Non-perturbative renormalization scheme in application to chiral perturbation theory in the nucleon sector

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Outline

- Introduction: LFχEFT
- Covariant Light-Front Dynamics
- Fock sector dependent renormalization scheme
- Taylor-Lagrange regularization scheme
- Application to ChPT
- Perspectives

Introduction

To understand the nucleon structure at low energy from a chiral effective Lagrangian we need an appropriate calculational scheme:

- relativistic
- non-perturbative
- well-controlled approximation scheme

We present **LFχEFT** – Light-Front Chiral Effective Field Theory

Light Front Chiral Effective Field Theory

Key points:

- **CLFD**

  explicitly covariant formulation of light-front dynamics

  [J.Carbonell et al. Phys.Rep. 300 (1998) 215]

- **FSDR**

  Fock sector dependent renormalization scheme

  [V. Karmanov et al. PRD 77 (2008) 085028]

- **TLRS**

  Taylor-Lagrange regularization scheme

  [P.Grangé et al. Phys. Rev. D 82 (2010) 025012]

Already done

CLFD + FSDR + Pauli-Villars

- Yukawa N=2, N=3
- QED N=2

  [V. Karmanov et al. PRD 77 (2008) 085028]
- ChPT N=2

CLFD + FSDR + TLRS

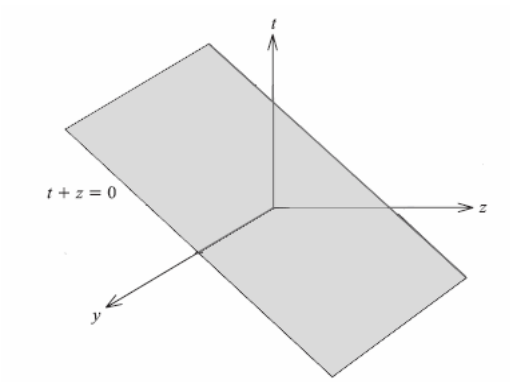
- Yukawa N=2

  [P.Grangé et al. Phys.Rev.D 80 (2009) 105012]

ChPT is the first “realistic” test of this approach

Covariant Light-Front Dynamics

Standard version of LFD



Covariant formulation

Rotational invariance
is broken!

Arbitrary position
of the LF plane

$$\omega \cdot x = 0$$

$$\omega^2 = 0$$

[J.Carbonell et al. Phys.Rep. 300 (1998) 215]

The state vector construction

Fock decomposition:

$$\phi_\omega^{J\sigma}(p) \equiv |1\rangle + |2\rangle + \dots + |n\rangle + \dots$$

$$\downarrow \qquad \downarrow \qquad \qquad \downarrow$$

$$\phi_1 \qquad \phi_2 \qquad \qquad \phi_n \longrightarrow$$ n-body light-front wave functions

Truncation of the Fock decomposition: $n \leq N$

- N – the maximal number of Fock sectors under consideration
- n – number of constituents in a given Fock sector

Upper index $\phi_n^{(N)}$: n-body light-front wave functions depend on the Fock space truncation

For example: $\phi_2^{(2)} \neq \phi_2^{(3)}$

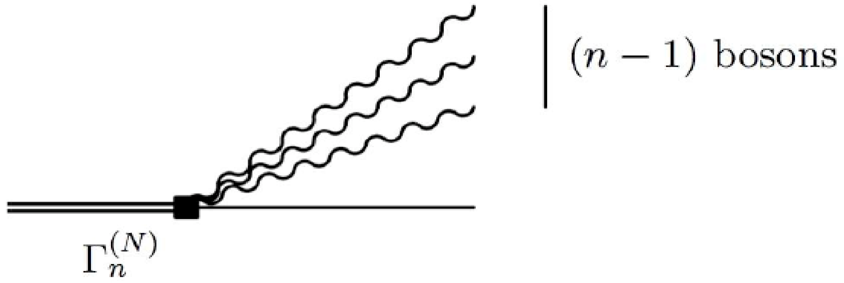
Vertex functions

Wave functions ⟷ vertex functions:

$$\bar{u}(k_1)\Gamma_n^{(N)}u(p) = \left(s_n - M^2\right)\phi_n^{(N)}$$

$$s_n = (k_1 + ... + k_n)^2$$

Graphical representation:



Vertex function for a physical fermion
made of a constituent fermion coupled to bosons

Vertex functions

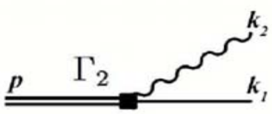
Vertex functions decomposition :

- invariant amplitudes constructed from the particle 4-momenta
- spin structures

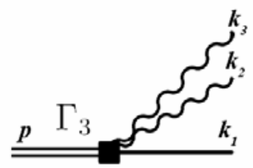
Yukawa model :

$$\bar{u}(k_1)\Gamma_1 u(p) = (m^2 - M^2) a_1 \bar{u}(k_1) u(p)$$

$$\bar{u}(k_1)\Gamma_2 u(p) = \bar{u}(k_1)\left[b_1 + b_2 \frac{M\ \not{\omega}}{\omega\cdot p}\right] u(p)$$



$$\bar{u}(k_1)\Gamma_3 u(p) = \bar{u}(k_1)\left[c_1 + c_2 \frac{M\ \not{\omega}}{\omega\cdot p} + C_{ps}\left(c_3 + c_4 \frac{M\ \not{\omega}}{\omega\cdot p}\right)\gamma_5\right] u(p)$$

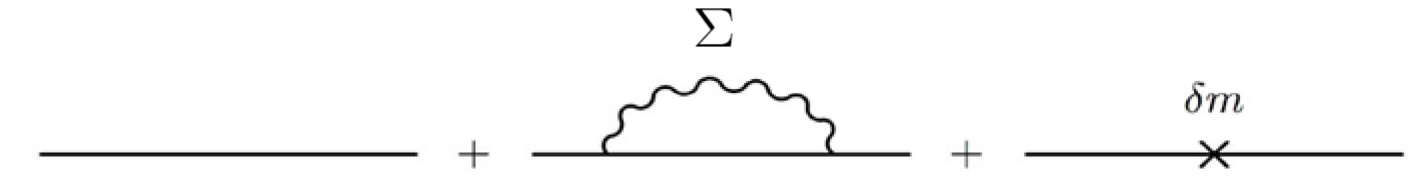


$$C_{ps} = \frac{1}{M^2\ \omega\cdot p}\ \epsilon^{\mu\nu\rho\lambda}\ k_{2\mu}\ k_{3\nu}\ p_\rho\ \omega_\lambda$$

a, b, c are scalar functions depending on dynamics

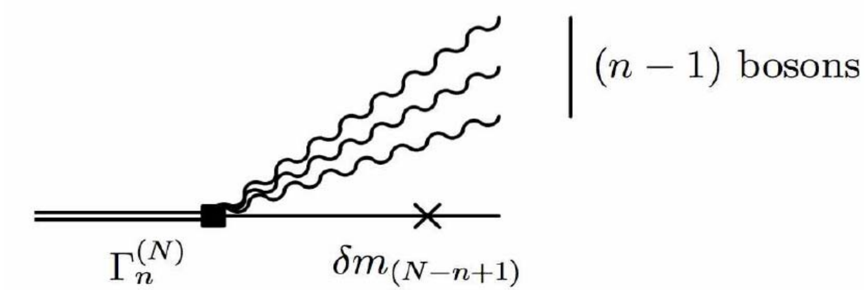
Renormalization scheme

Contribution to the physical fermion propagator



$$\delta m \to \delta m_2$$

The general case: dependence on the Fock sector



(maximal number of particles in which the fermion line can fluctuate)

The same strategy for bare coupling constant g_0

[V. Karmanov et al. PRD 77 (2008) 085028]

Renormalization scheme

Iterative scheme: from sector to sector

problem for $N=2$ ⟶ $\delta m_2, g_{02}$

problem for $N=3$ ⟶ $\delta m_3, g_{03}$

problem for $N=4$

⋮

⟶ $\delta m_N, g_{0N}$ for given N

Regularization

Infinite regularization schemes:

- Cut-off
- Dimensional regularization
- Pauli-Villars regularization scheme

All these schemes deal with infinitely large contributions

We use TLRS: systematic finite regularization scheme

Amplitudes depend on arbitrary finite scale

Basics of TLRS

$$\mathcal{A} = \int T(x)\mathrm{d}x \to \int T(x)f(x)\mathrm{d}x$$

$f(x)$ – super regular test function

- $f(x) = 1$ everywhere it is defined
- vanishes with all derivatives at boundaries

Support : $f(x \geq H) = 0$

Ordinary cut-off : $H=H_0$

We go beyond this ordinary cut-off

Basics of TLRS

$$\mathcal{A} = \int T(x)\mathrm{d}x \to \int T(x) f(x)\mathrm{d}x$$

Running boundary condition:

$$H(x) = \eta\, x\, g_\alpha(x)\,, \qquad \eta > 1\,, \quad 0 < \alpha < 1$$

Lagrange formula:

$$f(ax) = -\int_a^\infty \mathrm{d}t\, \partial_t\, f(xt)$$

When we put $\alpha \to 1^-$:

- $g_\alpha(x) \to 1$
- $H(x_{max}) \to \infty$
- $f(x) \to 1$ everywhere
- integration limit over t : $\quad x\,t \leq H(x)$

$$t \leq \eta$$

Basics of TLRS : how it works

$$\mathcal{A} = \int_0^\infty \frac{\mathrm{d}x}{x+a} \to \int_0^\infty \frac{\mathrm{d}x}{x+a} f(x)$$

New variable y : $x = ay$

$$\mathcal{A} = \int_0^\infty \frac{\mathrm{d}y}{y+1} f(ay)$$

New variable t : $z = yt$

$$\mathcal{A} = -\int_0^\infty \mathrm{d}z \int_a^\infty \mathrm{d}t\, \partial_t \frac{1}{z+t} f(z)$$

Integration domain : $z = yt \leq H(y)$

$t \leq \eta$

$$\mathcal{A} = \int_0^\infty \mathrm{d}z \left(\frac{1}{z+a} - \frac{1}{z+\eta} \right)$$

– Pauli-Villars type subtraction

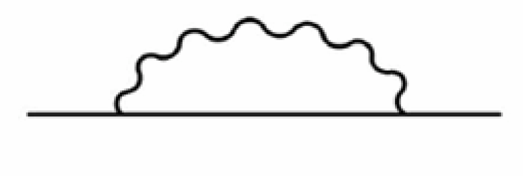
Final result : $\mathcal{A} = \ln \eta - \ln a$

η is an arbitrary finite positive number

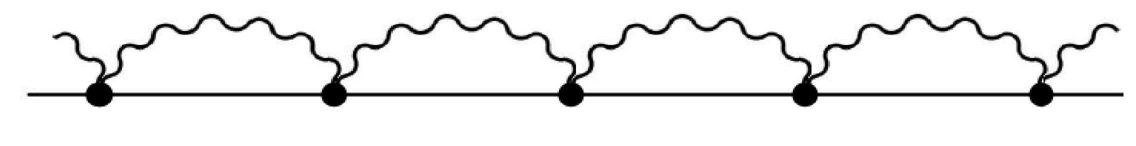
Application to Chiral Perturbation Theory N=2

Need of a non-perturbative framework to calculate
bound state properties

easy with πNN coupling



to be generalized for $\pi\pi NN$ case



ChPT Lagrangian

- Lagrangian is formulated in terms of u fields

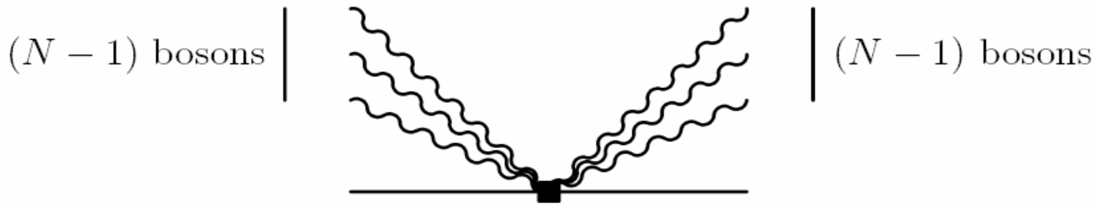
$$u = e^{i\frac{\vec{\tau}\cdot\vec{\pi}}{2F_0}}$$

F_0 is the pion decay constant

- Expansion in a finite number of degrees of pion field

$$\mathcal{L} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \ldots + \mathcal{L}^{(N)} + \ldots$$

- N-body Fock space truncation: $2(N-1)$ pions

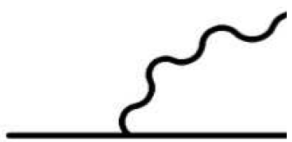


ChPT Lagrangian

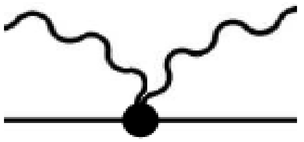
In our first study

$$\mathcal{L} = -\frac{g_A}{2F_0}\bar{\Psi}\gamma^\mu\gamma_5\vec{\tau}\cdot\partial_\mu\vec{\pi}\Psi - \frac{1}{4F_0^2}\bar{\Psi}\,\vec{\tau}\cdot(\vec{\pi}\times\partial_\mu\vec{\pi})\gamma^\mu\Psi$$

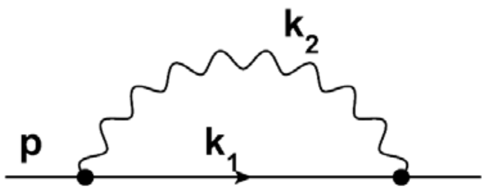
linear πNN interaction



contact $\pi\pi NN$ interaction



Self-energy calculation



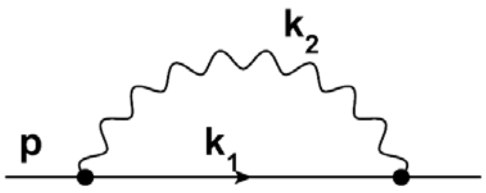
Attach test functions corresponding to internal propagators

$$\Sigma \to \Sigma \, f\left(\frac{k_1^2}{\Lambda^2}\right) \, f\left(\frac{k_2^2}{\Lambda^2}\right)$$

Introduce a new variable t and apply the Lagrange formula

$$\Sigma = -\frac{3g_A^2 M}{32F_0^2\pi^2} \int\limits_0^\infty \mathrm{d}\mathbf{R}_\perp^2 \int\limits_0^1 \mathrm{d}x \int\limits_a^\infty \mathrm{d}t \partial_t \frac{\mu^2}{\mathbf{R}_\perp^2 + t^2(x^2M^2 + \mu^2(1-x))} f\left(\right) f\left(\right)$$

Self-energy calculation



Proceed in the common way:

- consider running boundary condition
- put $\alpha \to 1^-$, $f \to 1$
- find integration limit $t \leq \eta$

The final result :

$$\Sigma = \frac{3g_A^2 M\mu^2}{32F_0^2\pi^2}\ln\eta - \frac{3g_A^2 M\mu^2}{32F_0^2\pi^2}\int_0^1 \mathrm{d}x \ln\frac{x^2M^2+\mu^2(1-x)}{M^2}$$

Chiral limit

The nucleon mass correction from the self-energy contribution :

$$M = M_0 + \Sigma$$

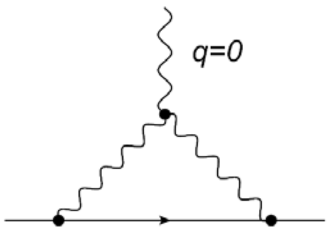
The full self-energy :

$$\Sigma = \frac{3g_A^2 M\mu^2}{32F_0^2\pi^2}\ln\eta + \frac{3g_A^2\mu^2}{32F_0^2\pi^2}\left(2M - \frac{\mu^2}{M}\ln\frac{\mu}{M} - \frac{\mu\sqrt{4M^2-\mu^2}}{M}\arctan\frac{\sqrt{4M^2-\mu^2}}{\mu}\right)$$

Non-analytic terms in the chiral limit :

$$\Sigma = \frac{3g_A^2}{32F_0^2}\left(-\frac{\mu^3}{\pi} - \frac{\mu^4\ln\frac{\mu}{M}}{\pi^2 M} + \frac{\mu^5}{8\pi M^2} + \ldots\right)$$

Scalar form factor



To calculate it we proceed in a common way:

- attach test function
- introduce new variable t
- apply the Lagrange formula

$$\sigma = -\frac{3g_A^2\mu^2 M}{32F_0^2\pi^2}\int \mathrm{d}\mathbf{R}_\perp^2\,\mathrm{d}x\int_a^\infty \mathrm{d}t\partial_t \frac{1}{\mathbf{R}_\perp^2+t^2\left(x^2M^2+\mu^2(1-x)\right)} f[\,]f[\,] \longrightarrow \sim\Sigma$$

$$+\frac{3g_A^2\mu^2 M}{32F_0^2\pi^2}\int \mathrm{d}\mathbf{R}_\perp^2\,\mathrm{d}x\int_a^\infty \mathrm{d}t\partial_t \frac{t^2\mu^2(1-x)}{\left[\mathbf{R}_\perp^2+t^2\left(x^2M^2+\mu^2(1-x)\right)\right]^2} f[\,]f[\,] \longrightarrow \text{convergent integral}$$

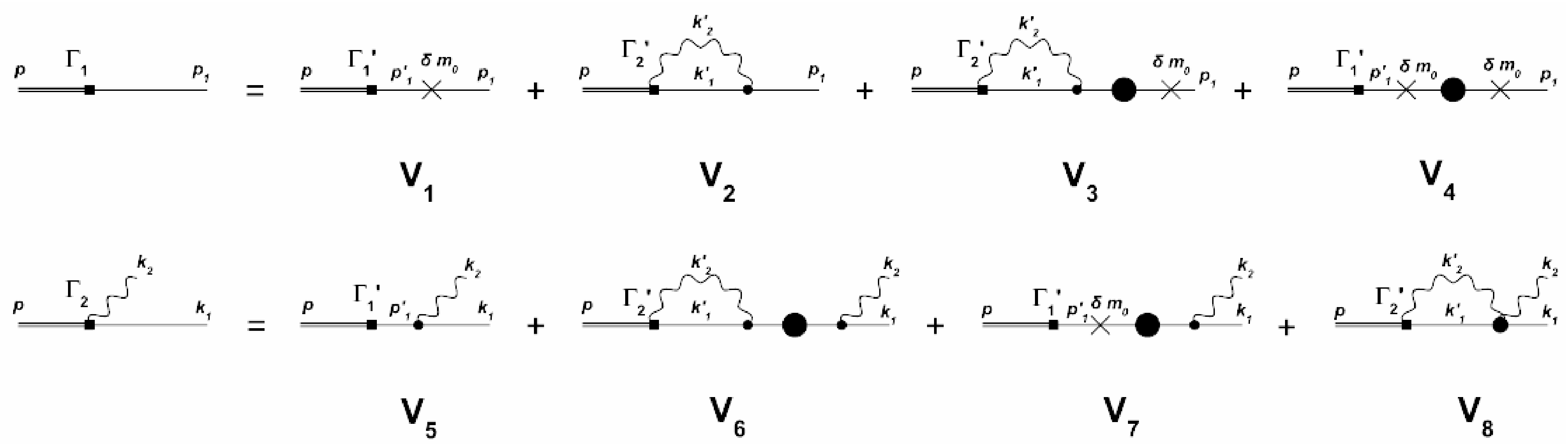
The final result

$$\sigma = \frac{3g_A^2 M\mu^2}{32F_0^2\pi^2}\ln\eta + \frac{3g_A^2\mu^2}{32F_0\pi^2}\left(2M - \frac{2\mu^2}{M}\ln\frac{\mu}{M} - \frac{2\mu(3M^2-\mu^2)}{M\sqrt{4M^2-\mu^2}}\arctan\frac{\sqrt{4M^2-\mu^2}}{\mu}\right)$$

coincides with the Feynman – Hell-Mann theorem: $\sigma = \mu^2\frac{\partial M}{\partial \mu^2}$

System of equations

In the two-body Fock space truncation

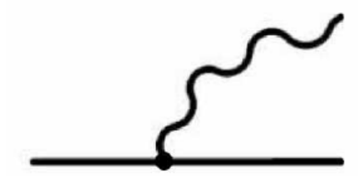


$$\bar{u}(p_1)\Gamma_1 u(p) = \bar{u}(p_1)(V_1 + V_2 + V_3 + V_4)u(p)$$
$$\bar{u}(k_1)\Gamma_2 u(p) = \bar{u}(k_1)(V_5 + V_6 + V_7 + V_8)u(p)$$

The term V_8 corresponds to the contact $\pi\pi NN$ interaction

Vertex functions representation

We choose the representation of vertex functions according to the vertex with an outgoing pion:



$$-ig_0(\not{k} - \not{\omega}\tau)\gamma_5$$

$$\begin{aligned}
\bar{u}(k_1)\Gamma_1 u(p) &= (m^2 - M^2)a_1\bar{u}(k_1)u(p)\ , \\
\bar{u}(k_1)\Gamma_2 u(p) &= -i\bar{u}(k_1)\left((\not{k}_2 - \not{\omega}\tau)\gamma_5\, b_1(\mathbf{R}_\perp, x) + \gamma_5\frac{M\ \not{\omega}}{\omega\cdot p}\, b_2(\mathbf{R}_\perp, x)\right)u(p) \\
\tau &= \frac{s - M^2}{2\omega\cdot p} \\
s &= (k_1 + k_2)^2
\end{aligned}$$

With this representation b_1 and b_2 are just scalars

Solution

$$\begin{cases} b_1 &= \dfrac{2Ma_1 g_0}{1 - \frac{f_0}{F_0^2} Z} \\ b_2 &= 0 \\ \delta m_0 &= -\dfrac{3g_0 b_1}{4a_1 F_0^2} Z \end{cases}$$

$$Z \sim \Sigma$$

Condition : $b_1 = g_A$

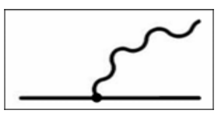
Bare coupling constant : $g_0 = \dfrac{g_A}{2Ma_1}\left(1 - \dfrac{f_0}{F_0^2} Z\right)$

Mass counterterm : $\delta m = -\dfrac{3g_A^2 M}{2F_0^2}\left(1 - \dfrac{f_0}{F_0^2} Z\right) Z$

Final results

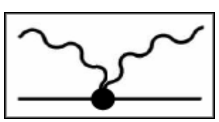
$$
\begin{cases}
b_1 = g_A \\
b_2 = 0 \\
\delta m = -\dfrac{3 g_A^2 M}{2 F_0^2} \left(1 - \dfrac{f_0}{F_0^2} Z \right) Z
\end{cases}
$$

Without contact interaction :



$$f_0 = 0 \,, \qquad \delta m = -\frac{3 g_A^2 M}{2 F_0^2} Z = \Sigma$$

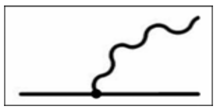
With contact $\pi\pi NN$ interaction :



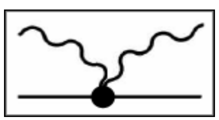
$$f_0 = 1 \,, \qquad \delta m = -\frac{3 g_A^2 M}{2 F_0^2} \left(1 - \frac{1}{F_0^2} Z \right) Z$$

Nucleon mass corrections

First non-analytic contributions with and without contact interaction



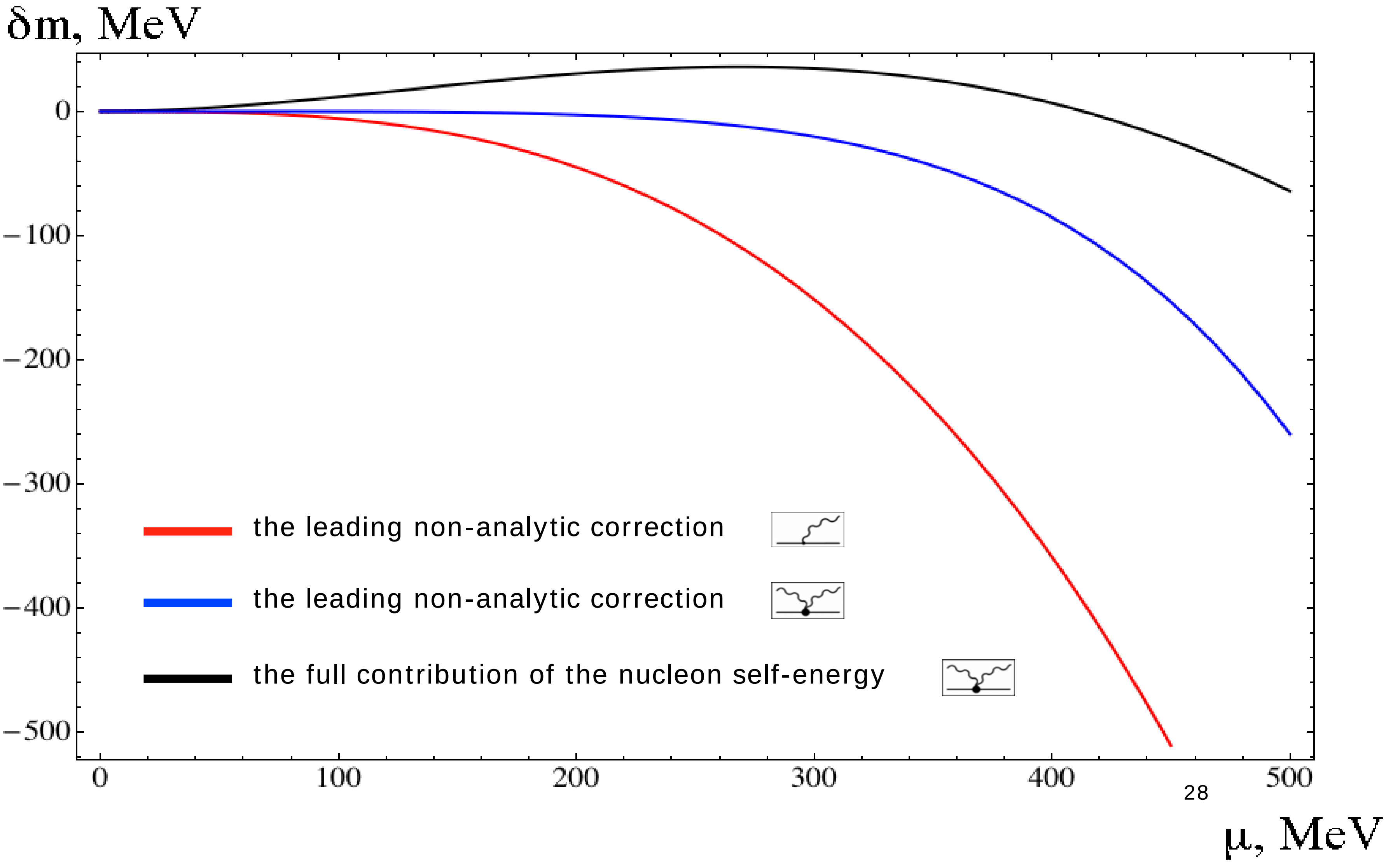
$$\delta m = \frac{3g_A^2}{32F_0^2}\left(-\frac{\mu^3}{\pi} - \frac{\mu^4 \ln\frac{\mu}{M}}{\pi^2 M} + \frac{\mu^5}{8\pi M^2}\right)$$



$$\delta m = \frac{3g_A^2}{32F_0^2}\left(-\frac{\mu^3}{\pi} - \frac{\mu^4 \ln\frac{\mu}{M}}{\pi^2 M} + \frac{\mu^5}{8\pi M^2} - \frac{f_0}{4F_0^2}\frac{\mu^5}{\pi^3}\right)$$

The first contribution of the contact interaction is of order $\mathcal{O}(\mu^5)$

Contributions to the nucleon mass



Perspectives

Calculations for ChPT $N=3$ (1 nucleon and 2 pions)

} σ and ρ resonances contributions

} Δ and Roper resonances contributions