PQCD predictions for hadron collider physics: getting beyond the leading order

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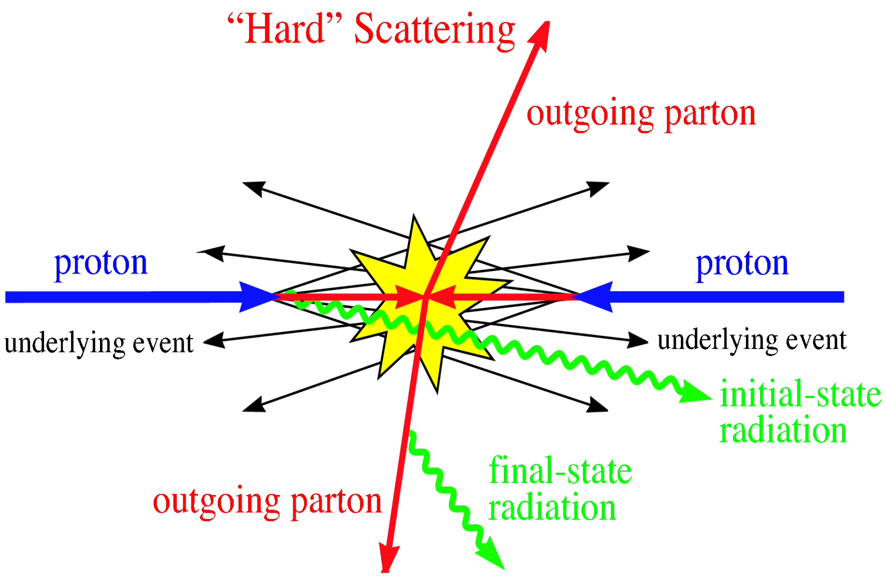
Outline

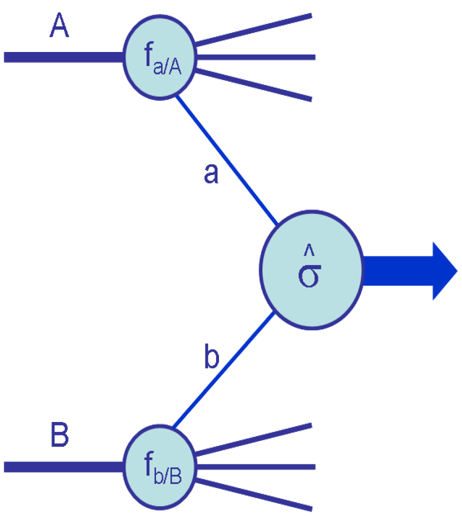
- The field of pQCD, as applied to hadron collider phenomenology, went through a remarkable transformation
  - new on-shell technology for one-loop computations
  - new phenomenological NLO QCD results for high-multiplicity processes
  - automation of one-loop computations – Madgraph/Alpgen/Comphep@NLO?
  - first NNLO results for fully differential quantities
  - active search for a general subtraction scheme@NNLO

My goal in this talk is to describe ideas that lead to these developments and give examples of their phenomenological relevance. Please note that this is not a review talk on perturbative QCD, so that all examples are personally-biased and not inclusive

The need for higher orders

- Experiments at the Tevatron and the LHC search for physics beyond the Standard Model in hard collisions, where all momentum transfers are large. Perturbative QCD is a systematic, improvable framework to describe such processes





$$\langle \mathcal{O} \rangle = \sum_{i,j} \int \mathrm{d}x_1 \mathrm{d}x_2 \; f_i(x_1) f_j(x_2) \; \mathrm{d}\sigma_{ij\to p} \; \mathcal{F}_{p\to\mathcal{O}} + \mathcal{O}(1/Q)$$

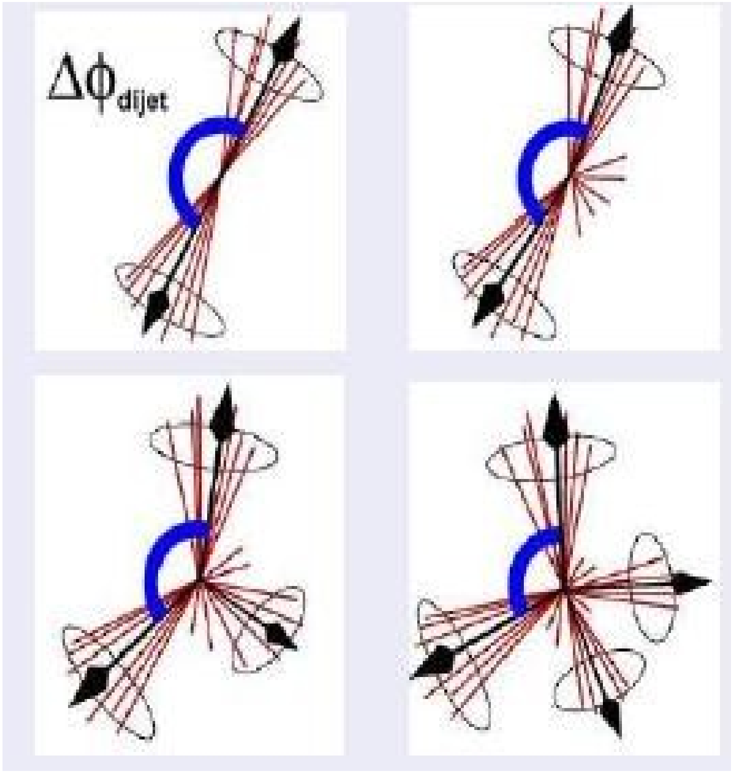
Parton distribution functions are non-perturbative universal objects. Parton scattering cross-sections are computable in perturbative QCD. Perturbative partons are evolved to hadrons using parton showers

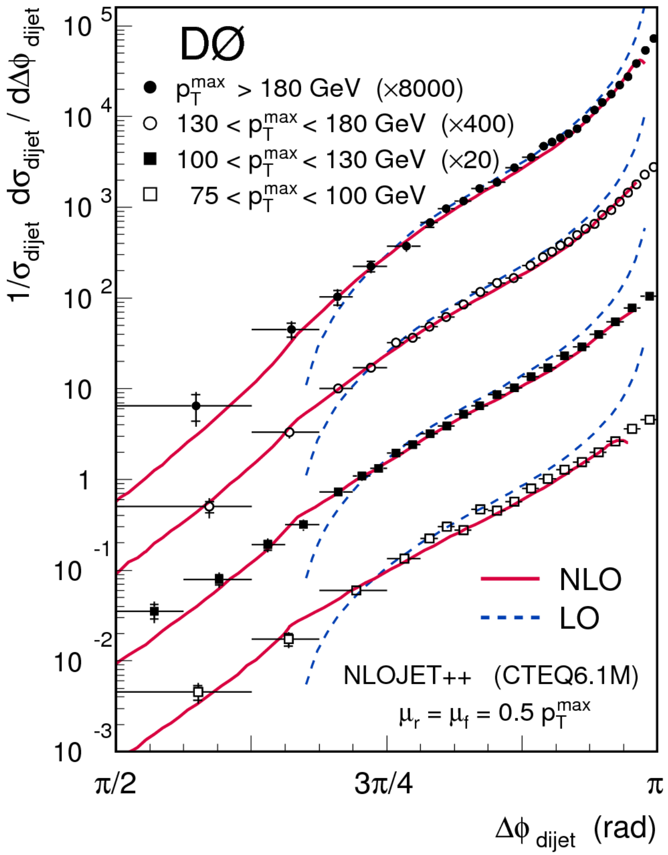
The benefit of higher orders

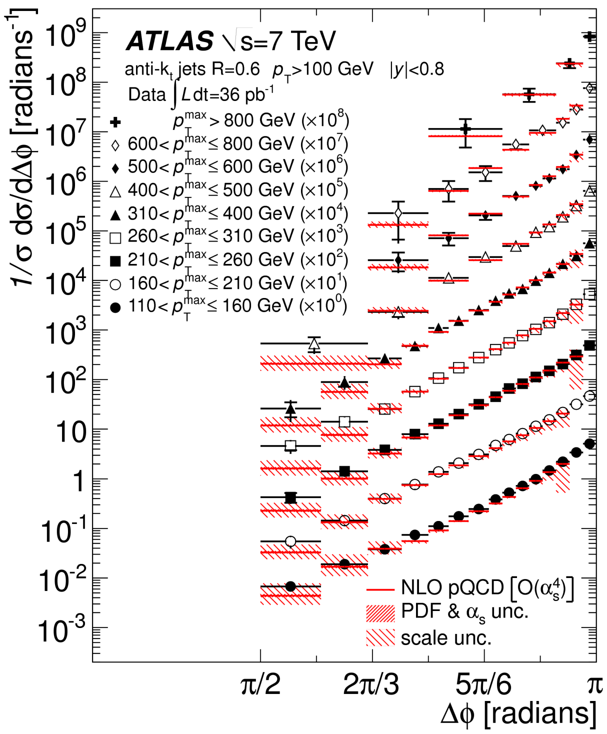
- Description of a particular process in higher orders of pQCD often leads to
  - reduced sensitivity to unphysical renormalization/factorization scales – control of the normalization
  - more realistic description of jets
  - a possibility of ``fool-proof'' extrapolation between different kinematic regions (data driven background estimates)
  - smaller PDF uncertainties and better compatibility between different PDF sets
- Apart from theoretical niceties,   the validity of pQCD description of hard hadron collisions – and related benefits of going to higher orders – has been   verified by the Tevatron and the early LHC data

Jet azimuthal correlations at the Tevatron and the LHC

- Jet angular correlations allow us to trace how things work when additional jets are being created in the hard process

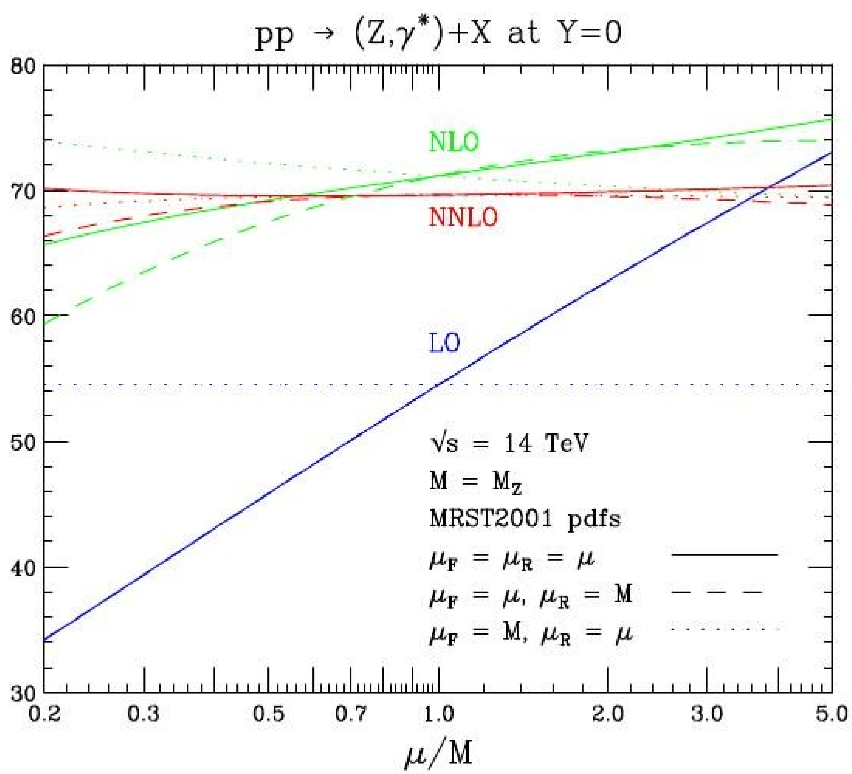


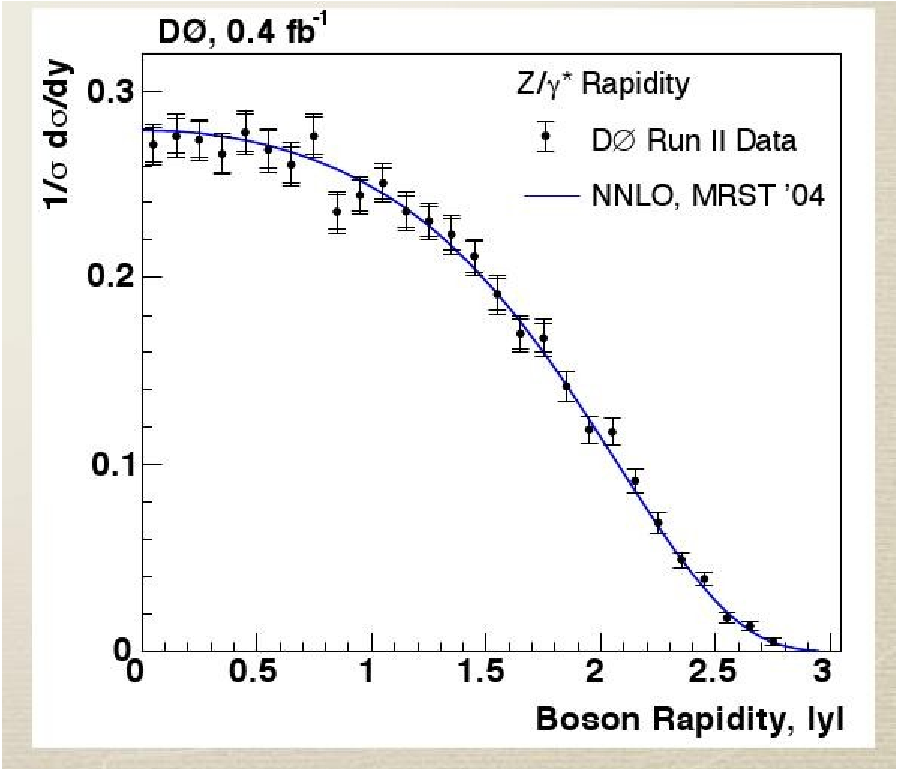




Z/gamma rapidity distributions

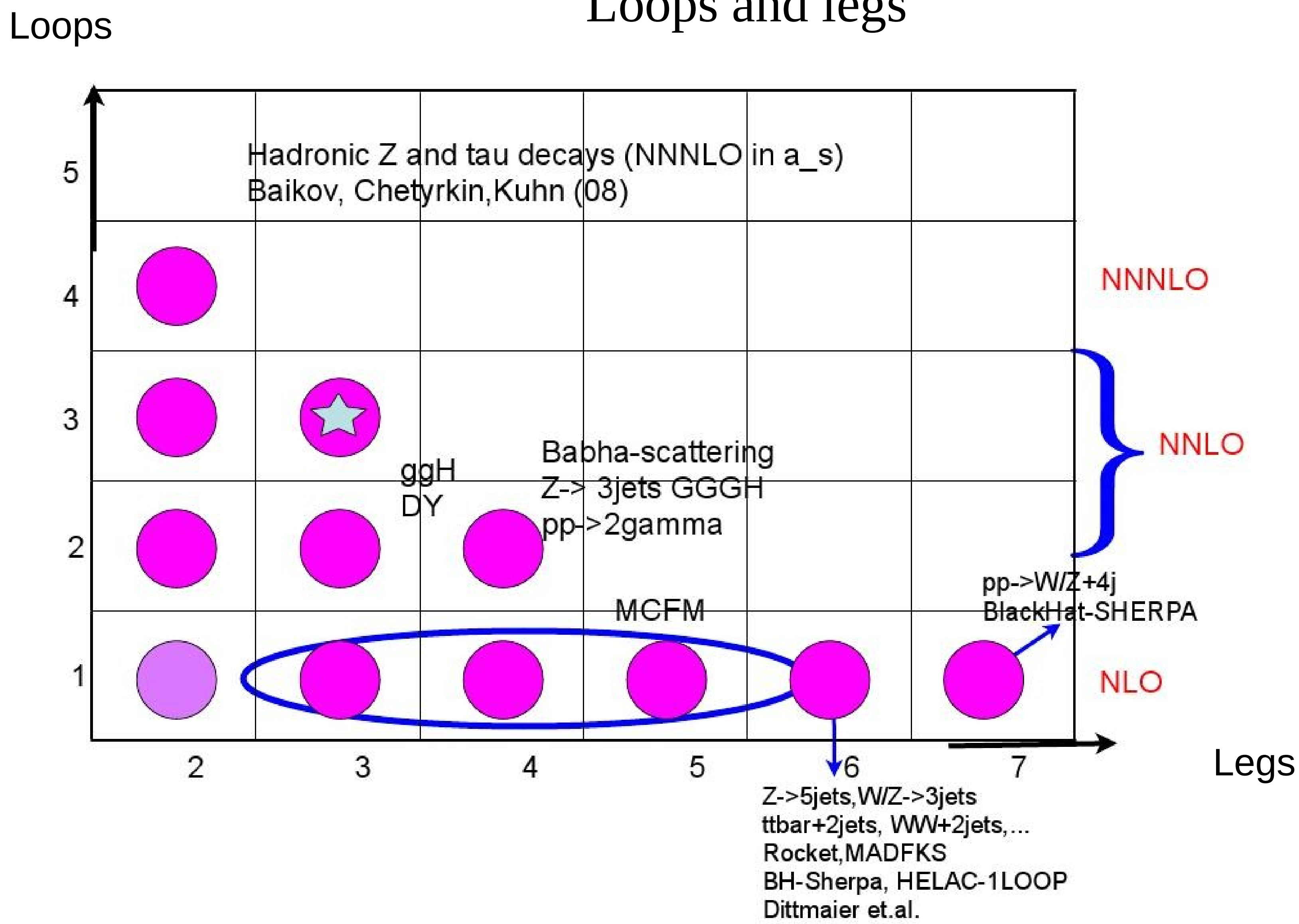
- Rapidity distributions of dileptons in hadron collisions are known through NNLO. Remarkable consistency with Tevatron measurements. Input for PDF constraints.





Anastasiou, Dixon, Petriello, K.M.,

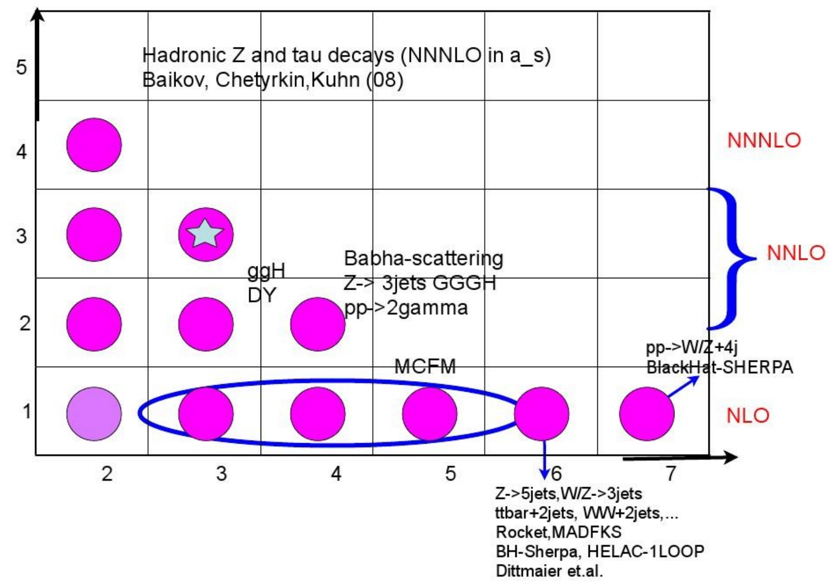
Loops and legs



Slide courtesy of Z. Kunszt

Loops and legs

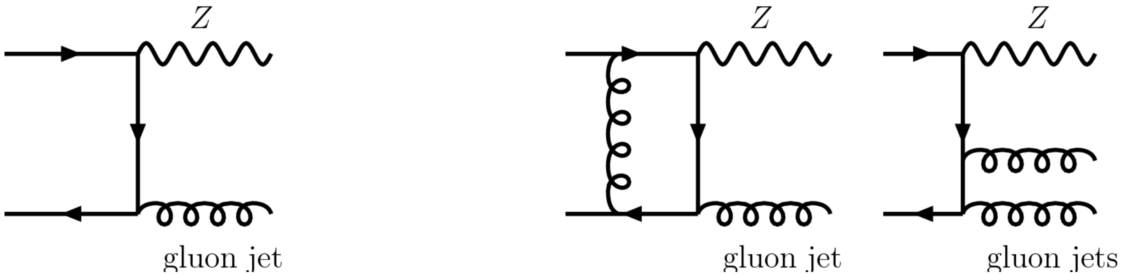
- For many years, progress in multi-loop computations was driven by the integration-by-parts technique and the Laporta algorithm
- These are great tools that are applicable to single-scale (inclusive) problems such as R(s), tau-decays, QCD beta-function, g-2, quark masses from sum rules, muon decay and the Fermi constant, DGLAP evolution kernels etc
- They were also succesfully applied for computing two-loop scattering amplitudes for 2-> 2 scattering processes
- We have benefited a lot from this technology and continue to use it, but description of hadron collisions requires dealing with perturbative computations for processes with large number of external particles and large number of kinematic scales
- Such results should be applicable to ``unintegerated", fully differential kinematic distributions in order to be useful; this leads to certain complications and requires a somewhat different approach



Anatomy of NLO computations

- Lets recall how this is achieved in case of next-to-leading order (NLO) computations

$$\mathrm{d}\sigma_{\geq n} = \mathrm{d}\sigma_n^{(0)} + \frac{\alpha_s}{2\pi}\left(\mathrm{d}\sigma_n^{(1)} + \mathrm{d}\sigma_{n+1}^{(0)}\right)$$



$$\mathrm{d}\sigma_{\geq n} = \mathrm{d}\sigma_n^{(0)} + \frac{\alpha_s}{2\pi}\left(\mathrm{d}\sigma_n^{(1)} + \mathrm{d}\tilde{\sigma}_n^{\mathrm{dip}} + \left[\mathrm{d}\sigma_{n+1}^{(0)} - \mathrm{d}\tilde{\sigma}_n^{\mathrm{dip}}\right]\right)$$

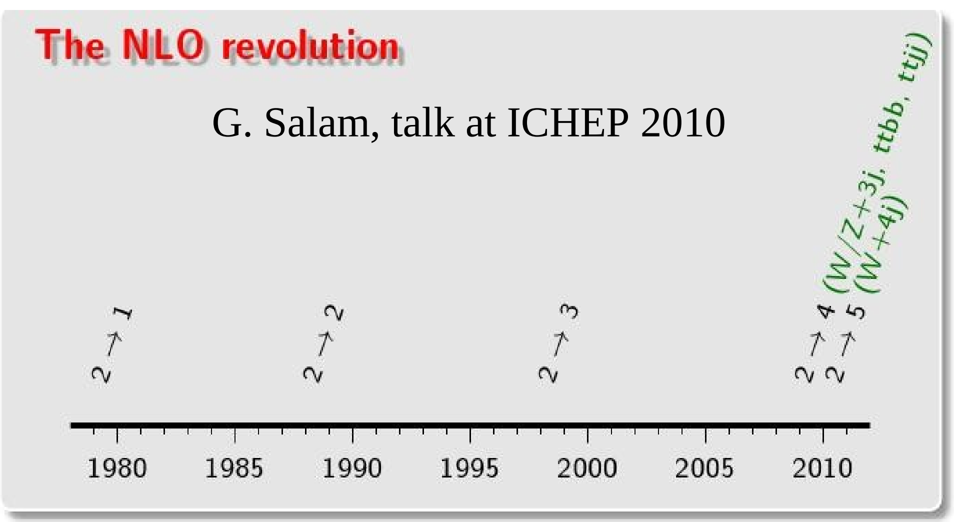
- The subtraction terms (Catani-Seymour, Frixione-Kunszt-Signer) are constructed to make the real emission matrix element squared integrable locally, provided a suitable infra-red safe definition of hadronic final state
- Large number of existing programs can handle real emission computations and generation of subtraction terms
- The problem – for a long time – was the computation of one-loop (!) matrix elements

One-loop computations

- One loop computations become a problem if we try to turn a ``solution of principle'' to a ``solution of practice''
- Practical one-loop computations are often performed along the following line
  - each one-loop diagram is a linear combination of tensor integrals;
  - each tensor integral can be expressed as a linear combination of scalar 1-point, 2-point, 3-point and 4-point scalar integrals
  - one-loop scalar integrals are known and have been tabulated
- Verdict : algorithm for one-loop computations exists, hence they are trivial
- This, of course, is almost right. The problem with this argumet is that
  - number of Feynman diagrams grows factorially;
  - number of terms produced by the tensor reduction grows very strongly;
  - numerical instabilities (Gram determinant problem)
- As the result – the standard procedure becomes hardly manageable if we go to higher multiplicity processes

Progress with NLO computations

- Difficulties with one-loop computations lead to a very slow progress in NLO QCD computations for large number of external particles
- The LHC physics is high-multiplicity physics, so it is essential to go to 2->4 or even 2-> 5 processes
- As an example, typical searches for supersymmetry require 4 jets and misssing energy, so Z+4 jets is an irreducible background. A NLO prediction for Z+4 jets was absolutely impossible until very recently



In recent three to four years new technology for NLO computations appeared that allowed us to take on 2->4 and $2 \to 5$ computations

An experimenter's wishlist April 2001

- Hadron collider cross-sections one would like to know at NLO

Run II Monte Carlo Workshop, April 2001

Single boson	Diboson	Triboson	Heavy flavour
$W+\leq 5j$	$WW+\leq 5j$	$WWW+\leq 3j$	$t\bar{t}+\leq 3j$
$W+b\bar{b}+\leq 3j$	$WW+b\bar{b}+\leq 3j$	$WWW+b\bar{b}+\leq 3j$	$t\bar{t}+\gamma+\leq 2j$
$W+c\bar{c}+\leq 3j$	$WW+c\bar{c}+\leq 3j$	$WWW+\gamma\gamma+\leq 3j$	$t\bar{t}+W+\leq 2j$
$Z+\leq 5j$	$ZZ+\leq 5j$	$Z\gamma\gamma+\leq 3j$	$t\bar{t}+Z+\leq 2j$
$Z+b\bar{b}+\leq 3j$	$ZZ+b\bar{b}+\leq 3j$	$WZZ+\leq 3j$	$t\bar{t}+H+\leq 2j$
$Z+c\bar{c}+\leq 3j$	$ZZ+c\bar{c}+\leq 3j$	$ZZZ+\leq 3j$	$t\bar{b}+\leq 2j$
$\gamma+\leq 5j$	$\gamma\gamma+\leq 5j$		$b\bar{b}+\leq 3j$
$\gamma+b\bar{b}+\leq 3j$	$\gamma\gamma+b\bar{b}+\leq 3j$		
$\gamma+c\bar{c}+\leq 3j$	$\gamma\gamma+c\bar{c}+\leq 3j$		
	$WZ+\leq 5j$		
	$WZ+b\bar{b}+\leq 3j$		
	$WZ+c\bar{c}+\leq 3j$		
	$W\gamma+\leq 3j$		
	$Z\gamma+\leq 3j$		

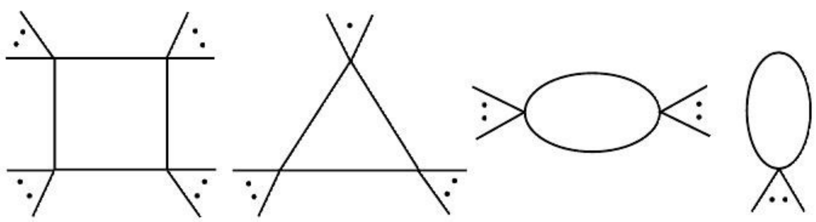
The change in the paradigm

- The remarkable progress illustrated on the previous slide occurred (at least partially) due to development of a radically new method for one-loop computations
- Instead of computing scattering amplitudes from Feynman diagrams, we construct them from on-shell gauge invariant tree-level scattering amplitudes
- The trick is a generalization of the old idea of unitarity where imaginary parts of scattering amplitudes are reconstructed from the unitairty cuts

$$i\left(T_{ij} - T^{+}_{ij}\right) = \sum T_{in} T^{+}_{nj}$$

- Exploit the fact that large fraction of any c

$$\mathcal{A}^{1-\mathrm{loop}} = \sum c_j I_j \qquad I_j =$$



$$\mathrm{Im}\left(\mathcal{A}^{1-\mathrm{loop}}\right) \propto \sum \left|\mathcal{A}^{\mathrm{tree}}\right|^2 \qquad \sum c_j \mathrm{Im}(I_j) \propto \sum \left|\mathcal{A}^{\mathrm{tree}}\right|^2$$

In the past few years, a procedure appeared that allows computation of the reduction coefficients directly from on-shell scattering amplitudes by-passing Feynman diagrams.

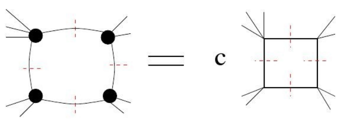
Modern unitarity techniques

- Unitarity techniques in the contemporary context were introduced by Bern, Dixon and Kosower in 1990s and used for a number of high-profile computations. For a long time, this was a collection of tricks and brilliant guesswork.

- Solid computational method emerged in the past four years
  - Quadrupole cuts freeze loop momentum and give the box reduction coefficient directly; Britto, Cachazo, Feng
  - The OPP tensor integral reduction technique; Ossola, Pittau, Papadopoulos
  - The OPP procedure meshes well with unitarity; Ellis, Kunszt, Giele
  - Generalized D-dimensional unitarity Giele, Kunszt, Melnikov

Box Coefficients from Quadruple Cuts

(RB, Cachazo, Feng)



Generalized Unitarity: Try replacing all four propagators by delta functions.

This operation isolates any given box.

In four dimensions, these four delta functions localize the integral completely. This computation is very easy!

The loop momentum solution

The box coefficients computed from quadruple cuts are given by

$$c = \frac{1}{2} \sum_{\mathcal{S}} A_1^{\text{tree}} A_2^{\text{tree}} A_3^{\text{tree}} A_4^{\text{tree}}$$

$\mathcal{S}$ is the set of all solutions of the on-shell conditions for the internal lines.

$$\mathcal{S} = \{\, \ell \,|\, \ell^2 = 0, \quad (\ell - K_1)^2 = 0, \quad (\ell - K_1 - K_2)^2 = 0, \quad (\ell + K_4)^2 = 0 \}$$

Can these equations always be solved?

In complexified momentum space, there are exactly 2 solutions.

(Note: nonvanishing 3-point amplitudes.)

From R. Britto talk, LoopFest 2008

OPP reduction

- The OPP procedure is central for all existing implementations of the unitarity method.
- It is a novel approach to the reduction of one-loop tensor integrals to scalar integrals

$$\mathrm{Int}_N = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\mathrm{Num}(k)}{\prod_j^N D_i(k)} = \sum_j c_j I_j \qquad D_i = (p_i + k)^2 - m_i^2$$

- OPP pointed out that computation of the reduction coefficients requires limited information about the function $\mathrm{Num}(k)$
- In fact, we need to know it only for such values of the loop momenta for which certain combinations of inverse propagators vanish (all combinations should be considered)

From OPP to generalized unitarity

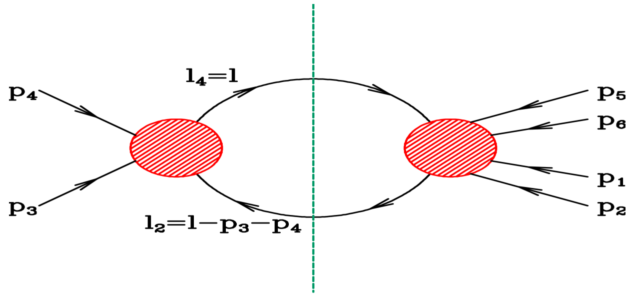
- The OPP procedure applied to full one-loop amplitudes leads to an unitarity-based framework for one-loop calculations

Ellis, Giele, Kunszt

$$\mathcal{A}^{1\ \mathrm{loop}} = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\mathrm{Num}_D(k,p)}{\prod_i D_i} = \sum c_j I_j$$

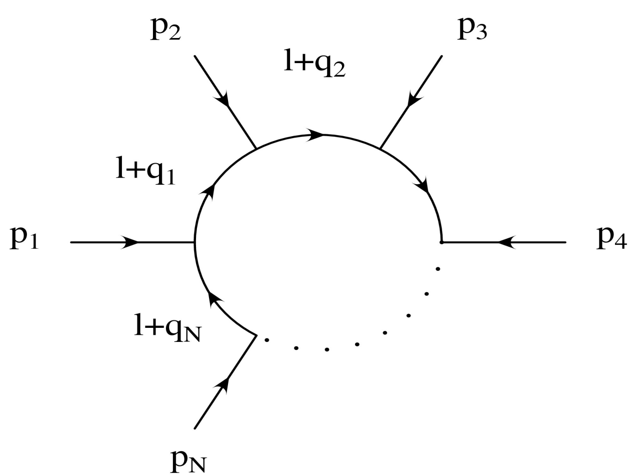
- The OPP procedure determines reduction coefficients from loop momenta for which combinations of inverse Feynman propagators vanish. If this occurs, some virtual particles go on their mass-shells and the one-loop amplitude factorizes into products of tree-amplitudes

$$\frac{\mathrm{Num}(k,p)}{\prod D_i} \to \prod_i A_i^{\mathrm{tree}}(\{p\},\{k_i\})$$

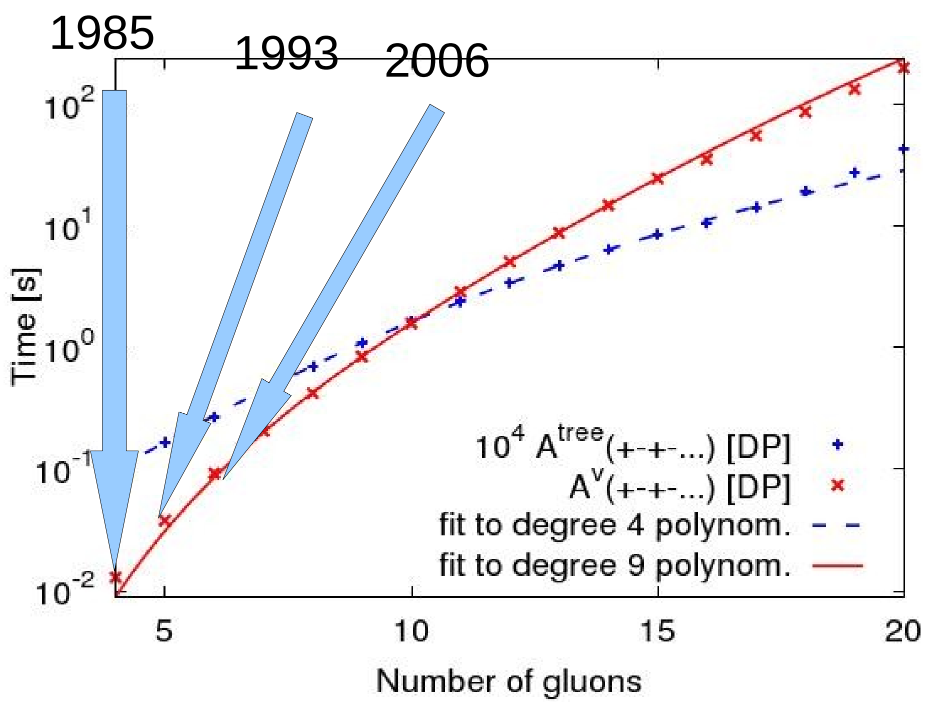


- Those tree amplitudes are conventional BUT, as a rule, have to be evaluated at a complex on-shell momenta.

The power of unitarity: gluon amplitudes



$$20! \approx 2.4 \times 10^{18}$$ diagrams



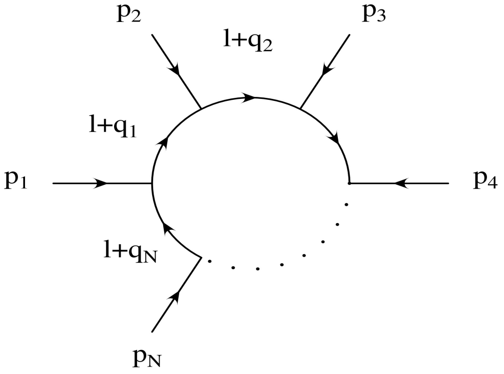
Giele, Zanderighi

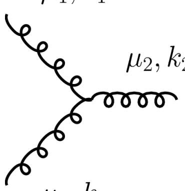
N-gluon amplitudes can be calculated for arbitrary N. Explicit numerical results available for N through 20. Factorial growth in the number of Feynman diagrams makes this computation impossible with traditional methods.

The algorithm: getting loops from trees

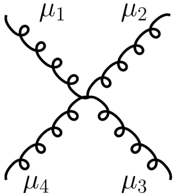
- How to construct an algorithm that starts with tree scattering amplitudes and delivers one-loop scattering amplitudes?
- A unique way of writing the itegrand exists in non-abelian gauge quantum field theories

$$\mathcal{A} = \sum_{\text{perm}} \text{Tr}(T^{a_1} T^{a_2} .. T^{a_n}) \mathcal{A}_n(g_1, g_2, ... g_n)$$





$$= \quad i\frac{g_s}{\sqrt{2}}\left[(k_1 - k_2)_{\mu_3}\, g_{\mu_1\mu_2} + (k_2 - k_3)_{\mu_1}\, g_{\mu_2\mu_3} + (k_3 - k_1)_{\mu_2}\, g_{\mu_3\mu_1}\right]$$



$$i\frac{g_s^2}{2}\left[2 g_{\mu_1\mu_3}\, g_{\mu_2\mu_4} - g_{\mu_1\mu_4}\, g_{\mu_2\mu_3} - g_{\mu_1\mu_2}\, g_{\mu_3\mu_4}\right]$$

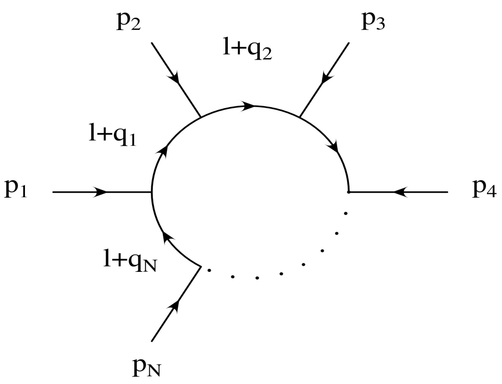
Each color-ordered amplitude follows from a parent diagram

Any diagram that contributes to a particular color-ordered amplitude is obtained by pinching and pulling lines in the parent diagram

Parent diagram possesses a well-defined set of propagators which is not changed by pinching and pulling

The algorithm

- For numerical implementation
  - specify all possible cuts that lead to non-vanishing contributions in dimensional regularization, starting with the quadruple cut
  - loop momentum on the cut assumes complex values
  - each cut produces a sum of products of certain number of tree amplitudes
  - tree-amplitudes for complex on-shell momenta are computed using Berends-Giele recursion relations
  - products of tree amplitudes provide reduction coefficients for master integrals
  - For proper treatment of ultraviolet structure of the theory, one needs to perform this procedure in higher-dimensional (integer) space-time. For pure Yang-Mills, for example, D=5 and D=6 is sufficient to reconstruct the full one-loop scattering amplitude from on-shell unititarity cuts.



The procedure allows us to obtain an answer for a one-loop scattering amplitude without having to deal with Feynman diagrams AND off-shell degrees of freedom including ghosts !

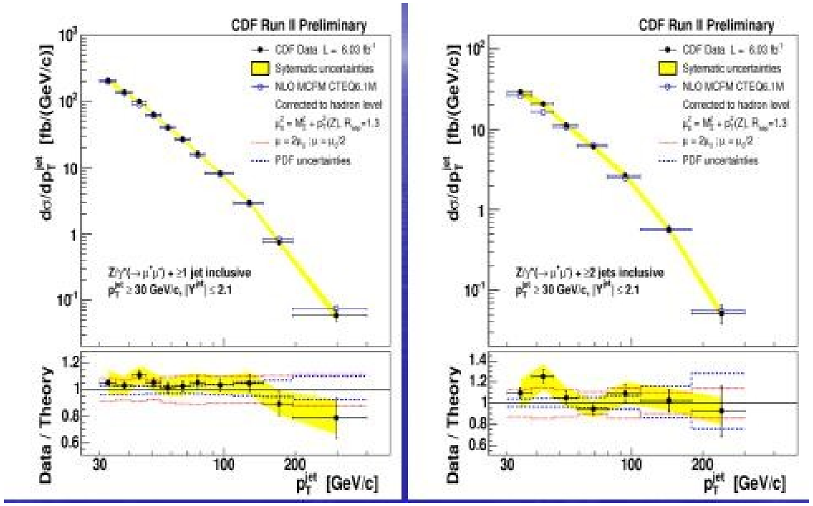
The standard: MCFM

MCFM Summary - v. 3.4

J. Campbell, R.K. Ellis

$p\bar{p} \to W^{\pm}/Z$	$p\bar{p} \to W^{+} + W^{-}$
$p\bar{p} \to W^{\pm} + Z$	$p\bar{p} \to Z + Z$
$p\bar{p} \to W^{\pm} + \gamma$	$p\bar{p} \to W^{\pm}/Z + H$
$p\bar{p} \to W^{\pm} + g^{\star}\,(\to b\bar{b})$	$p\bar{p} \to Zb\bar{b}$
$p\bar{p} \to W^{\pm}/Z + 1$ jet	$p\bar{p} \to W^{\pm}/Z + 2$ jets
$p\bar{p}(gg) \to H$	$p\bar{p}(gg) \to H + 1$ jet
$p\bar{p}(VV) \to H + 2$ jets	

- MCFM aims to provide a unified description of a number of processes at NLO accuracy.
- Various leptonic and/or hadronic decays of the bosons are included as further sub-processes.
- MCFM version 2.0 is part of the CDF code repository.



MCFM Information

- Version 3.4 available at: ***http://mcfm.fnal.gov***
- Improvements over previous release:
  - more processes
  - better user interface
  - support for PDFLIB, Les Houches PDF accord
  - ntuples as well as histograms
  - unweighted events
  - Pythia/Les Houches generator interface (LO)
- Coming attractions:
  - even more processes, photon fragmentation etc.

Automation and craftsmanship

- It appears that new paradigm for NLO computations makes the automation of NLO computations possible.

	Process	μ	n_{lf}	Cross section (pb) LO	NLO
a.1	$pp \to t\bar{t}$	m_{top}	5	123.76 ± 0.05	162.08 ± 0.12
a.2	$pp \to tj$	m_{top}	5	34.78 ± 0.03	41.03 ± 0.07
a.3	$pp \to tjj$	m_{top}	5	11.851 ± 0.006	13.71 ± 0.02
a.4	$pp \to t\bar{b}j$	$m_{top}/4$	4	25.62 ± 0.01	30.96 ± 0.06
a.5	$pp \to t\bar{b}jj$	$m_{top}/4$	4	8.195 ± 0.002	8.91 ± 0.01
b.1	$pp \to (W^+ \to)e^+\nu_e$	m_W	5	5072.5 ± 2.9	6146.2 ± 9.8
b.2	$pp \to (W^+ \to)e^+\nu_e\, j$	m_W	5	828.4 ± 0.8	1065.3 ± 1.8
b.3	$pp \to (W^+ \to)e^+\nu_e\, jj$	m_W	5	298.8 ± 0.4	300.3 ± 0.6
b.4	$pp \to (\gamma^*/Z \to)e^+e^-$	m_Z	5	1007.0 ± 0.1	1170.0 ± 2.4
b.5	$pp \to (\gamma^*/Z \to)e^+e^-\, j$	m_Z	5	156.11 ± 0.03	203.0 ± 0.2
b.6	$pp \to (\gamma^*/Z \to)e^+e^-\, jj$	m_Z	5	54.24 ± 0.02	56.69 ± 0.07
c.1	$pp \to (W^+ \to)e^+\nu_e b\bar{b}$	$m_W + 2m_b$	4	11.557 ± 0.005	22.95 ± 0.07
c.2	$pp \to (W^+ \to)e^+\nu_e t\bar{t}$	$m_W + 2m_{top}$	5	0.009415 ± 0.000003	0.01159 ± 0.00001
c.3	$pp \to (\gamma^*/Z \to)e^+e^- b\bar{b}$	$m_Z + 2m_b$	4	9.459 ± 0.004	15.31 ± 0.03
c.4	$pp \to (\gamma^*/Z \to)e^+e^- t\bar{t}$	$m_Z + 2m_{top}$	5	0.0035131 ± 0.0000004	0.004876 ± 0.000002
c.5	$pp \to \gamma t\bar{t}$	$2m_{top}$	5	0.2906 ± 0.0001	0.4169 ± 0.0003
d.1	$pp \to W^+W^-$	$2m_W$	4	29.976 ± 0.004	43.92 ± 0.03
d.2	$pp \to W^+W^-\, j$	$2m_W$	4	11.613 ± 0.002	15.174 ± 0.008
d.3	$pp \to W^+W^+\, jj$	$2m_W$	4	0.07048 ± 0.00004	0.1377 ± 0.0005
e.1	$pp \to HW^+$	$m_W + m_H$	5	0.3428 ± 0.0003	0.4455 ± 0.0003
e.2	$pp \to HW^+\, j$	$m_W + m_H$	5	0.1223 ± 0.0001	0.1501 ± 0.0002
e.3	$pp \to HZ$	$m_Z + m_H$	5	0.2781 ± 0.0001	0.3659 ± 0.0002
e.4	$pp \to HZ\, j$	$m_Z + m_H$	5	0.0988 ± 0.0001	0.1237 ± 0.0001
e.5	$pp \to Ht\bar{t}$	$m_{top} + m_H$	5	0.08896 ± 0.00001	0.09869 ± 0.00003
e.6	$pp \to Hb\bar{b}$	$m_b + m_H$	4	0.16510 ± 0.00009	0.2099 ± 0.0006
e.7	$pp \to Hjj$	m_H	5	1.104 ± 0.002	1.036 ± 0.002

Table 2: Results for total rates, possibly within cuts, at the 7 TeV LHC, obtained with MADFKS and MADLOOP. The errors are due to the statistical uncertainty of Monte Carlo integration. See the text for details.

MadLoop, Hirshi, et al

Madgraph to generate diagrams and OPP reduction procedure

Automatic construction of FKS dipoles

Parton shower (MC@NLO) is automatic

Craftsmanship still required to deal with highest multiplciity processes and processes with unusual features

$$pp \to t\bar{t}b\bar{b} \qquad pp \to W(Z) + 3j$$

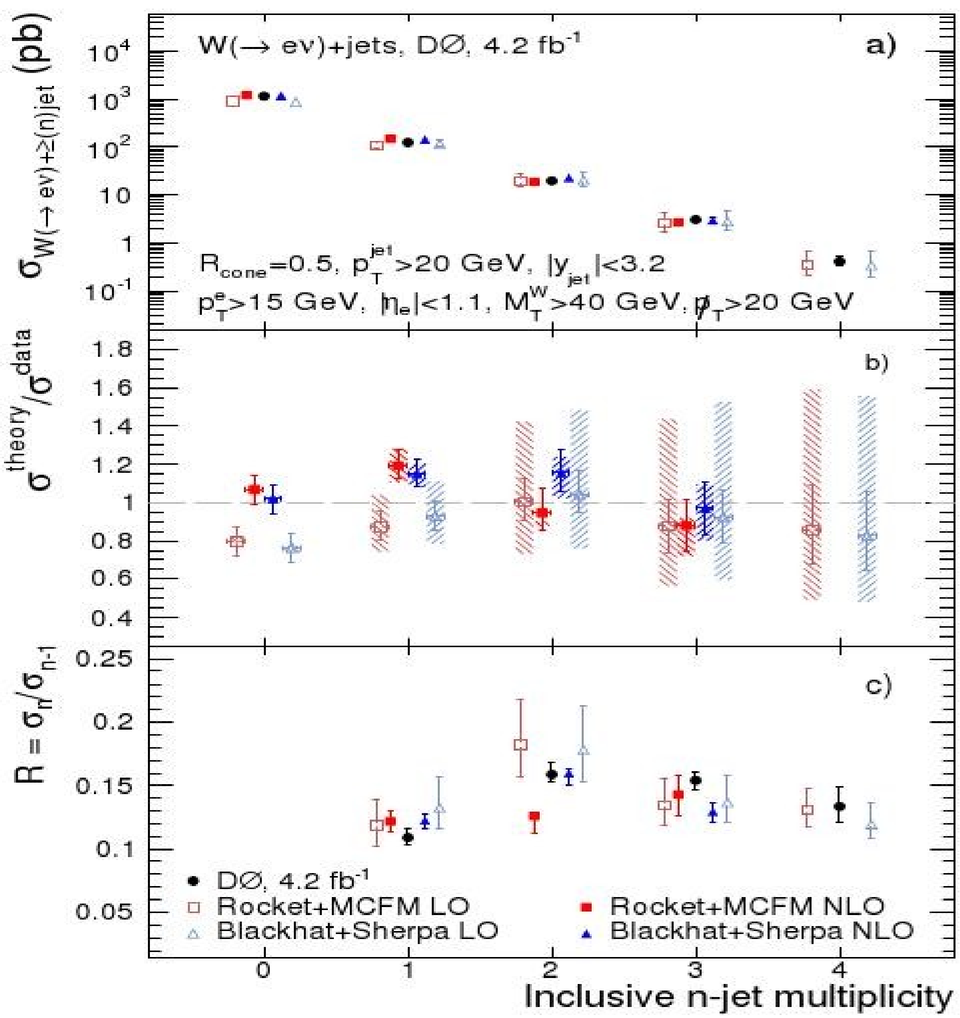
$$pp \to t\bar{t}jj \qquad pp \to W^+W^-b\bar{b}$$

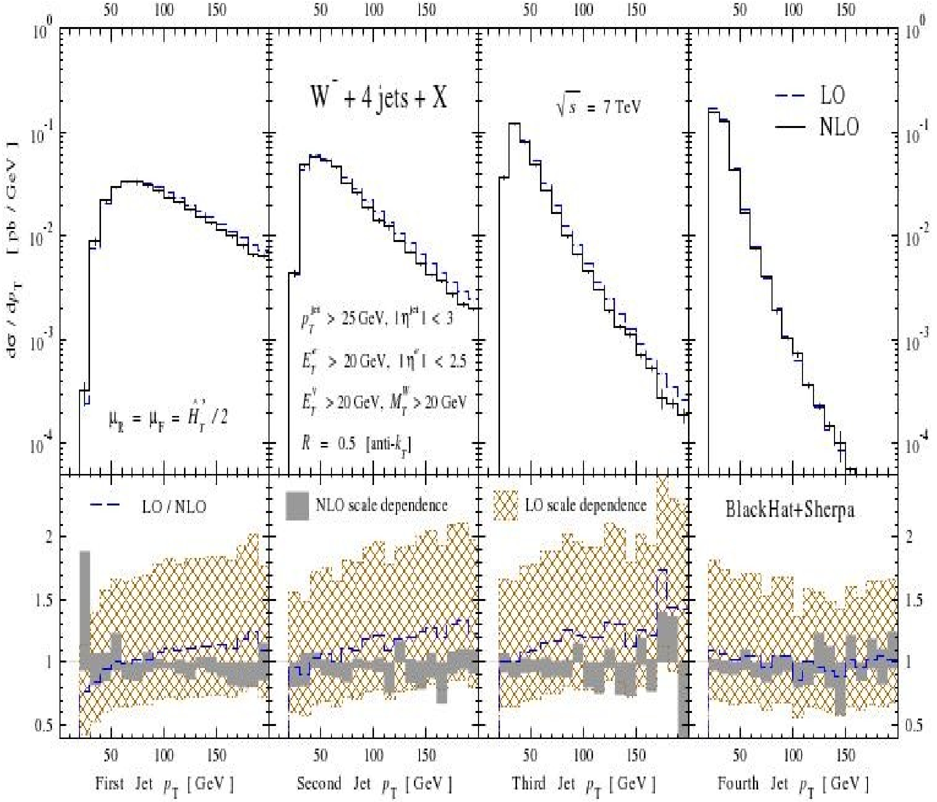
$$pp \to W(Z) + 4j \quad pp \to W^+W^+jj$$

Bern, Dixon, Kosower, Berger, Forde, Maitre, Febres-Cordero, Gleisberg, Papadopoulos, Ossola, Pittau, Czakon, Worek, Bevilacqua, Ellis, Kunszt, Giele, Zanderighi, Melia, Rountsh, Denner, Dittmaier, Pozzorini, Kallweit

W/Z + jets @ NLO

- D0 compares W+jets spectra with NLO QCD predictions
- Predictions for W+4jets at the LHC; transverse momenta distributions of four jets



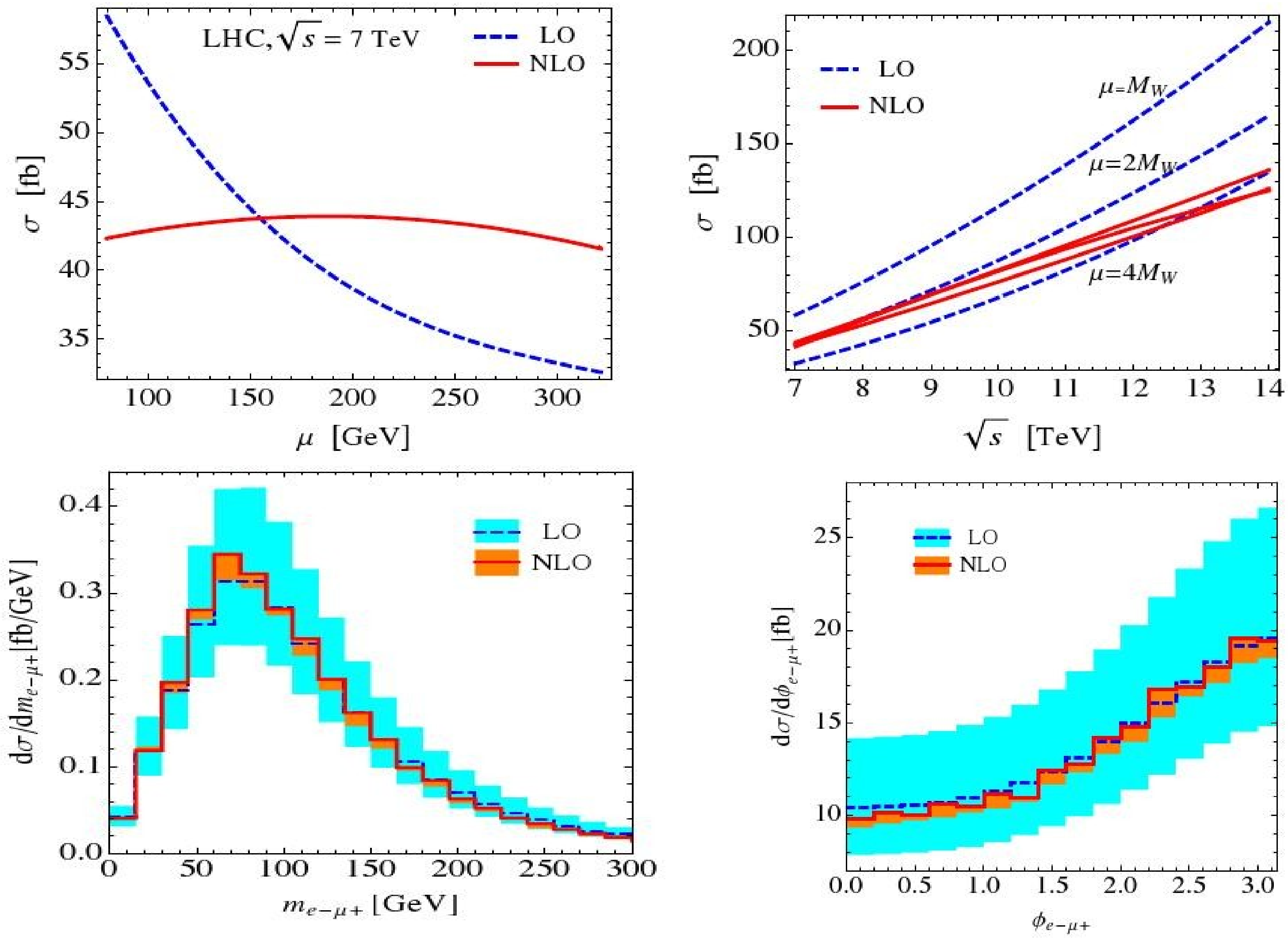


Blackhat + Sherpa collaboration

A computation of that complexity was unthinkable, just a few years ago

W+W-+jj @NLO

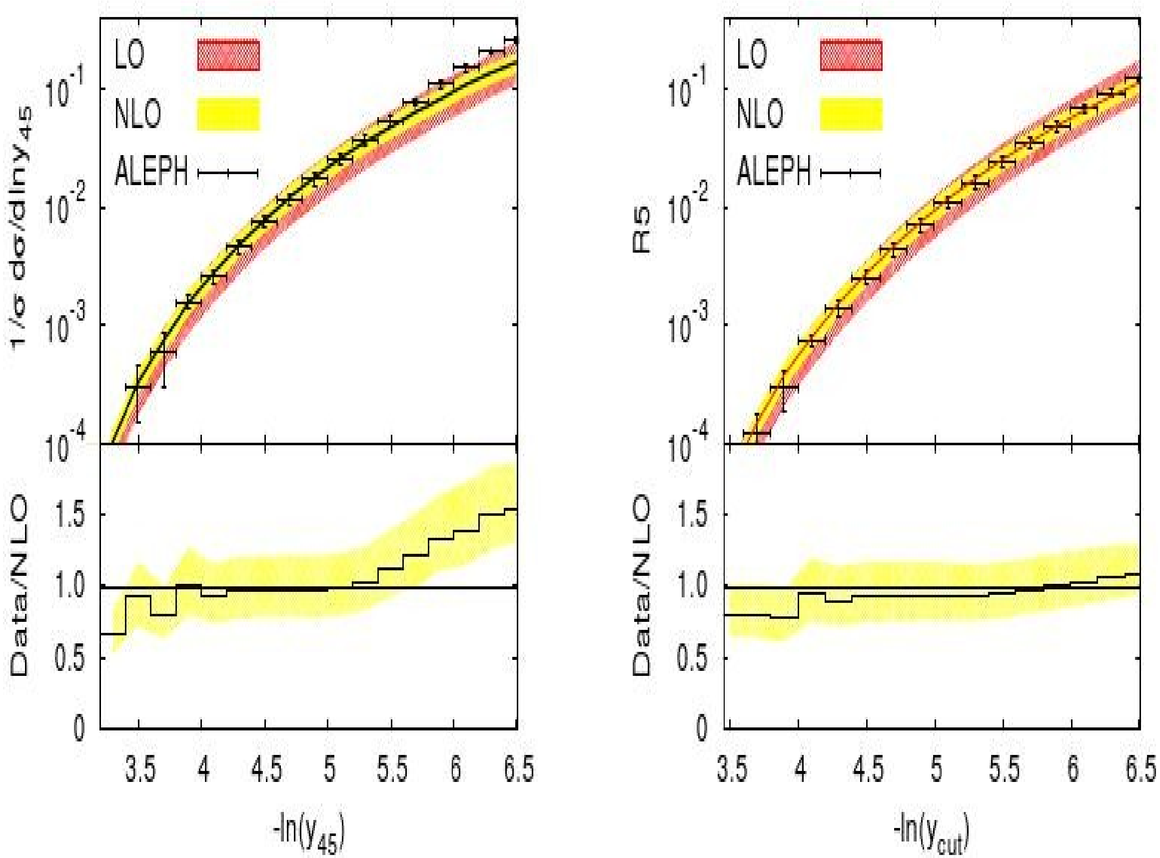
- The background process to Higgs searches in weak boson fusion

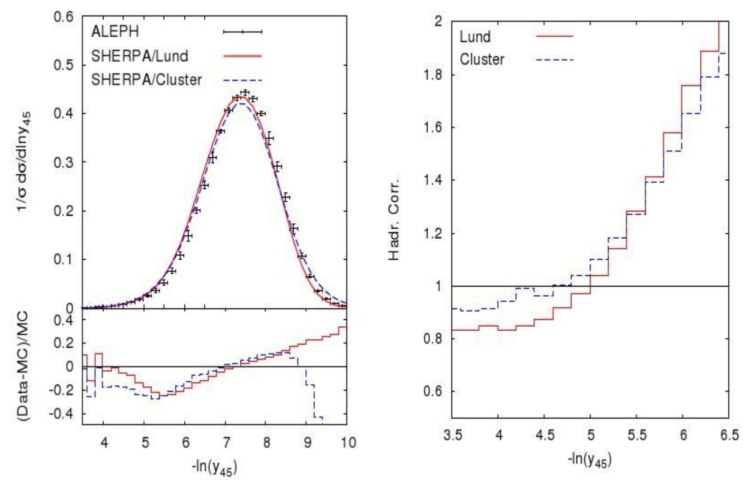


Melia, K.M., Rontsch, Zanderighi

e+e- → 5 jets at LEP

- Outside fo the hadron collider physics context – production of 5 jets at LEP.
- Highest exclusive jet multiplicity studied at LEP. Great sensitivity to the strong coupling constant. Small hadronization effects if done properly.





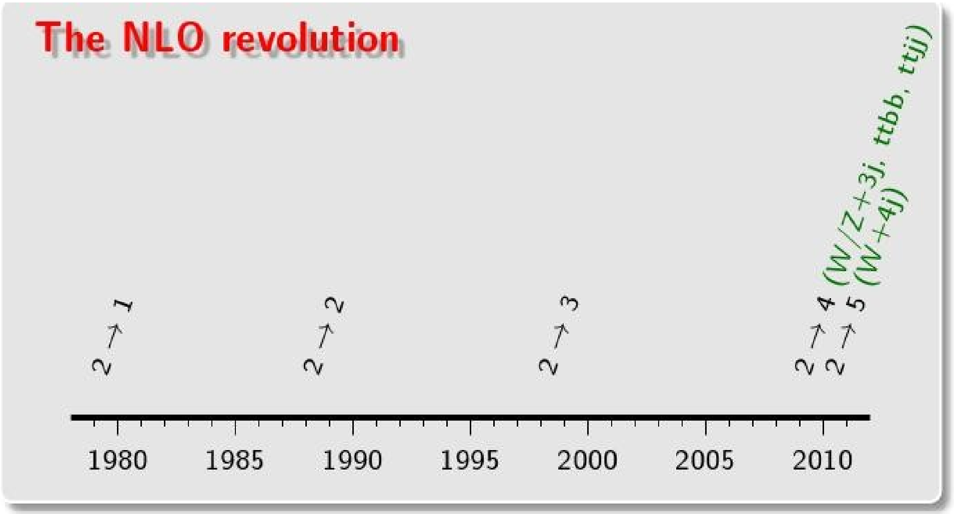
	LEP1, hadr. $\sigma_{\mathrm{tot}}^{-1}\mathrm{d}\sigma/\mathrm{d}y_{45}$, R_5	LEP1, no hadr. $\sigma_{\mathrm{tot}}^{-1}\mathrm{d}\sigma/\mathrm{d}y_{45}$, R_5
stat.	$+0.0002$ -0.0002	$+0.0002$ -0.0002
syst.	$+0.0027$ -0.0029	$+0.0027$ -0.0029
pert.	$+0.0062$ -0.0043	$+0.0068$ -0.0047
fit range	$+0.0014$ -0.0014	$+0.0005$ -0.0005
hadr.	$+0.0012$ -0.0012	–
$\alpha_s(M_Z)$	$0.1159^{+0.0070}_{-0.0055}$	$0.1163^{+0.0073}_{-0.0055}$

$$\alpha_s(M_Z) = 0.1156^{+0.0041}_{-0.0034}$$

Frederix, Frixione, Zandrighi, K.M.

Iintermediate summary: the NLO revolution

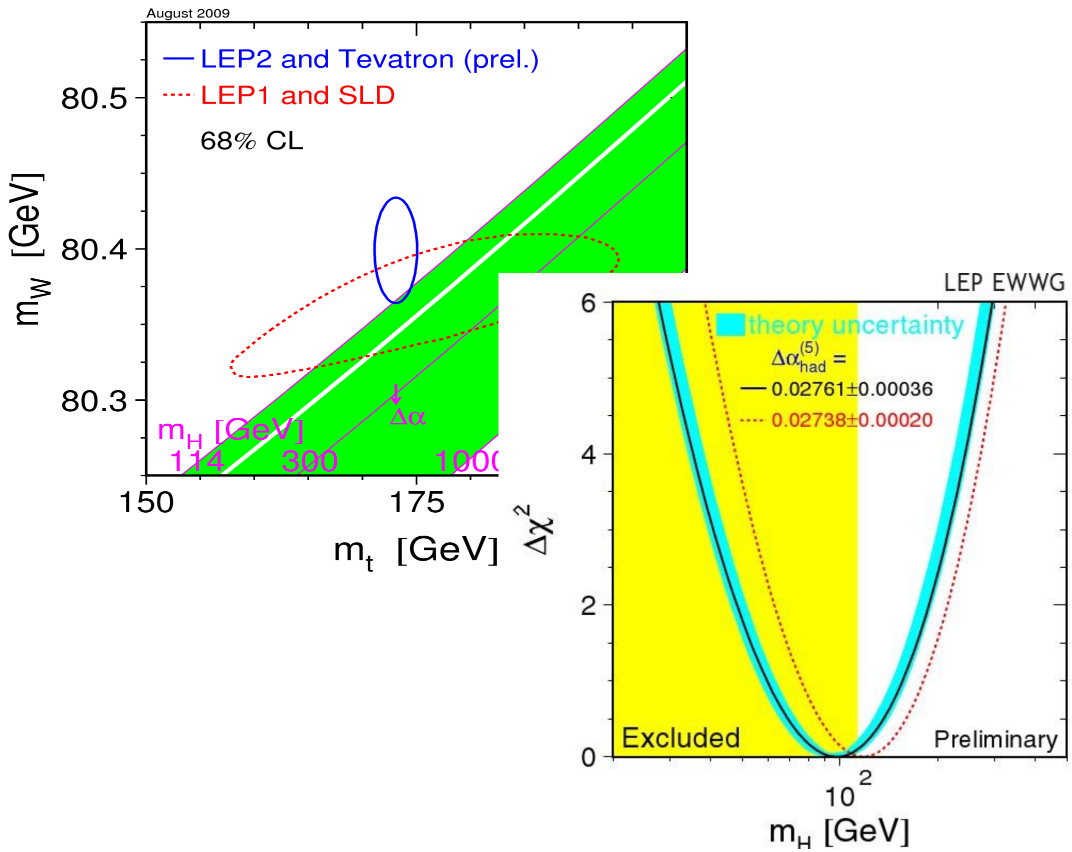
- Last three years – spectacular progress in NLO QCD computations for collider physics (mostly for the LHC and the Tevatron)
- Refinements of traditional diagrammatic techniques and new, unitarity-based technology, allowed us to obtain NLO QCD results for processes with large multiplicity final states for a variety of processes of importance for the LHC
- It appears that new methods for one-loop computations are sufficiently robust to allow automation, similar to what has happened with leading order computations at the end of 1990s. First results look very encouraging

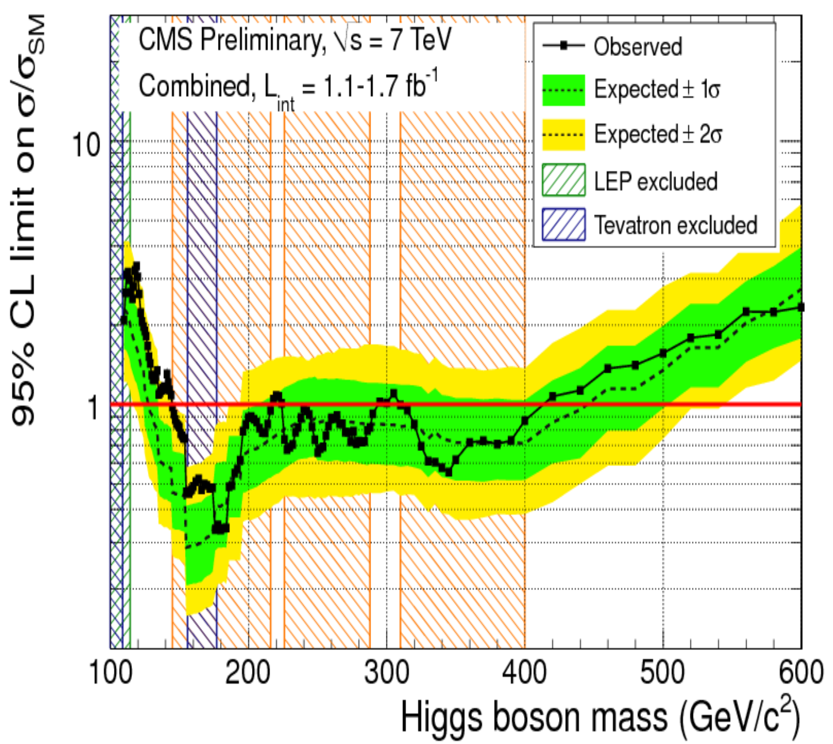


Process ($V \in \{Z, W, \gamma\}$)	Comments
Calculations completed since Les Houches 2005	
1. $pp \to VV$jet	WWjet completed by Dittmaier/Kallweit/Uwer [3]; Campbell/Ellis/Zanderighi [4] and Binoth/Karg/Kauer/Sanguinetti (in progress)
2. $pp \to$ Higgs+2jets	NLO QCD to the gg channel completed by Campbell/Ellis/Zanderighi [5]; NLO QCD+EW to the VBF channel completed by Ciccolini/Denner/Dittmaier [6,7]
3. $pp \to VVV$	ZZZ completed by Lazopoulos/Melnikov/Petriello [8] and WWZ by Hankele/Zeppenfeld [9]
Calculations remaining from Les Houches 2005	
4. $pp \to t\bar{t}b\bar{b}$	relevant for $t\bar{t}H$
5. $pp \to t\bar{t}$+2jets	relevant for $t\bar{t}H$
6. $pp \to VVb\bar{b}$,	relevant for VBF $\to H \to VV, t\bar{t}H$
7. $pp \to VV$+2jets	relevant for VBF $\to H \to VV$ VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld [10–12]
8. $pp \to V$+3jets	various new physics signatures
NLO calculations added to list in 2007	
9. $pp \to b\bar{b}b\bar{b}$	Higgs and new physics signatures
Calculations beyond NLO added in 2007	
10. $gg \to W^*W^*\ \mathcal{O}(\alpha^2\alpha_s^3)$	backgrounds to Higgs
11. NNLO $pp \to t\bar{t}$	normalization of a benchmark process
12. NNLO to VBF and Z/γ+jet	Higgs couplings and SM benchmark
Calculations including electroweak effects	
13. NNLO QCD+NLO EW for W/Z	precision calculation of a SM benchmark

Adding one order in pQCD : NNLO

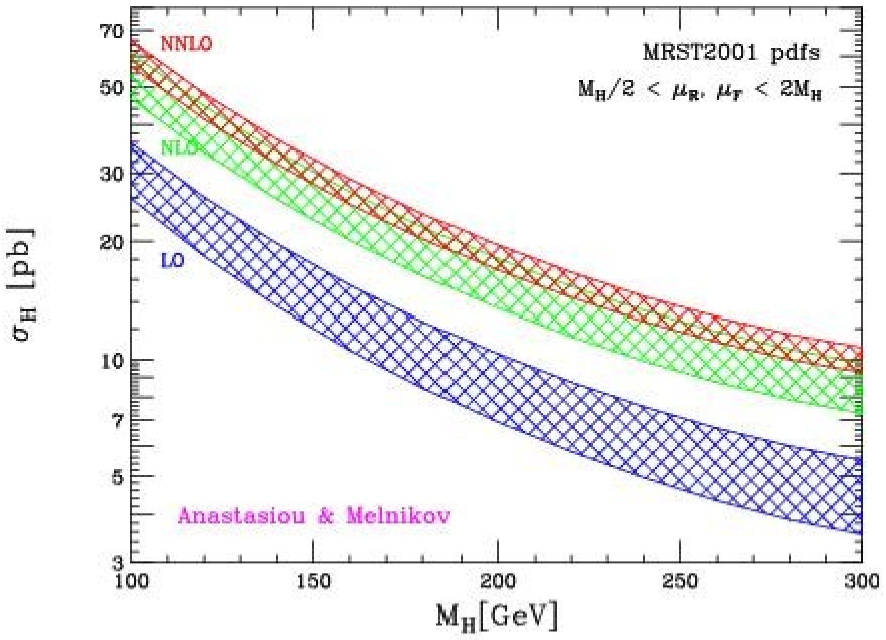
- There are cases when NLO computations are insufficient because of either achievable precision (W,Z production) or because NLO effects are large (Higgs production)
- In the former case – the NNLO QCD effects influence the Higgs boson exclusion indirectly, through the W-mass in precision electroweak; in the latter case – directly, by affecting production cross-sections

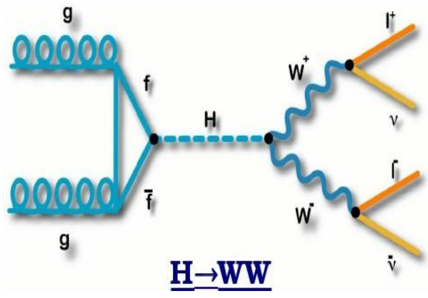




The search for the Higgs boson: pp → H → WW

- NNLO QCD corrections to this process, in the large top mass approximation, were computed nearly ten years ago. Both NLO and NNLO QCD effects are large.
- Usefulness of corrections to the total cross-section unclear
  - experimental results are divided into 0-jet, 1-jet, 2-jet bins
  - a cut on the transverse mass of the W-bosons is introduced to suppress the background
  - spin correlations of leptons are used to discriminate against the background



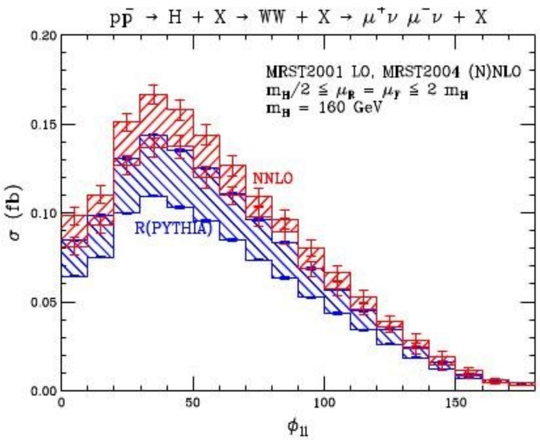


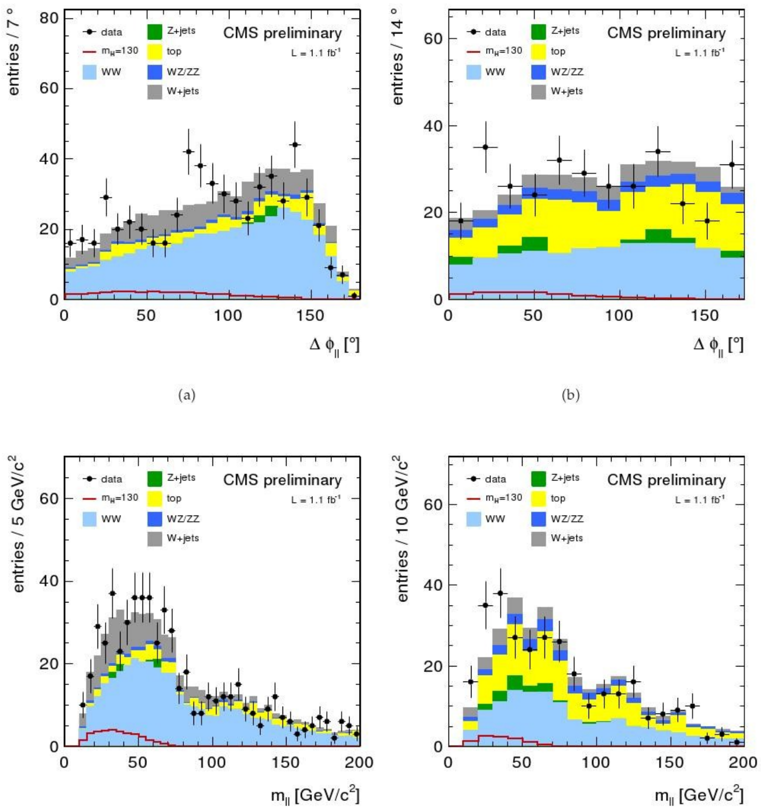
NNLO computations for unintegrated kinematics of the final state are required

The search for Higgs boson : pp → H → WW

- Such computations have been done; the results are used in the experimental analysis and allow us to draw serious conclusions (bump significance)

$\sigma_{acc}/\sigma_{incl}$	Trigger	+ Jet-Veto	+ Isolation	All Cuts
NNLO ($\mu = m_H/2$)	44.7%	39.4% (88.1%)	36.8% (93.4%)	27.8% (75.5%)
NNLO ($\mu = 2\, m_H$)	44.9%	41.8% (93.1%)	40.7% (97.4%)	31.0% (76.2%)
MC@NLO ($\mu = m_H/2$)	44.4%	38.1% (85.8%)	35.3% (92.5%)	26.5% (75.2%)
MC@NLO ($\mu = 2\, m_H$)	44.8%	38.8% (86.7%)	35.9% (92.5%)	27.0% (75.2%)
HERWIG	46.7%	40.8% (87.4%)	37.8% (92.7%)	28.6% (75.7%)
PYTHIA	46.6%	37.9% (81.3%)	32.2% (85.0%)	24.4% (75.8%)

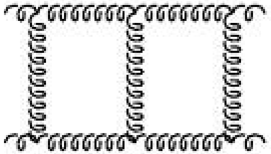


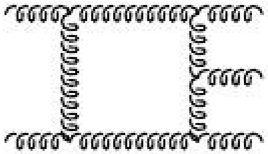


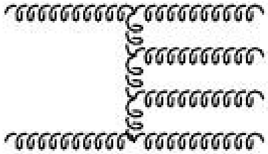
Anastasiou, Dissertori, Grazzini, Stoeckli, Webber

Anatomy of NNLO

- We have flexible tools to describe 2 → 1 processes (pp → W, pp → H) through NNLO in perturbative QCD. We would like to extend those results to cover 2 → 2 processes as well.
- For a variety of reasons, we may be interested in pp → jj, pp → tt, pp → Zj, pp → Hj etc.
- Some 2 → 2 processes, such as pp → W+W- and pp → gamma gamma do not require the full power of the NNLO technology
- How far are we from first physics results on 2 → 2 scattering @ NNLO ?
- For 2 → 2 @ NNLO we require
  - 2 → 2 scattering amplitudes for at two loops
  - 2 → 3 scattering amplitudes @ one-loop, integrated over the phase-space of the unresolved parton
  - 2 → 4 scattering amplitude integrated over the phase-space of two unresolved partons







Large number of 2 → 2 scattering amplitudes at two-loops is available since 2001, we definitely can compute 2 → 3 amplitudes at NLO and clearly 2 → 4 scattering amplitudes for most basic processes are well-known – so what is the problem ?

Why NNLO is non-trivial if loop contributions are known?

- The reason we have not done that are infra-red / collinear divergencies

$$\mathrm{d}\sigma \sim \mathrm{d}\sigma_{VV} + \mathrm{d}\sigma_{RV} + \mathrm{d}\sigma_{RR}$$

- Each of these contributions leave in a different phase-space and each is infra-red divergent. They must be combined before numerical integration is attempted, but how to do this efficiently is unknown – it is a matter of active research
- Two main lines of thought

Subtractions;

NLO analog: Catani-Seymour

Applied at NNLO to e+e- $\rightarrow$ 3j

Sector decomposition

NLO analog: FKS

Applied to pp $\rightarrow$ H, pp $\rightarrow$ W,Z

- Subtractions terms are (still) very difficult to construct
- Sector decomposition – difficult to keep phase-space parametrization ``local", i.e. original applications of sector decompositions attempted to find nice global parametrization of the final state particles phase-space

Sector decomposition and FKS

- The approach to NLO computations by Frixione, Kunszt and Signer (FKS) is an efficient procedure to deal with infra-red divergencies at NLO. It is based on two simple observations
  - a phase-space for N+1 final states particles that contributes to a N-jet observable can be partitioned into sectors in such a way that, at any sector, one and only one identified particle can become soft or at most two identified particles can become collinear;
  - for each such sector, a phase-space parametrization that trivializes extraction of singularities, is obvious
- A recent suggestion to apply similar considerations to NNLO computations seems very promising !

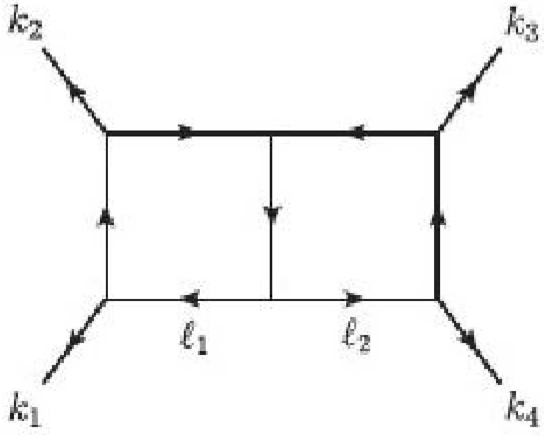
  Czakon

  - pre-partitioning of the phase-space
  - choice of a suitable parametrization in each of the pre-sectors
  - sector decomposition and the extraction of singular limits

NNLO beyond 2 → 2 ?

- If the program of developing suitable infra-red/collinear divergences extraction algorithms succeeds, it will be very general and, in fact, applicable to higher multiplcities
- The bottleneck for getting physics will be the two-loop high-multiplicity diagrams (2 → 3 and higher)
- Does the OPP/generalized unitarity algorithm for tensor reduction exist in higher-loops?

$$P^{**}_{2,2}[f(\ell_1,\ell_2)] = \int \frac{d^D\ell_1}{(2\pi)^D}\frac{d^D\ell_2}{(2\pi)^D}\frac{f(\ell_1,\ell_2)}{\ell_1^2(\ell_1-k_1)^2(\ell_1-K_{12})^2(\ell_1+\ell_2)^2\ell_2^2(\ell_2-k_4)^2(\ell_2-K_{34})^2}$$



Very first studies of the maximal cut for a 2->2 two-loop box diagram appeared recently

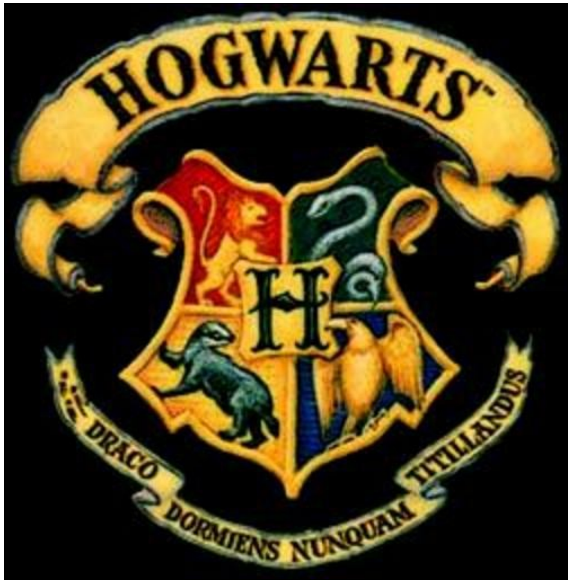
Mastrolia, Ossola, Kosower, Larsen

Conclusions

- During the past ten years the field of pQCD computations for hadron collider phsyics went through a remarkable transformation
- A unitarity-based paradigm for NLO computations has been developed; using amplitudes (i.e. only physical degrees of freedom) and produces tangible physics results that were unthinkable before
- Automation of NLO computations appears to be within reach
- NNLO results for fully differential computations became a reality and are heavily used in the experimental studies
- Parton showers are combined with NLO QCD computations and with high-multipliticy leading order computations

Conclusions

- Many advances were driven by simple ideas
  - Berend-Giele recursion
  - Integration by parts, Laporta algorithm
  - Asymptotic expansions
  - Sector decomposition
  - Britto-Cachazo-Feng-Witten
  - OPP
- Highly non-trivial ideas are being developed in the context of scattering amplitude in N=4 SYM
  - new symmetries
  - recursion relations for the integrands
  - General solutions for tree-level scattering amplitudes
  - twistors and other unusual mathematical structures



Can any of these ideas turn into something that is useful in practice?

Conclusions

- Top quark forward-backward asymmetry
- Feature in Wjj
- Demise of the CKM
- Proton charge radius in muonic hydrogen
- Muon anomalous magnetic moment