Results of numerical simulations for pair production of unstable particles in MPT in NNLO

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The problem:

A description of productions and decays of fundamental unstable particles
for colliders subsequent to LHC ($\Longrightarrow$ ILC)
generally should be made with NNLO accuracy

(i) gauge cancellations and unitarity;
(ii) enough high accuracy of computation of resonant contributions

Existing methods:

- Pole expansion/DPA: Laurent expansion around complex poles + conventional PT for residues / LEP1, LEP2 /
- Complex mass scheme (CMS): complex-valued renormalized mass $\Rightarrow$ complex-valued Weinberg angle etc. / A.Denner, S.Dittmaier, M.Roth, etc. /
- Pinch-technique method $\Rightarrow$ huge volume of extra calculations /J.Papavassiliou, A.Pilaftsis, D.Binosi, etc. /

NLO

Modified perturbation theory (MPT):

direct expansion of the cross-section *in powers of the coupling constant* with the aid of distribution-theory methods

Asymptotic expansion in α $\Rightarrow$ gauge invariance should be maintained
The accuracy of description of resonant contributions = ?

To clear up this question, I do numerical simulation in the MPT up to the NNLO

/ M.Nekrasov, Mod.Phys.Lett. A 26 (2011) 223,1807 /

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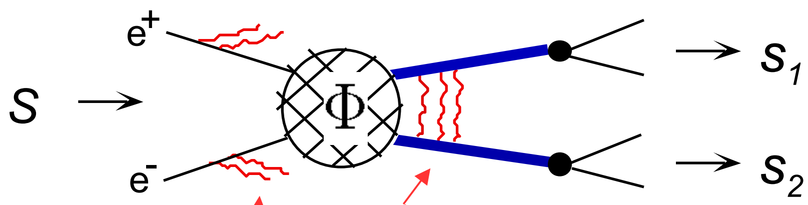
Pair production and decay, double-resonant contributions



$$\sigma(s) = \int_{s_{\min}}^{s} \frac{\mathrm{d}s'}{s}\, \phi(s'/s; s)\, \hat{\sigma}(s'), \qquad \hat{\sigma}(s) = \iint \mathrm{d}s_1\, \mathrm{d}s_2\;\; \hat{\sigma}(s\,; s_1, s_2)\,(1+\delta_c)$$

$$\hat{\sigma}(s\,; s_1, s_2) = \frac{1}{s^2}\, \theta(\sqrt{s} - \sqrt{s_1} - \sqrt{s_2}\,)\, \sqrt{\lambda(s, s_1, s_2)}\; \Phi(s; s_1, s_2)\, \rho(s_1)\, \rho(s_2)$$

Expansion in powers of α
in the sense of distributions

$$\rho(s_i) = \frac{M_i \Gamma_i}{\pi} \frac{1}{\left|s_i - M_i^2 + \alpha\, \Sigma(s_i)\right|^2}$$

Basic ingredients of MPT

- Asymptotic expansion of BW factors in powers of α   / F.Tkachov,1998 /

$$\rho(s) = \frac{M\Gamma_0}{\pi}\,\frac{1}{|s - M^2 + \Sigma(s)|^2} = \delta(s - M^2) + PV\,\mathcal{T}\,[\,\rho(s)\,] + \sum_n c_n(\alpha)\,\delta^{(n)}(s - M^2)$$

Taylor in α

Polynomial in α

NNLO :

3-loop

$$= \delta(s - M^2) + \frac{M\Gamma_0}{\pi}\left[PV\frac{1}{(s-M^2)^2} - PV\,\frac{2\alpha\,\mathrm{Re}\Sigma_1(s)}{(s - M^2)^3}\right] + \sum_{n=0}^{2} c_n(\alpha)\,\delta^{(n)}(s - M^2) + O(\alpha^3)$$

- Analytic regularization of the kinematic factor   / M.Nekrasov,2007 /

$$\sqrt{\lambda(s, s_1, s_2)} \longrightarrow \lim_{\nu \to 1/2} \left\{\lambda(s, s_1, s_2)\right\}^{\nu}$$

analytic calculation of "singular" integrals

- Conventional-perturbation-theory for "test" function Φ

Coefficients $c_n(\alpha)$

NNLO:

$$\rho(s) = \delta(s-M^2) + \frac{M\Gamma_0}{\pi}\left[PV\frac{1}{(s-M^2)^2} - PV\frac{2\alpha\,\mathrm{Re}\Sigma_1(s)}{(s-M^2)^3}\right] + \sum_{n=0}^{2} c_n(\alpha)\,\delta^{(n)}(s-M^2)$$

$$I_n = \mathrm{Im}\Sigma_n(M^2)\,,\quad I'_n = \mathrm{Im}\Sigma'_n(M^2)\,,\ \cdots$$

$$R_n = \mathrm{Re}\Sigma_n(M^2)\,,\quad R'_n = \mathrm{Re}\Sigma'_n(M^2)\,,$$

$$\Sigma = \alpha\Sigma_1 + \alpha^2\Sigma_2 + \alpha^2\Sigma_3$$

OMS conventional : $R_n = R'_n = 0$

$$c_0 = -\alpha\frac{I_2}{I_1} + \alpha^2\left(\frac{I_2^2}{I_1^2} - \frac{I_3}{I_1} - \frac{1}{2}I_1 I_1''\right)$$

$$c_1 = -\alpha^2\,(I_1 I_1')\,, \qquad c_2 = -\alpha^2 I_1^2$$

$\overline{\text{OMS}}$: (pole scheme) / M.Nekrasov, 2002 / / B.Kniel & A.Sirlin, 2002 /

$$R_1 = R'_1 = 0\,, \qquad R_2 = -I_1 I'_1\,, \quad R'_2 = -I_1 I''_1/2$$

$$c_0 = -\alpha\frac{I_2}{I_1} + \alpha^2\left[\frac{I_2^2}{I_1^2} - \frac{I_3}{I_1} - (I'_1)^2\right]$$

$$c_1 = 0, \qquad c_2 = -\alpha^2 I_1^2$$

M = pole mass,
gauge-invariant and scheme-independent
(observable mass)

Unitarity: $\alpha I_1 = M\Gamma_0\,,\qquad \alpha^2 I_2 = M\alpha\Gamma_1\,,\qquad \alpha^3 I_3 = M\alpha^2\Gamma_2 + \Gamma_0^3/(8M)$

$$\Gamma = \Gamma_0 + \alpha\Gamma_1 + \alpha^2\Gamma_2 + \cdots$$

Singular integrals, scheme of calculations

Dimensionless variables

$s \to x$, $s_i \to x_i$

$$\sqrt{s} = 2M + \frac{M}{2}x\,, \quad \sqrt{s_i} = M_i + \frac{M}{2}x_i \qquad M \equiv \frac{M_1+M_2}{2}$$

$$\hat{\sigma}(x) = \iint \mathrm{d}x_1\,\mathrm{d}x_2\;(x-x_1-x_2)_+^{\nu}\;\rho(x_1)\,\rho(x_2)\;\Phi(x\,;x_1,x_2)$$

$$\left\{PV\frac{1}{x_1^{n_1}},\;\delta^{(n_1-1)}(x_1)\right\}\quad\left\{PV\frac{1}{x_2^{n_2}},\;\delta^{(n_2-1)}(x_2)\right\}$$

$$x^k\,PV\frac{1}{x^n} = \frac{1}{x^{n-k}}$$

$$x^k\,\delta^{(n-1)}(x) \sim \delta^{(n-k-1)}(x)$$

$$0 \le k < n$$

at given n_1 and n_2 :

$$\Phi(x\,;x_1,x_2) = \sum_{k_1=0}^{n_1-1}\sum_{k_2=0}^{n_2-1}\frac{x_1^{k_1}}{k_1!}\frac{x_2^{k_2}}{k_2!}\,\Phi^{(k_1,k_2)}(x\,;0,0) + \Delta\Phi(x\,;x_1,x_2)$$

at $\nu = 1/2$

$$A^{\nu}_{l_1 l_2}(x) = \iint \mathrm{d}x_1\,\mathrm{d}x_2\,(x-x_1-x_2)_+^{\nu}\,\delta^{(l_1-1)}(x_1)\,\delta^{(l_2-1)}(x_2) \sim (x)_+^{5/2-l_1-l_2} + \text{‘reg’}$$

$$B^{\nu}_{l_1 l_2}(x) = \iint \mathrm{d}x_1\,\mathrm{d}x_2\,(x-x_1-x_2)_+^{\nu}\,PV\frac{1}{x_1^{l_1}}\,\delta^{(l_2-1)}(x_2) \sim (-x)_+^{5/2-l_1-l_2} + \text{‘reg’}$$

$$C^{\nu}_{l_1 l_2}(x) = \iint \mathrm{d}x_1\,\mathrm{d}x_2\,(x-x_1-x_2)_+^{\nu}\,PV\frac{1}{x_1^{l_1}}\,PV\frac{1}{x_2^{l_2}} \sim (x)_+^{5/2-l_1-l_2} + \text{‘reg’}$$

$$\sigma(x) = \int \mathrm{d}x'\;\phi(x',x)\hat{\sigma}(x')$$

$$\int \mathrm{d}x\;x_+^{\nu}\,\varphi(x) \underset{def}{=} \int_0^{\infty}\mathrm{d}x\;x^{\nu}\left\{\varphi(x) - \sum_{k=0}^{N-1}\frac{x^k}{k!}\,\varphi^{(k)}(0)\right\}$$

$$-N-1 < \mathrm{Re}\,\nu < -N$$

Numerical calculations & estimate of errors

- Fortran code with double precision
- Simpson method for calculating absolutely convergent integrals (relative accuracy $\delta_0 = 10^{-5}$)
- Linear patches for resolving 0/0-indeterminacies (x/x, x^2/x^2, …)

additional errors:

δ_1: due to patches themselves ⇒ $\delta_1 \sim \varepsilon^2 \varphi''_0/\varphi_0$

ε - actual size of the patch

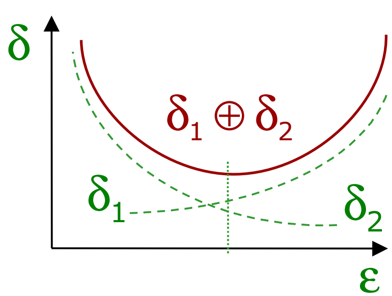
integrand

δ_2: due to the loss of decimals near indeterminacy points:

x/x: $\dfrac{f(x)-f(0)}{x} \Rightarrow \dfrac{\varepsilon f'(0)}{\varepsilon}$ $\varepsilon = 10^{-N}$ $\delta_2 \sim 10^{-(D-N)} \dfrac{f_0}{f'_0}$

x^2/x^2: $\dfrac{f(x)-f(0)-xf'(0)}{x^2} \Rightarrow \dfrac{\varepsilon^2 f''(0)/2}{\varepsilon^2}$ $\varepsilon^2 = 10^{-N}$ $\delta_2 \sim 10^{-(D-N)} \dfrac{2f_0}{f''_0}$

…



minimization of errors $\delta_1 \oplus \delta_2$ (numerical estimate) ⇒ $N = 8$ at $D = 15$ ← double precision

Overall error: $\delta = \delta_0 \oplus \delta_1 \oplus \delta_2 < 10^{-3}$ ← NNLO

Specific models for testing MPT

- Test function Φ :

$$e^+e^- \to (\gamma, Z) \to t\bar{t} \to W^+b\, W^-\bar{b}$$

$$e^+e^- \to (\gamma, Z) \to W^+W^- \to 4f$$

Born approximation

- Breigt-Wigner factors :

$$\Sigma = \alpha\Sigma_1 + \alpha^2\Sigma_2 + \alpha^3\Sigma_3$$ ⟵ three-loop contributions to self-energy

- Universal soft massless-particles contributions :

Flux function in leading-log approximation:

$$\phi(z;s) = \beta_e(1-z)^{(\beta_e-1)} - \frac{1}{2}\beta_e(1+z)\,, \qquad \beta_e = \frac{2\alpha}{\pi}\left(\ln\frac{s}{m_e^2} - 1\right)$$

Coulomb singularities through one-gluon/photon exchanges:

$$\delta_c = \kappa\,\frac{\alpha_s\pi}{2\beta}\left[1 - \frac{2}{\pi}\arctan\left(\frac{|\beta_M|^2-\beta^2}{2\beta\,\mathrm{Im}\beta_M}\right)\right]$$

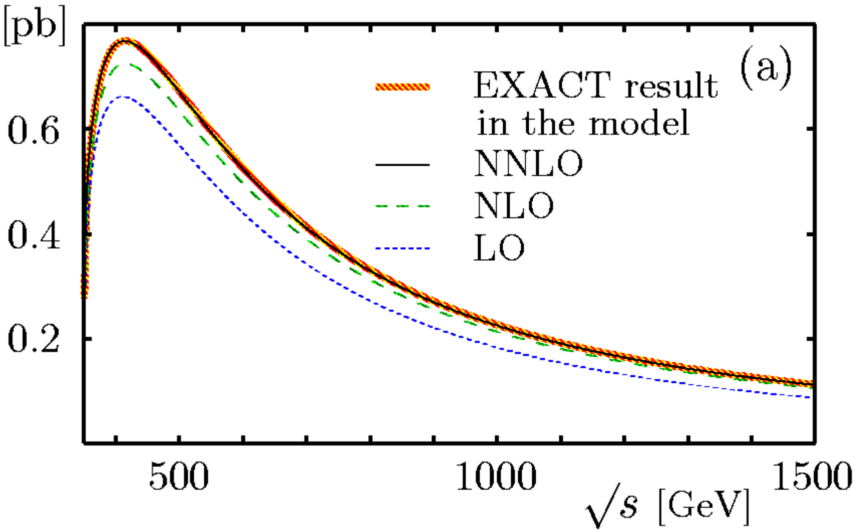
$$\beta = s^{-1}\sqrt{\lambda(s,s_1,s_2)}$$
$$\beta_M = \sqrt{1-4(M^2-\mathrm{i}M\Gamma)/s}$$

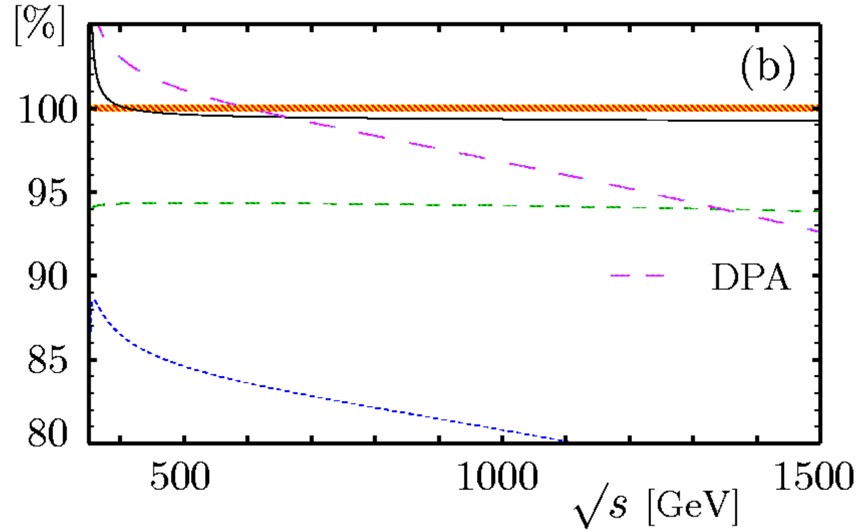
Results of calculations. The case of top-quarks

Total cross-section σ(s) :

Mt = 175 GeV

MW = 80.4 GeV   Mb = 0

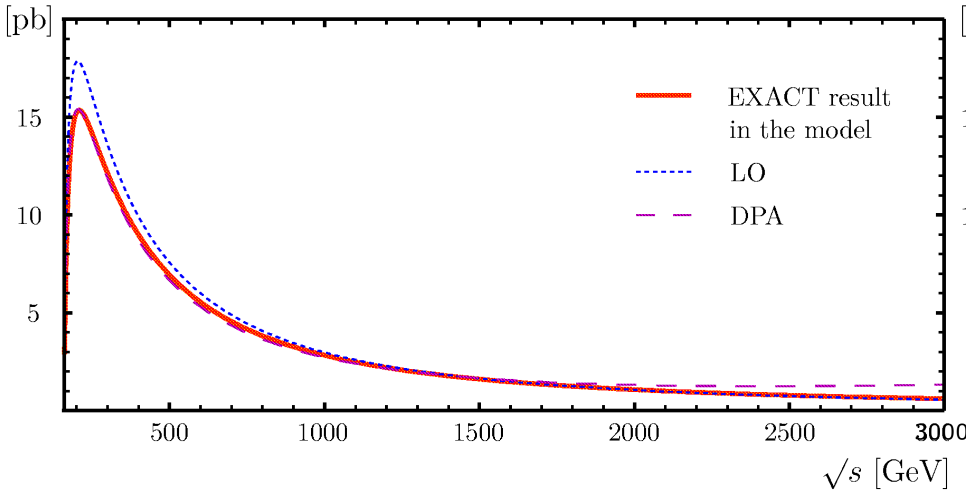


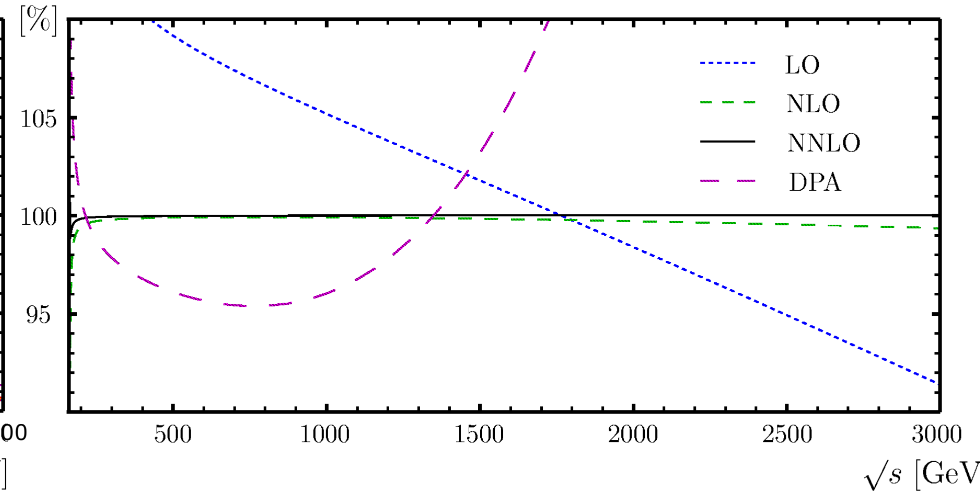


√s [GeV]	σ[pb]	σ_{LO}	σ_{NLO}	σ_{NNLO}
500	0.6724 100%	0.5687 84.6%	0.6344 94.3%	0.6698 99.6%
1000	0.2255 100%	0.1821 80.8%	0.2124 94.2%	0.2240 99.3%
1500	0.1122 100%	0.0867 77.3%	0.1053 93.8%	0.1113 99.2%

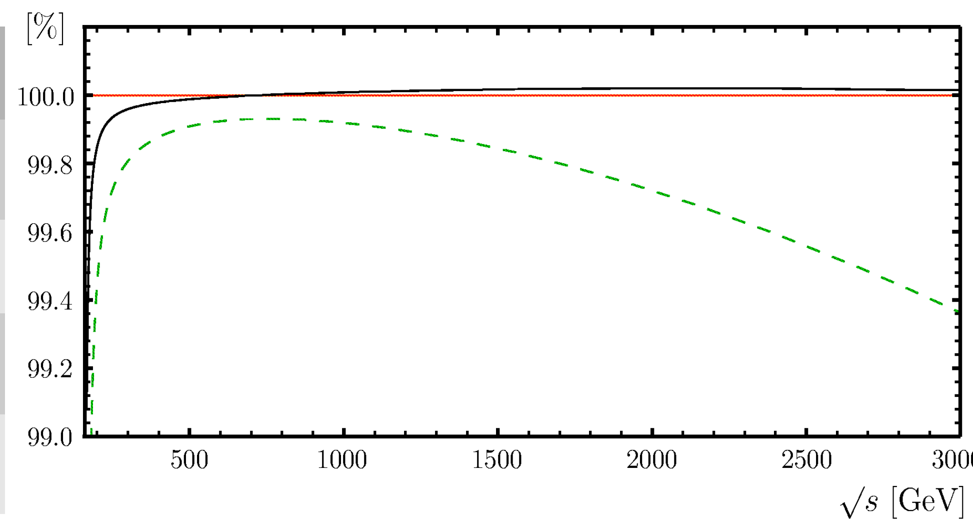
Results of calculations. The case of W-bosons

Total cross-section σ(s) : MW = 80.40 GeV MZ = 91.19 GeV





√s	σ[pb]	σLO	σNLO	σNNLO
200	15.258 100%	17.839 116.92%	15.175 99.46%	15.235 99.85%
500	6.9355 100%	7.5657 109.09%	6.9294 99.91%	6.9342 99.98%
1000	2.8286 100%	2.9733 105.12%	2.8263 99.92%	2.8285 100.00%
3000	0.61023 100%	0.55733 91.33%	0.60625 99.35%	0.61026 100.00%



Conclusion

In the case of pair production and decays of unstable particles:

- The existence of MPT expansion practically has been shown
  (working FORTRAN code up to NNLO is presented)

- MPT stably works at the energies near the maximum of the cross-section and higher

- At ILC energies NNLO in MPT provides accuracy of
  - a few 0.1%, in the case of top quarks *)
  - less than 0.1%, in the case of W bosons

MTV is a good candidate for support at the ILC the pair production and decay of fundamental unstable particles

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*) The higher precision is possible if proceeding to NNNLO
or if using NNLO of MPT for calculation of loop contributions only,
on the analogy of actual practice of application of DPA