Nucleon-to-resonance transition form factors in a vector-meson-dominance model

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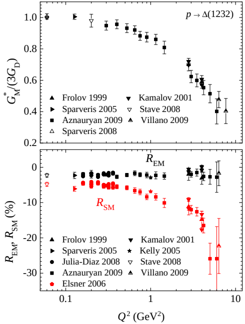
Experimental data on NR-transition FFs

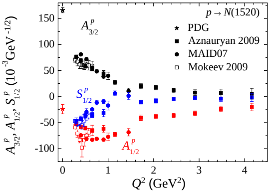
The experiments have been carried out in JLab, MAMI, ELSA...
Experimentalists study the electroproduction of the resonances with spins 1/2, 3/2 and 5/2: $\Delta(1232)$, $N(1440)$, $N(1520)$, $N(1535)$, $N(1680)$...
The available data:

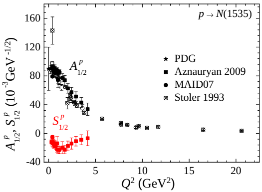
- inclusive: for $\Delta(1232)$ up to $Q^2 = 9$ GeV^2, $N(1535)$ up to $Q^2 = 20$ GeV^2;
- exclusive: for $\Delta(1232)$ up to $Q^2 = 8$ GeV^2, for $N(1440)$, $N(1520)$, $N(1535)$ up to $Q^2 = 4$ GeV^2, for $N(1680)$ up to $Q^2 = 1.5$ GeV^2.

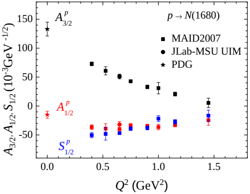
Expected data:

- new measurements up to $Q^2 = 12$—16 GeV^2 (JLab);
- time-like FFs (SLAC).









Two subproblems

1. It is necessary to find a mathematically consistent Lagrangian of nucleon interactions with higher-spin baryon resonances, photons, pions, and vector mesons. The Lagrangian and FFs should be defined unambiguously by symmetry conditions. This should determine polynomial Q^2-dependencies in the observables.

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2. The Q^2-dependencies of the FFs should be reproduced in some particular dynamical model.

Covariant constraints for the RS fields

Nucleon resonances with spin $J = \ell + \frac{1}{2} \geqslant \frac{3}{2}$ are described as the symmetric tensor-spinor Rarita—Schwinger (RS) fields $\Psi_{\mu_1 \ldots \mu_\ell}$. The spin content of the RS field $\Psi_{\mu_1 \ldots \mu_\ell}$:

$$J, \left(J - \frac{1}{2}\right)^{\pm}, \ldots \left(\frac{1}{2}\right)^{\pm}.$$

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$$J, \left(J - \frac{1}{2}\right)^{\pm}, \dots \left(\frac{1}{2}\right)^{\pm}.$$

To eliminate redundant degrees of freedom, subsidiary conditions are imposed on the field:

$$\partial^\lambda \Psi_{\lambda \mu_2 \dots \mu_\ell} = 0 \qquad \text{transversality}$$

$$\gamma^\lambda \Psi_{\lambda \mu_2 \dots \mu_\ell} = 0 \qquad \text{tracelessness}$$

Free spin-3/2 field

The free-field Lagrangian for the vector-spinor field Ψ^μ:

$$\mathscr{L}_{\mathrm{ff}}(A) = \bar{\Psi}^\mu \left[i\Gamma_{\mu\nu\lambda}(A)\partial^\lambda - M\Gamma_{\mu\nu}(A) \right] \Psi^\nu,$$

$$\Gamma_{\mu\nu\lambda}(A) = g_{\mu\nu}\gamma_\lambda - A^*\gamma_\mu g_{\nu\lambda} - A\gamma_\nu g_{\mu\lambda} + \left(\frac{3}{2}|A|^2 - \operatorname{Re} A + \frac{1}{2} \right) \gamma_\lambda\gamma_\nu,$$

$$\Gamma_{\mu\nu}(A) = g_{\mu\nu} - \left(3|A|^2 - 3\operatorname{Re} A + 1 \right) \gamma_\mu\gamma_\nu,$$

where $A \neq \frac{1}{2}$ is a complex parameter.

Free field equation and constraints:

$$(i\gamma^\lambda\partial_\lambda - M)\Psi_\mu = 0,$$

$$\partial^\lambda\Psi_\lambda = 0 = \gamma^\lambda\Psi_\lambda.$$

Symmetries of the free spin-3/2 field

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Point invariance of the equivalent class of the Lagrangians:

$$\Psi'_\mu = \Theta^{(A,A')}_{\mu\nu}\Psi^\nu, \qquad \Theta^{(A,A')}_{\mu\nu} = g_{\mu\nu} + \frac{A'-A}{2\left(2A-1\right)}\gamma_\mu\gamma_\nu,$$

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Gauge invariance of the RS field in the massless limit:

$$\Psi'_\mu = \Psi_\mu + \partial_\mu\theta(x) \quad \text{for} \quad \gamma_\mu\partial^\mu\theta(x) = 0,$$
$$\mathscr{L}'_{\mathrm{ff}}(A) = \mathscr{L}_{\mathrm{ff}}(A).$$

Constraints for the interacting RS fields

Minimal EM coupling

$$\mathscr{L}_{int} = -eA_\lambda \bar{\Psi}^\mu \gamma^\lambda \Psi_\mu$$

breaks the free-field symmetries, modifies the constraints reducing their number, which leads to the excitation of unphysical DsOF and different pathologies.

G. Velo, D. Zwanziger, Phys. Rev. 186 (1969) 1337;
K. Johnson, E.C.G. Sudarshan, Ann. Phys. (N.Y.) 13 (1961) 126;
V. Pascalutsa, Phys. Rev. D 58 (1998) 096002.

For the trilinear nucleon-resonance interactions with pions, photons, and vector mesons we can solve all consistency problems by requiring invariance of the Lagrangian under the point and gauge transformations $\Psi'_\mu = \Psi_\mu + \partial_\mu \theta_1(x) + \gamma_\mu \theta_2$, *i.e.* the vector-spinor source J_μ should be traceless and transversal, $\gamma_\lambda J^\lambda = 0 = \partial_\lambda J^\lambda$.

Nucleon-resonance interaction Lagrangian

NRV-Lagrangian (spin-3/2) consists of invariants such as

$$\bar{\Psi}^{\mu\nu,\bar{\alpha}}\Gamma_{[\mu\nu][\lambda\sigma]\bar{\alpha}\bar{\beta}}N^{,\bar{\beta}}V^{\lambda\sigma}, \text{ where } \Psi^{\mu\nu}=\partial^{\mu}\Psi^{\nu}-\partial^{\nu}\Psi^{\mu},$$

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$\bar{\alpha}$, $\bar{\beta}$ are multi-indicies. Symmetry properties of the coupling matrices:

$$\Gamma_{[\mu\nu][\lambda\sigma]\bar{\alpha}\bar{\beta}} = -\Gamma_{[\nu\mu][\lambda\sigma]\bar{\alpha}\bar{\beta}} = -\Gamma_{[\mu\nu][\sigma\lambda]\bar{\alpha}\bar{\beta}},$$

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The coupling matrix for the spin-$\frac{3}{2}$:

$$\Gamma_{[\mu\nu][\lambda\sigma]} = -\frac{1}{6}\left(\sigma_{\mu\nu}\sigma_{\lambda\sigma} + 3\sigma_{\lambda\sigma}\sigma_{\mu\nu}\right).$$

NRV-Lagrangian $(J_R = 3/2)$

$$\mathscr{L} = \frac{ig_1}{2M_N^2}\bar{\Psi}^{\mu\nu}\Gamma_{[\mu\nu][\lambda\sigma]}\gamma_R N V^{\lambda\sigma} + \frac{g_2}{2M_N^2 M_R}\bar{\Psi}^{\mu\nu,\rho}\Gamma_{[\mu\nu][\lambda\sigma]}\gamma_\rho\gamma_R N V^{\lambda\sigma} +$$
$$+ \frac{ig_3}{2M_N^2 M_R^2}\bar{\Psi}^{\mu\nu,\rho}\big(\Gamma_{[\mu\nu][\lambda\rho]}g_{\sigma\omega} - \Gamma_{[\mu\nu][\sigma\rho]}g_{\lambda\omega} + \Gamma_{[\mu\nu][\lambda\omega]}g_{\sigma\rho}$$
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$NR\pi$-vertex ($J_R = 3/2$)

$NR\pi$-Lagrangian is also defined uniquely by the symmetry:

$$\mathscr{L} = \frac{f}{m_\pi M_R^2} \bar{\Psi}^{[\mu\nu],\lambda} \Gamma_{[\mu\nu][\lambda\sigma]} \gamma_R \gamma_5 N \pi^{,\sigma};$$

V. Shklyar, H. Lenske, Phys. Rev. C 80 (2009) 058201

Field tensor-spinors for arbitrarily high spin

Gauge-invariant RS field tensor spinors:

$$\ell = 1: \quad \Psi^{[\mu_1\nu_1]} = \partial^{\mu_1}\Psi^{\nu_1} - \partial^{\nu_1}\Psi^{\mu_1};$$

$$\ell = 2: \quad \Psi^{([\mu_1\nu_1][\mu_2\nu_2])} = \frac{1}{2}\big(\partial^{\mu_1}\partial^{\mu_2}\Psi^{\nu_1\nu_2} - \partial^{\mu_1}\partial^{\nu_2}\Psi^{\nu_1\mu_2} - $$
$$- \partial^{\nu_1}\partial^{\mu_2}\Psi^{\mu_1\nu_2} - \partial^{\nu_1}\partial^{\nu_2}\Psi^{\mu_1\mu_2}\big);$$

$$\ell: \quad \Psi^{([\mu_1\nu_1][\mu_2\nu_2]\ldots[\mu_\ell\nu_\ell])} = \frac{1}{2^{\ell-1}}\left(\partial^{\mu_1}\partial^{\mu_2}\ldots\partial^{\mu_\ell}\Psi^{\nu_1\nu_2\ldots\nu_\ell} + \ldots\right).$$

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$$\ell: \quad \Psi^{([\mu_1\nu_1][\mu_2\nu_2]\dots[\mu_\ell\nu_\ell])} = \frac{1}{2^{\ell-1}}\left(\partial^{\mu_1}\partial^{\mu_2}\dots\partial^{\mu_\ell}\Psi^{\nu_1\nu_2\dots\nu_\ell} + \dots\right).$$

Some multi-indices:

$$A^a = [\mu_a\nu_a]$$
$$\bar{A} = ([\mu_1\nu_1][\mu_2\nu_2]\dots[\mu_\ell\nu_\ell]),$$
$$\bar{A}^a = ([\mu_1\nu_1]\dots[\mu_{a-1}\nu_{a-1}][\mu_{a+1}\nu_{a+1}]\dots[\mu_\ell\nu_\ell]).$$

The couplings for arbitrary spin

The expansion of the traceless coupling:

$$\Gamma_{\bar{A}\bar{\alpha}} = \Gamma^{(\ell)}_{\bar{A}\bar{B}} \left[\mathsf{R}^{\bar{B}}{}_{\bar{\alpha}} + \gamma_\rho \mathsf{S}^{\rho\bar{B}}{}_{\bar{\alpha}} + \sigma_{\rho\omega} \mathsf{T}^{[\rho\omega]\bar{B}}{}_{\bar{\alpha}} \right].$$

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Recurrence relation for the coupling matrix (arbitrary ℓ):

$$\Gamma^{(\ell)}_{\bar{A}\bar{B}} = \frac{3}{2(2\ell+1)\ell^2} \sum_{a,b=1}^{\ell} \Bigg[(\ell+1)\Gamma^{(\ell-1)}_{\bar{A}^a\bar{B}^b}\Gamma^{(1)}_{A^aB^b} + \\ + (\ell-1)\Gamma^{(\ell-1)}_{\bar{A}^a\bar{B}^b}\Gamma^{(1)}_{B^bA^a} + \sum_{b\neq c=1}^{\ell} \Gamma^{(\ell-1)}_{\bar{A}^aA^a\bar{B}^{bc}}\Gamma^{(1)}_{B^bB^c} \Bigg].$$

NRV-Lagrangian for an arbitrary ℓ

$$\mathscr{L}_1 = \frac{i^{3\ell+2} g_1}{2M_N^{3\ell-1}} \bar{\Psi}^{\bar{A},\lambda_2\ldots\lambda_\ell} \Gamma^{(\ell)}_{\bar{A}\bar{B}} \gamma_R N^{,\sigma_2\ldots\sigma_\ell} V^{\lambda_1\sigma_1},$$

$$\mathscr{L}_2 = \frac{i^{3\ell+3} g_2}{2M_N^{3\ell-1} M_R} \bar{\Psi}^{\bar{A},\lambda_2\ldots\lambda_\ell\rho} \Gamma^{(\ell)}_{\bar{A}\bar{B}} \gamma_\rho \gamma_R N^{,\sigma_2\ldots\sigma_\ell} V^{\lambda_1\sigma_1},$$

$$\begin{aligned}\mathscr{L}_3 = \frac{i^{3\ell+2} g_3}{2M_N^{3\ell-1} M_R^2} & \bar{\Psi}^{\bar{A},\lambda_2\ldots\lambda_\ell\rho} \big(\Gamma^{(\ell)}_{\bar{A}\bar{B}^1\lambda_1\rho} g_{\sigma_1\omega} - \Gamma^{(\ell)}_{\bar{A}\bar{B}^1\sigma_1\rho} g_{\lambda_1\omega} \\ & + \Gamma^{(\ell)}_{\bar{A}\bar{B}^1[\lambda_1\omega]} g_{\sigma_1\rho} - \Gamma^{(\ell)}_{\bar{A}\bar{B}^1[\sigma_1\omega]} g_{\lambda_1\rho} - \Gamma^{(\ell)}_{\bar{A}\bar{B}^1[\lambda_1\sigma]} g_{\rho\omega_1}\big) \times \\ & \times N^{,\sigma_2\ldots\sigma_\ell\omega} V^{\lambda_1\sigma_1}.\end{aligned}$$

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The consequences of the point and gauge invariance:

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- there is only one universal matrix as a symmetry “carrier”;
- classification of the Lagrangian vertexes in terms of differential order;
- unified Lagrangian structure for an arbitrary high resonance spins.

Helicity amplitudes

$$A_{3/2} = \mp\sqrt{N_\ell}\Big[\left(Q^2 \pm \mu_\pm M_N\right) F_1 + \mu_\pm M_R F_2 - \left(Q^2 + \mu_\pm M_R\right) F_3\Big],$$

$$A_{1/2} = -\sqrt{\frac{\ell N_\ell}{\ell+2}}\Big[\mu_\pm M_R F_1 + \left(Q^2 \pm \mu_\pm M_N\right) F_2 \mp \mu_\pm M_N F_3\Big],$$

$$S_{1/2} = \mp\sqrt{\frac{\ell N_\ell}{2(\ell+2)}} Q_+ Q_- \left[F_1 - F_2 + \frac{Q^2 + M_R^2 + M_N^2}{2M_R^2} F_3\right],$$

$$\text{where } N_\ell(Q^2) = \frac{2^\ell (\ell!)^2 \pi\alpha Q_\pm^{2(\ell-1)} Q_\mp^{2\ell}}{(2\ell)! M_N^{4\ell+1}(M_R^2 - M_N^2)},$$

$$\mu_\pm = M_R \pm M_N, \qquad Q_\pm = \sqrt{Q^2 + \mu_\pm^2}.$$

The upper and bottom signs correspond to
$J^P = (3/2)^\pm,\ (5/2)^\mp,\ \dots$

pQCD constraints

$$A_{3/2}(Q^2) \sim \frac{1}{Q^5 L^{n_1}(Q^2)}, \qquad A_{1/2}(Q^2) \sim \frac{1}{Q^3 L^{n_2}(Q^2)},$$
$$S_{1/2}(Q^2) \sim \frac{1}{Q^3 L^{n_3}(Q^2)},$$

where $L^{n_f}(Q^2) = \ln^{n_f}(Q^2/\Lambda^2)$ and $n_2 - n_3 \approx 2$ (Idilbi 2004, Carlson 1986).
For $J = \ell + 1/2$ we get:

$$F_1(Q^2) \sim \frac{1}{Q^{6+2\ell} L^{n_1}(Q^2)}, \qquad F_2(Q^2) \sim \frac{1}{Q^{4+2\ell} L^{n_2}(Q^2)},$$
$$F_3(Q^2) \sim \frac{1}{Q^{6+2\ell} L^{n_3}(Q^2)}.$$

pQCD constraints

It is interesting that the following asymptotic identities hold:

$$A_{3/2}(Q^2) = \mp\sqrt{N_\ell(Q^2)}Q^2 F_1(Q^2),$$

$$A_{1/2}(Q^2) = -\sqrt{\frac{\ell N_\ell(Q^2)}{\ell+2}}Q^2 F_2(Q^2),$$

$$S_{1/2}(Q^2) = \mp\sqrt{\frac{\ell N_\ell(Q^2)}{2(\ell+2)}}\frac{Q^4}{2M_R^2}F_3(Q^2)$$

for $n_3 > n_1$.

Elastic nucleon FFs

The Lagrangian:

$$\mathscr{L} = e \sum_V g_{1(V)} \bar{N} \gamma_\mu V^\mu N - \frac{i}{2M_N} \sum_V g_{2(V)} \bar{N} \sigma_{\mu\nu} V^{\mu\nu} N.$$

Sachs FFs, the Dirac and Pauli FFs:

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2), \quad G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M_N^2} F_2(Q^2);$$

$$F_f(Q^2) = \sum_V \frac{g_{f(V)} m_V^2}{Q^2 + m_V^2}.$$

High-Q^2 asymptotic relations:

$$G_M(Q^2) = F_1(Q^2), \qquad G_E(Q^2) = \frac{Q^2}{2M_N^2} F_2(Q^2),$$

$$F_1(Q^2) \sim \frac{1}{Q^4 \ln^2 (Q^2/\Lambda^2)}, \qquad F_2(Q^2) \sim \frac{1}{Q^6}.$$

Multi-pole vector-meson–dominance model

Формфакторы:

$$F_f^{(p,n)}(Q^2) = \frac{1}{2}\left[\sum_{k=1}^{K} \frac{\varkappa_{kf}^{(\omega)}(Q^2) m_{(\omega)k}^2}{Q^2 + m_{(\omega)k}^2} \pm \sum_{k=1}^{K} \frac{\varkappa_{kf}^{(\rho)}(Q^2) m_{(\rho)k}^2}{Q^2 + m_{(\rho)k}^2}\right].$$

$\rho\omega$-meson families (PDG):

$$\rho(770)\ \omega(782), \qquad \rho(1450)\ \omega(1420), \qquad \rho(1700)\ \omega(1650),$$
$$\rho(1900)\omega(1960), \qquad \rho(2150)\ \omega(2205).$$

pQCD constraints on the model

Q^2-dependence of meson-baryon couplings is chosen a universal phenomelogical logarithmic function

$$\varkappa_{kf}^{(\omega,\,\rho)}(Q^2) \equiv \frac{\varkappa_{kf}^{(\omega,\,\rho)}(0)}{L_f^{(\omega,\,\rho)}(Q^2)},$$

где

$$L_f^{(\omega\,\rho)} = \left[1 + b_f^{(\omega,\,\rho)} \ln\left(1 + \frac{Q^2}{\Lambda^2}\right) + a_f^{(\omega,\,\rho)} \ln\left(1 + \frac{Q^2}{\Lambda^2}\right)^2\right]^{\frac{n_f}{2}}.$$

pQCD constraints on the model

$$F_f(Q^2) = \frac{1}{L_f(Q^2)} \sum_{k=1}^{K} \frac{\varkappa_{kf}(0) m_k^2}{Q^2 + m_k^2} =$$
$$= \frac{1}{L_f(Q^2)} \left[\sum_{k=1}^{K} \varkappa_{kf}(0) - \frac{1}{Q^2} \sum_{k=1}^{K} m_k^2 \varkappa_{kf}(0) + \ldots \right],$$

The superconvergence relations for the meson parameters

$$\sum_{k=1}^{K} m_{(\omega,\rho)k}^{2n} \varkappa_{kf}^{(\omega,\rho)}(0) = 0,$$

где $n = 2,\ 3, \ldots\ 4 + \ell$ при $f = 1,\, 3$ и $n = 2, \ldots\ 3 + \ell$ при $f = 2$ для электророждения резонансов со спином $J = \ell + \frac{1}{2}$.

$N\Delta(1232)$-transition

$\Delta^*(1232)$: $J^P = \frac{3}{2}^+$

Magnetic FF:

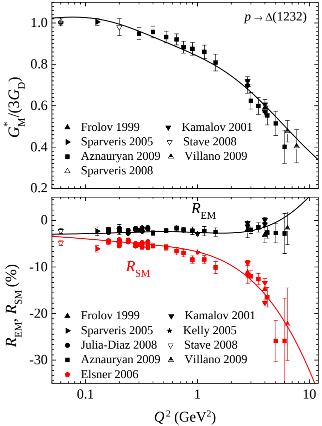
$$G^*_{\mathrm{M}} = -\left[\frac{M_N^3(M_R^2 - M_N^2)}{2\pi\alpha(M_R + M_N)^2 Q_-^2}\right]^{1/2} \times$$

$$\times \left(A_{1/2} + \sqrt{3}A_{3/2}\right),$$

The ratio of the electric and Coulomb amplitudes to the magnetic one:

$$R_{\mathrm{EM}} = \frac{A_{1/2} - A_{3/2}/\sqrt{3}}{A_{1/2} + \sqrt{3}A_{3/2}},$$

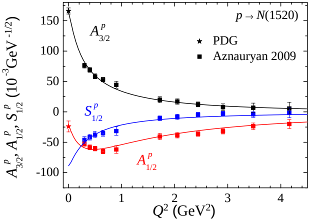
$$R_{\mathrm{SM}} = \frac{\sqrt{2}S_{1/2}}{A_{1/2} + \sqrt{3}A_{3/2}}.$$

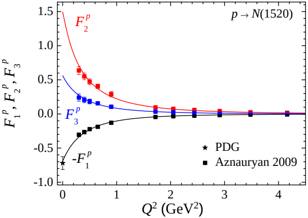


Magnetic FF and the amplitude ratios for the transition $p\gamma^* \to \Delta(1232)$ ($\chi^2/DOF = 1.51$)

The $N^*(1520)$ and $N^*(1680)$

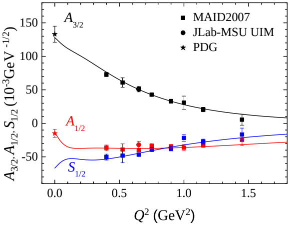
$N^*(1520)$: $J^P = \frac{3}{2}^-$

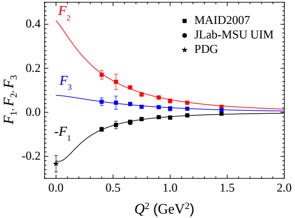




Helicity amplitudes and point and gauge invariant FFs for the transition $p\gamma^* \to N^*_+(1520)$ ($\chi^2/DOF = 1.05$)

$N^*(1680)$: $J^P = \frac{5}{2}^+$





Helicity amplitudes and point and gauge invariant FFs for the transition $p\gamma^* \to N^*_+(1680)$ ($\chi^2/DOF = 0.87$)

The $N^*(1440)$ and $N^*(1535)$

Helicity amplitudes:

$$A_{1/2}(Q^2) = \sqrt{2N}\left[Q^2 F_1(Q^2) + M_N\left(M_R \pm M_N\right) F_2(Q^2)\right],$$

$$S_{1/2}(Q^2) = \pm\sqrt{N}\frac{Q_+ Q_-}{2M_R}\left[(M_R \pm M_N) F_1(Q^2) - M_N F_2(Q^2)\right],$$

$$\text{where } N = \frac{\pi\alpha[Q^2 + (M_R \mp M_N)^2]}{M_N^5(M_R^2 - M_N^2)}.$$

Upper signs are for $J^P = \frac{1}{2}^+$, bottom ones are for $J^P = \frac{1}{2}^-$.
pQCD scaling ($n_1 - n_2 \approx 2$):

$$F_1(Q^2) \sim \frac{1}{Q^6 L^{n_1}(Q^2)}, \qquad F_2(Q^2) \sim \frac{1}{Q^6 L^{n_2}(Q^2)}.$$

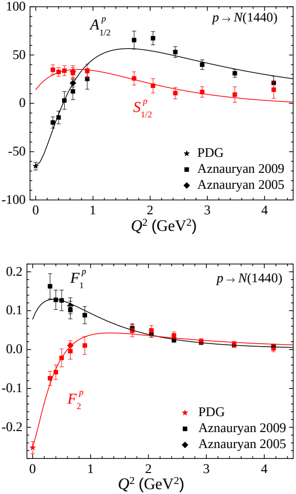
Asymptotic relations:

$$A_{1/2}(Q^2) \sim \sqrt{2N} Q^2 F_1(Q^2), \qquad S_{1/2}(Q^2) \sim \pm\sqrt{N}\frac{Q^2}{2M_R} F_2(Q^2).$$

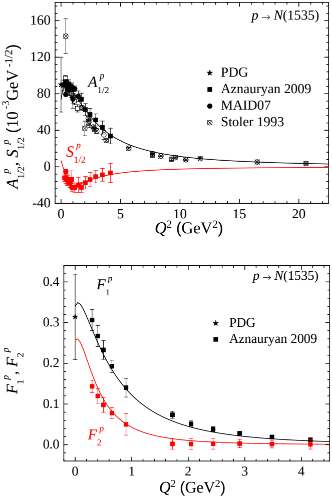
The $N^*(1440)$ and $N^*(1535)$

$N^*(1440)$: $J^P = \frac{1}{2}^+$

$N^*(1535)$: $J^P = \frac{1}{2}^-$



Helicity amplitudes and FFs for the transition $p\gamma^* \to N^*_+(1440)$
$(\chi^2/DOF = 0.97)$



Helicity amplitudes and FFs for the transition $p\gamma^* \to N^*_+(1535)$
$(\chi^2/DOF = 0.65)$

$N\Delta(1232)$-transition

$\Delta^*(1232)$: $J^P = \frac{3}{2}^+$

Magnetic FF:

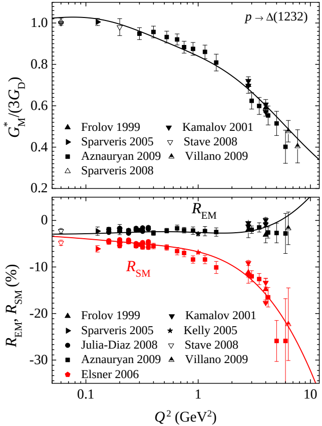
$$G^*_{\rm M} = -\left[\frac{M_N^3(M_R^2 - M_N^2)}{2\pi\alpha(M_R + M_N)^2 Q_-^2}\right]^{1/2} \times$$

$$\times \left(A_{1/2} + \sqrt{3}A_{3/2}\right),$$

The ratio of the electric and Coulomb amplitudes to the magnetic one:

$$R_{\rm EM} = \frac{A_{1/2} - A_{3/2}/\sqrt{3}}{A_{1/2} + \sqrt{3}A_{3/2}},$$

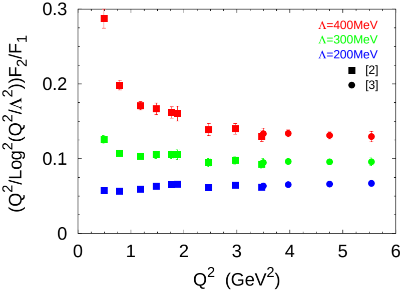
$$R_{\rm SM} = \frac{\sqrt{2}S_{1/2}}{A_{1/2} + \sqrt{3}A_{3/2}}.$$



Magnetic FF and the amplitude ratios for the transition $p\gamma^* \to \Delta(1232)$ ($\chi^2/DOF = 1.51$)

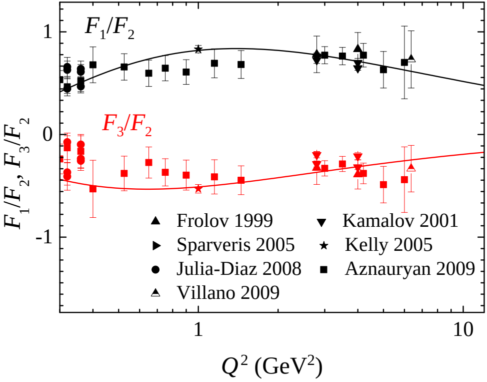
“Scaling” of the ratios of the elastic form factors

$$\frac{F_2(Q^2)}{F_1(Q^2)} \propto \frac{1}{Q^2} \ln^2 \frac{Q^2}{\Lambda^2}$$



A. V. Belitsky, X. Ji, F. Yuan, Phys. Rev. Lett. **91**, 092003 (2003)

“Scaling” of the ratios of the $N\Delta(1232)$ FFs



$$\frac{F_f(Q^2)}{F_2(Q^2)} \propto \frac{1}{Q^2} \ln^{N_f} \frac{Q^2}{\Lambda^2}, \quad f = 1,\ 3$$

for $Q^2 \geqslant 0.4$ GeV^2, $\Lambda = 0.29$ GeV, $N_3 = 2$, $N_1 = 2.7$.

$N\Delta(1232)$-transition

$\Delta^*(1232)$: $J^P = \frac{3}{2}^+$

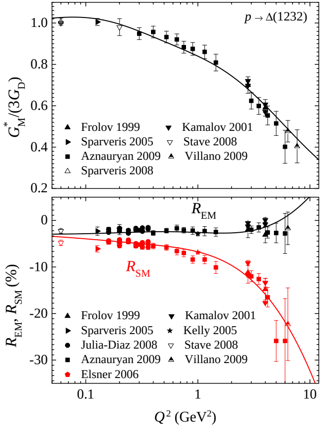
Magnetic FF:

$$G_{\mathrm{M}}^* = -\left[\frac{M_N^3(M_R^2 - M_N^2)}{2\pi\alpha(M_R + M_N)^2 Q_-^2}\right]^{1/2} \times$$
$$\times \left(A_{1/2} + \sqrt{3}A_{3/2}\right),$$

The ratio of the electric and Coulomb amplitudes to the magnetic one:

$$R_{\mathrm{EM}} = \frac{A_{1/2} - A_{3/2}/\sqrt{3}}{A_{1/2} + \sqrt{3}A_{3/2}},$$

$$R_{\mathrm{SM}} = \frac{\sqrt{2}S_{1/2}}{A_{1/2} + \sqrt{3}A_{3/2}}.$$



Magnetic FF and the amplitude ratios for the transition $p\gamma^* \to \Delta(1232)$ ($\chi^2/DOF = 1.51$)

Results and conclusions

- We have constructed effective Lagrangians for the NRV-interactions that possess the gauge and point invariance of the RS field. The symmetry ensures mathematical coherence of the theory and fixes all three terms of the minimally local Lagrangian. The point and gauge invariance unifies the structure and properties of the Lagrangians for arbitrarily high spins.

- The multi-pole vector-meson–dominance model constrained by high-Q^2 pQCD predictions is in a good agreement with the available data on the transitions to the resonances $\Delta(1232)$, $N(1440)$, $N(1535)$, $N(1520)$, and $N(1680)$.

Results and conclusions

- The ratios of the point and gauge invariant form factors $N \to \Delta(1232)$ exhibit asymptotic scaling behavior at momentum transfers as low as 0.4 GeV^2. While the high-Q^2 scaling of the FFs is well understood as a consequence of the asymptotic freedom, the dynamics leading to the low-Q^2 scaling of the FF ratios in the nonperturbative domain of QCD is still to be established both qualitatively and quantitatively.

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