B_1) = \sum_{\{B_{i1}\}} \delta_{B_1 \; \sum_{i1=1}^{N_1} B_{i1}} \prod_{i1=1}^{N_1} p_{B_1}(B_{i1});$$

$$P_{N_2}(B_2) = \sum_{\{B_{i2}\}} \delta_{B_2 \; \sum_{i2=1}^{N_2} B_{i2}} \prod_{i2=1}^{N_2} p_{B_2}(B_{i2});$$

$$P_{N_1}(F_1) = \sum_{\{F_{i1}\}} \delta_{F_1 \; \sum_{i1=1}^{N_1} F_{i1}} \prod_{i1=1}^{N_1} p_{F_1}(F_{i1});$$

$$P_{N_2}(F_2) = \sum_{\{F_{i2}\}} \delta_{F_2 \; \sum_{i2=1}^{N_2} F_{i2}} \prod_{i2=1}^{N_2} p_{F_2}(F_{i2}).$$

$n - n$ correlations

Summation

$$
\langle {n_F}^2 \rangle \equiv \sum_{C} \omega\left(C\right) \langle {n_F}^2 \rangle_C = \sum_{n_F} {n_F}^2 \sum_{N_1,N_2} \omega\left(N_1, N_2\right) P_{N_1}\left(F1\right) P_{N_2}\left(F2\right) = \sum_{N_1,N_2} \omega\left(N_1, N_2\right)
$$

$$
\sum_{\{F_{i1},F_{i2}\}} \left(\sum_{i1=1}^{N_1} \left(F_{i1}\right)^2 + \sum_{i2=1}^{N_2} \left(F_{i2}\right)^2 + \sum_{i1\neq j1, i1, j1=1}^{N_1} F_{i1}F_{j1} + \sum_{i2\neq j2, i2, j2=1}^{N_2} F_{i2}F_{j2} + 2\sum_{i1=1}^{N_1}\sum_{i2=1}^{N_2} F_{i1}F_{i2} \right) \times
$$

$$
\times \prod_{i1=1}^{N_1} p_{F1}\left(F_{i1}\right) \prod_{i2=1}^{N_2} p_{F2}\left(F_{i2}\right) = \sum_{N_1,N_2} \omega\left(N_1, N_2\right) \left(N_1 \overline{\mu_{F1}^2} + N_2 \overline{\mu_{F2}^2} + \left({N_1}^2 - N_1\right) \overline{\mu}_{F1}^2 + \right.
$$

$$
\left. + \left({N_2}^2 - N_2\right) \overline{\mu}_{F2}^2 + 2N_1 N_2 \overline{\mu}_{F1}\,\overline{\mu}_{F2} \right) = \overline{N_1}\,\overline{\mu_{F1}^2} + \overline{N_2}\,\overline{\mu_{F2}^2} + \left(\overline{{N_1}^2} - \overline{N_1}\right) \overline{\mu}_{F1}^2 + \left(\overline{{N_2}^2} - \overline{N_2}\right) \overline{\mu}_{F2}^2
$$

$$
+2\overline{N_1 N_2}\,\overline{\mu}_{F1}\,\overline{\mu}_{F2} = \overline{{N_1}^2}\,\overline{\mu}_{F1}^2 + \overline{{N_2}^2}\,\overline{\mu}_{F2}^2 + 2\overline{N_1 N_2}\,\overline{\mu}_{F1}\,\overline{\mu}_{F2} + \overline{N_1} D_{\mu_{F1}} + \overline{N_2} D_{\mu_{F2}}
$$

$n - n$ correlations

Generating functions

For any distribution $p(k)$ $\left(p(k) \geq 0, \sum_{k=0}^{\infty} p(k) = 1\right)$ GF is introduced:

$$h(z) \equiv \sum_{k=0}^{\infty} p(k)\, z^k$$

Let GF of distributions from one emitter:

$$\varphi_1(z_F, z_B), \quad \varphi_2(z_F, z_B)$$

Then resulting GF:

$$H(z_F, z_B) = \sum_{N_1, N_2} \omega(N_1, N_2) \left(\varphi_1(z_F, z_B)\right)^{N_1} \left(\varphi_2(z_F, z_B)\right)^{N_2}$$

$n - n$ correlations

Generating functions

Differentiating GF, one can find moments of distribution:

$$\frac{\partial\varphi_1}{\partial z_F}|_{z_F=z_B=1} = \overline{\mu}_{F1}, \quad \frac{\partial\varphi_2}{\partial z_F}|_{z_F=z_B=1} = \overline{\mu}_{F2}, \quad \frac{\partial\varphi_1}{\partial z_B}|_{z_F=z_B=1} = \overline{\mu}_{B1},$$

$$\frac{\partial\varphi_2}{\partial z_B}|_{z_F=z_B=1} = \overline{\mu}_{B2}, \quad \frac{\partial^2\varphi_1}{\partial z_F^2}|_{z_F=z_B=1} = \overline{\mu_{F1}^2} - \overline{\mu}_{F1}, \quad \frac{\partial^2\varphi_2}{\partial z_F^2}|_{z_F=z_B=1} = \overline{\mu_{F2}^2} - \overline{\mu}_{F2}$$

$$\frac{\partial^2\varphi_1}{\partial z_B^2}|_{z_F=z_B=1} = \overline{\mu_{B1}^2} - \overline{\mu}_{B1}, \quad \frac{\partial^2\varphi_2}{\partial z_B^2}|_{z_F=z_B=1} = \overline{\mu_{B2}^2} - \overline{\mu}_{B2}$$

Correlation coefficient. Coincidence of two methods

$$b^{abs}{}_{n-n} = \frac{\langle n_B\, n_F\rangle - \langle n_B\rangle\langle n_F\rangle}{\langle {n_F}^2\rangle - \langle n_F\rangle^2} = \frac{\left(\frac{\partial^2 H}{\partial z_B \partial z_F} - \frac{\partial H}{\partial z_B}\frac{\partial H}{\partial z_F}\right)|_{z_B=z_F=1}}{\left(\frac{\partial^2 H}{\partial {z_F}^2} + \frac{\partial H}{\partial z_F} - \left(\frac{\partial H}{\partial z_F}\right)^2\right)|_{z_B=z_F=1}}.$$

$p_t - n$ correlations

$$\langle p_{tB}\rangle = \sum_{C}\omega\left(C\right)\langle p_{tB}\rangle_{C} = \sum_{B_1,B_2}\frac{\overline{k_1}B_1+\overline{k_2}B_2}{B_1+B_2}\sum_{N_1,N_2}\omega\left(N_1,N_2\right)P_{N_1}\left(B_1\right)P_{N_2}\left(B_2\right)$$

$$\langle p_{tB}\, n_F\rangle = \sum_{C}\omega\left(C\right)\langle p_{tB}\rangle_{C}\langle n_F\rangle_{C} \;=\sum_{B_1,B_2}\frac{\overline{k_1}B_1+\overline{k_2}B_2}{B_1+B_2}\sum_{N_1,N_2}\omega\left(N_1,N_2\right)\times$$

$$\times\left(N_1\overline{\mu}_{F1}+N_2\overline{\mu}_{F2}\right)P_{N_1}\left(B_1\right)P_{N_2}\left(B_2\right);$$

$$cov\left(p_{tB},n_F\right)=\sum_{N_1,N_2}\omega\left(N_1,N_2\right)\left(N_1\overline{\mu}_{F1}+N_2\overline{\mu}_{F2}\right)\sum_{B_1,B_2}\frac{\overline{k_1}B_1+\overline{k_2}B_2}{B_1+B_2}\times$$

$$\times\left[P_{N_1}\left(B_1\right)P_{N_2}\left(B_2\right)-\sum_{N_1',N_2'}\omega\left(N_1',N_2'\right)P_{N_1'}\left(B_1\right)P_{N_2'}\left(B_2\right)\right]=\sum_{N_1,N_2}\omega\left(N_1,N_2\right)\times$$

$$\times\left(N_1\overline{\mu}_{F1}+N_2\overline{\mu}_{F2}\right)\sum_{B_1,B_2}\frac{\overline{k_1}B_1+\overline{k_2}B_2}{B_1+B_2}\left[P_{N_1}\left(B_1\right)P_{N_2}\left(B_2\right)-P\left(B_1,B_2\right)\right].$$

$p_t - n$ correlations

Possible approximation

$$\sum_{B_1,B_2} \frac{\overline{k_1}B_1 + \overline{k_2}B_2}{B_1 + B_2} P_{N_1}(B_1) P_{N_2}(B_2) = \frac{\sum_{B_1,B_2} \left(\overline{k_1}B_1 + \overline{k_2}B_2\right) P_{N_1}(B_1) P_{N_2}(B_2)}{\sum_{B_1,B_2} (B_1 + B_2) P_{N_1}(B_1) P_{N_2}(B_2)} =$$

$$= \frac{\overline{k_1}\langle B_1\rangle_{N_1} + \overline{k_2}\langle B_2\rangle_{N_2}}{\langle B_1\rangle_{N_1} + \langle B_2\rangle_{N_2}};$$

$$\sum_{B_1,B_2} \frac{\overline{k_1}B_1 + \overline{k_2}B_2}{B_1 + B_2} P(B_1, B_2) = \frac{\sum_{B_1,B_2} \left(\overline{k_1}B_1 + \overline{k_2}B_2\right) P(B_1, B_2)}{\sum_{B_1,B_2} (B_1 + B_2) P(B_1, B_2)} =$$

$$= \frac{\overline{k_1}\langle B_1\rangle + \overline{k_2}\langle B_2\rangle}{\langle B_1\rangle + \langle B_2\rangle}.$$

If $\overline{k_1} = \overline{k_2}$, there is a reduction, that is there are no correlations

$p_t - n$ correlations

$$\langle B_1 \rangle_{N_1} + \langle B_2 \rangle_{N_2} = \langle n_B \rangle_C = N_1 \overline{\mu}_{B1} + N_2 \overline{\mu}_{B2}; \langle B_1 \rangle + \langle B_2 \rangle = \langle n_B \rangle = \overline{N_1}\,\overline{\mu}_{B1} + \overline{N_2}\,\overline{\mu}_{B2},$$

$$\frac{\overline{\mu}_{B2}}{\overline{\mu}_{B1}} = \frac{\overline{\mu}_{F2}}{\overline{\mu}_{F1}},$$

$$\frac{N_1 \overline{\mu}_{F1} + N_2 \overline{\mu}_{F2}}{\langle B_1 \rangle_{N_1} + \langle B_2 \rangle_{N_2}} = \frac{N_1 \overline{\mu}_{F1} + N_2 \overline{\mu}_{F2}}{N_1 \overline{\mu}_{B1} + N_2 \overline{\mu}_{B2}} = \frac{\overline{\mu}_{F1}}{\overline{\mu}_{B1}}.$$

It can be seen that under this assumption correlations vanish even with $\overline{k_1} \neq \overline{k_2}$, that is this approximation is too rough

$p_t - n$ correlations

In order to avoid numerical summation, it is proposed to make such an approximation:

$$\langle p_{tB} \rangle = \overline{k_1} \frac{\langle B_1 \rangle}{\langle B_1 + B_2 \rangle} + \overline{k_2} \frac{\langle B_2 \rangle}{\langle B_1 + B_2 \rangle}$$

$$\langle p_{tB} n_F \rangle = \overline{k_1} \frac{\langle B_1 n_F \rangle}{\langle B_1 + B_2 \rangle} + \overline{k_2} \frac{\langle B_2 n_F \rangle}{\langle B_1 + B_2 \rangle}$$

$p_t - n$ correlations

Correlation coefficient

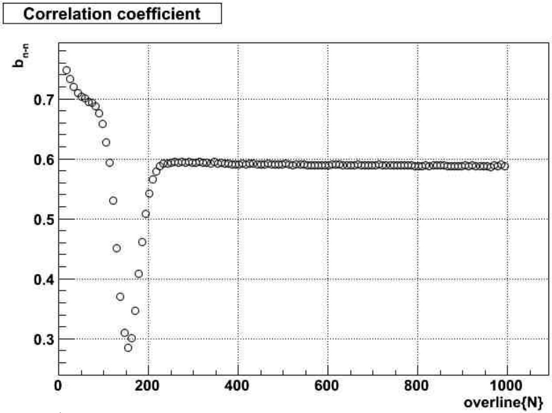
- $$b^{rel}{}_{p_t-n} = b^{abs}{}_{p_t-n} \frac{\overline{\mu}_{F1}\,\overline{N}\left(1-r+\frac{r}{\sqrt{2}}\right)^2}{\overline{k_1}\left(1-r+\frac{r}{2^{\frac{1}{4}}}\right)} = \frac{1-r+\frac{r}{\sqrt{2}}}{1-r+\frac{r}{2^{\frac{1}{4}}}} \times$$
$$\times \frac{r^2\left(V_N-2\right)\frac{2^2-2^{\frac{3}{2}}-2^{\frac{7}{4}}+2^{\frac{5}{4}}}{4} + r\left(V_N\left(-2+2^{-\frac{1}{2}}+2^{-\frac{1}{4}}\right)+2-\sqrt{2}-2^{\frac{3}{4}}+2^{\frac{1}{4}}\right)+V_N}{r^2\left(V_N-2\right)\left(\frac{3}{2}-\sqrt{2}\right)+r\left(\sqrt{2}-1\right)\left(\sqrt{2}-1-\sqrt{2}V_N-\frac{W_{\mu_{F1}}}{\sqrt{2}}\right)+V_N+W_{\mu_{F1}}}.$$

$p_t - n$ correlations

Correlation coefficient

- $$b^{rel}{}_{p_t-n} = b^{abs}{}_{p_t-n} \frac{\overline{\mu}_{F1}\,\overline{N}\left(1-r+\frac{r}{\sqrt{2}}\right)^2}{\overline{k_1}\left(1-r+\frac{r}{2^{\frac{1}{4}}}\right)} = \frac{1-r+\frac{r}{\sqrt{2}}}{1-r+\frac{r}{2^{\frac{1}{4}}}} \times$$
$$\times \frac{r^2\left(V_N-2\right)\frac{2^2-2^{\frac{3}{2}}-2^{\frac{7}{4}}+2^{\frac{5}{4}}}{4}+r\left(V_N\left(-2+2^{-\frac{1}{2}}+2^{-\frac{1}{4}}\right)+2-\sqrt{2}-2^{\frac{3}{4}}+2^{\frac{1}{4}}\right)+V_N}{r^2\left(V_N-2\right)\left(\frac{3}{2}-\sqrt{2}\right)+r\left(\sqrt{2}-1\right)\left(\sqrt{2}-1-\sqrt{2}V_N\right)+V_N+W_{\mu_{F1}}}.$$

$n - n$ correlations



$$r = \frac{1}{1 + e^{-0.25\cdot(N-150)}},\ P(N)\ is\ NB\ with\ \overline{N}\ and\ D_N = 2 \cdot \overline{N} + 10$$

Comparison with Monte-Carlo simulations