Localization of scalar fields on Branes with an Asymmetric geometries in the bulk

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Plan our talk

1. Description of the model of a minimal interaction of a gravity with scalar matter fields.

2. Full action up to quadratic order in fluctuations in a vicinity of a background metric.

3. Separation of equations for the physical degrees of freedom (a specially chosen gauge).

4. Scalar sector in gauge $\phi = 0$.

5. Branon mass spectrum in the theory with a potential ϕ^4 .

6. Asymmetric background solutions and the defect of the cosmological constant.

7. Conclusion and remarks.

Formulation of the model

$$X^A = \left(x^\mu, z\right), \quad x^\mu = \left(x^0, x^1, x^2, x^3\right), \quad \eta^{AB} = \mathrm{diag}\left(+, -, -, -, -\right)$$

$$S[g, \Phi] = \int d^5X \sqrt{|g|}\mathcal{L}(g, \Phi)$$

$$\mathcal{L} = \left\{-\frac{1}{2}M_*^3 R + \frac{1}{2}\partial_A\Phi\partial^A\Phi - V\left(\Phi\right)\right\}$$

$$R_{AB} - \frac{1}{2}g_{AB}R = \frac{1}{M_*^3}T_{AB}, \qquad D^2\Phi = -\frac{\partial V}{\partial\Phi}, \quad D^2 = D_C D^C$$

Conformally flat form $g_{AB} = A^2\left(z\right)\eta_{AB}$

$$T_{AB} = \partial_A\Phi\partial_B\Phi - g_{AB}\left(\frac{1}{2}\partial_C\Phi\partial^C\Phi - V\left(\Phi\right)\right)$$

EOM:

$$\left(\frac{A'}{A^2}\right)' = -\frac{\Phi'^2}{3M_*^3 A},$$

$$-2A^5 V(\Phi) = 3M_*^3\left(A^2 A'' + 2A(A')^2\right),$$

$$(A^3\Phi')' = A^5 \frac{\delta V}{\delta\Phi}.$$

only two of these equations are independent

Small fluctuations around the background metric:

coordinate transformation $X \to X = \tilde{X} + \tilde{\zeta}^A(X)$

$$\tilde{g}_{AB}(X) = g_{AB}(X) - \tilde{\zeta}^C_{,A} g_{CB}(X) - \tilde{\zeta}^C_{,B} g_{AC}(X) - g_{AB,C}(X)\tilde{\zeta}^C + O\left(\zeta^2\right)$$
$$= g_{AB}(X) - \tilde{\zeta}_{A;B} - \tilde{\zeta}_{B;A} + O\left(\tilde{\zeta}^2\right),$$

Let us define the fluctuations of the metric $h_{AB}(X)$ and the scalar field $\phi(X)$ on the background solutions of the equations of motion,

$$g_{AB}(X) = A^2(z)\left(\eta_{AB} + h_{AB}(X)\right); \quad \Phi(X) = \Phi(z) + \phi(X)$$

$$h_{5\mu} \equiv v_\mu, h_{55} \equiv S \quad \tilde{\zeta}_\mu = A^2\zeta_\mu, \tilde{\zeta}_5 = A\zeta_5,$$

infinitesimal gauge transformations:

$$h_{\mu\nu} \to h_{\mu\nu} - \left(\zeta_{\mu,\nu} + \zeta_{\nu,\mu} - \frac{2A'}{A^2}\eta_{\mu\nu}\zeta_5\right), \qquad v_\mu \to v_\mu - \left(\frac{1}{A}\zeta_{5,\mu} + \zeta'_\mu\right),$$

$$S \to S - \frac{2}{A}\zeta'_5, \qquad \phi \to \phi + \zeta_5\frac{\Phi'}{A},$$

action to quadratic order in fluctuations

The full action to the quadratic order represents the sum

$$\mathcal{L}_{(2)} = \mathcal{L}_h + \mathcal{L}_\phi + \mathcal{L}_S + \mathcal{L}_V$$

$$\sqrt{|g|}\mathcal{L}_h \equiv -\frac{1}{2}M_*^3A^3\left\{-\frac{1}{4}\,h_{\alpha\beta,\nu}h^{\alpha\beta,\nu} - \frac{1}{2}\,h^{\alpha\beta}_{,\beta}h_{,\alpha} + \frac{1}{2}\,h^{\alpha\nu}_{,\alpha}h^{\beta}_{\nu,\beta} + \frac{1}{4}\,h_{,\alpha}h^{,\alpha} + \frac{1}{4}h'_{\mu\nu}h'^{\mu\nu} - \frac{1}{4}h'^2\right\},$$

$$\sqrt{|g|}\mathcal{L}_\phi \equiv \frac{1}{2}A^3(\phi_{,\mu}\phi^{,\mu} - \phi'^2) - \frac{1}{2}\,A^5\frac{\delta^2V}{\delta\Phi^2}(\Phi)\phi^2 + \frac{1}{2}A^3\Phi'h'\phi,$$

$$\sqrt{|g|}\mathcal{L}_S \equiv \frac{1}{4}\left(-A^5VS^2+S\left(M_*^3A^3\left(h^{\mu\nu}_{,\mu\nu} - h^{,\mu}_{,\mu}\right)+M_*^3\left(A^3\right)'h'+2(A^3\Phi'\phi)'-4A^3\Phi'\phi'\right)\right),$$

$$\sqrt{|g|}\mathcal{L}_V \equiv -\frac{1}{8}M_*^3A^3v_{\mu\nu}v^{\mu\nu} + \frac{1}{2}v^\mu\left[-M_*^3A^3\left(h^{,\nu}_{\mu\nu} - h_{,\mu}\right)' + 2A^3\Phi'\phi_{,\mu} + M_*^3\left(A^3\right)'S_{,\mu}\right],$$

$$v_{\mu\nu} = v_{\mu,\nu} - v_{\nu,\mu},\; h = h_{\mu\nu}\eta^{\mu\nu}$$

Separation of equations for the physical degrees of freedom

$$h_{\mu\nu} = b_{\mu\nu} + F_{\mu,\nu} + F_{\nu,\mu} + E_{,\mu\nu} + \eta_{\mu\nu}\psi,$$

$b_{\mu\nu}$ and F_μ obey the relation $b^{;\mu}_{\mu\nu} = b = 0 = F^{;\mu}_{\mu}$

$$\zeta_\mu = \zeta^\perp_\mu + \partial_\mu C, \qquad \partial^\mu \zeta^\perp_\mu = 0; \qquad v_\mu = v^\perp_\mu + \partial_\mu \eta, \qquad \partial^\mu v^\perp_\mu = 0$$

vector fields are transformed as follows, $F_\mu \to F_\mu - \zeta^\perp_\mu, \quad v^\perp_\mu \to v^\perp_\mu - \zeta'^\perp_\mu,$

The scalar components:

η, E, ψ, S, ϕ change under gauge transformations in the following way,

$$\eta \to \eta - \frac{1}{A}\zeta_5 - C'; \qquad E \to E - 2C,$$

$$\psi \to \psi + \frac{2A'}{A^2}\zeta_5, \qquad S \to S - \frac{2}{A}\zeta_5', \qquad \phi \to \phi + \frac{\Phi'}{A}\zeta_5$$

using the parametrization we can calculate the components of the quadratic action,

$$h^{\mu\nu}_{,\mu\nu} - h^{;\mu}_{,\mu} = -3\Box\psi; \qquad h^{;\nu}_{\mu\nu} - h_{,\mu} = \Box F_\mu - 3\psi_{,\mu}$$

$$h \equiv h^\mu_\mu = \Box E + 4\psi; \quad h^{\alpha\beta}_{,\beta} = \Box(F^\alpha + E^{,\alpha}) + \psi^{,\alpha}; \quad h^{\alpha\beta}_{,\alpha\beta} = \Box^2 E + \Box\psi;$$

a partial separation of degrees of freedom,

$$
\begin{aligned}
\mathcal{L}_{(2)} \quad = \quad & \frac{1}{8}M_*^3A^3\ \left\{\ b_{\mu\nu,\sigma}b^{\mu\nu,\sigma} - (b')_{\mu\nu}(b')^{\mu\nu} - f_{\mu\nu}f^{\mu\nu}\right\} \\
& +\frac{3}{4}M_*^3A^3\ \left\{-\psi_{,\mu}\psi^{,\mu} + \psi_{,\mu}S^{,\mu} + 2(\psi')^2 + 4\frac{A'}{A}\psi'S\right\} \\
& +\frac{1}{2}A^3\ \left\{\phi_{,\mu}\phi^{,\mu} - (\phi')^2 - \ A^2\frac{\delta^2V}{\delta\Phi^2}\phi^2 - \frac{1}{2}A^2V(\Phi)S^2\right. \\
& \left. +4\Phi'\psi'\phi + S\left(-\Phi'\phi' + \frac{1}{2}A^2\frac{\delta V}{\delta\Phi}\phi\right)\right\} \\
& +\frac{3}{4}M_*^3A^3\ \Box(E' - 2\eta)\left(\frac{A'}{A}S + \psi' + \frac{2\Phi'}{3M_*^3}\phi\right),
\end{aligned}
$$

where $f_\mu \equiv F'_\mu - v^{\perp}_\mu, \quad f_{\mu\nu} \equiv f_{\mu,\nu} - f_{\nu,\mu}$

Now let's examine two gauge choices for scalar sector. The first one preserves the structure of perturbation theory, $S = 0$.The second one is defined by setting $\phi = 0$. In this case the branon field is described by ψ in a non-perturbative setting.

Scalar sector in gauge $\phi = 0$

$$\mathcal{L}_{(2)}(v_\mu = \phi = 0) = \mathcal{L}_b + \mathcal{L}_f + \mathcal{L}_{S,\psi},$$

$$\sqrt{|g|}\mathcal{L}_{\phi=0} = \frac{3}{4}M_*^3 A^3\Big(-\psi_{,\mu}\psi^{,\mu} + 2(\psi')^2 + \psi_{,\mu}S^{,\mu} + 4\frac{A'}{A}\psi' S + \frac{2(A')^2 + AA''}{2A^2}S^2\Big)$$

we used the identity $M_*^3(A^3)'' = -2A^5 V(\Phi)$ *and integration by parts*

For the normalization of the kinetic term it is useful to redefine the field

$$\psi = \Omega^{-1}\hat{\psi},$$

$$\sqrt{|g|}\mathcal{L}_\psi = \hat{\psi}_{,\mu}\hat{\psi}^{,\mu} - \hat{\psi}(-\partial_z^2 + V(z))\hat{\psi}, \qquad V(z) = \frac{\Omega''}{\Omega} \qquad \Omega^2 = \frac{A^5}{4}\frac{\Phi'^2}{A'^2}$$

This equation allows to calculate the mass spectrum of scalar branons

in gaussian normal coordinates $x_\mu, y,$

$$ds^2 = A^2(z)\left(dx_\mu dx^\mu - dz^2\right) = \exp\left(-2\rho(y)\right)dx_\mu dx^\mu - dy^2.$$

$$z = \int \exp\rho(y)\,dy, \qquad A(z) = \exp\left(-\rho(y)\right)$$

the action has a form,

$$S = \int d^5X \sqrt{|g|}\mathcal{L}_\psi = \int d^4x dy \exp(\rho) \left(\hat{\psi}_{,\mu}\hat{\psi}^{,\mu} - \exp(-2\rho)\hat{\psi}\left(-\partial_y^2 + \rho'\partial_y + V\right)\hat{\psi}\right)$$

For a canonical normalization of the kinetic term a field should be redefined, $\hat{\psi} = \exp(-\rho/2)\tilde{\psi}$. Then the action looks as,

$$S = \int d^5X \left(\tilde{\psi}_{,\mu}\tilde{\psi}^{,\mu} - \tilde{\psi}\hat{D}_y\tilde{\psi}\right),$$

$$\begin{aligned} \hat{D}_y &= \exp(-2\rho)\left(-\partial_y^2 + 2\rho'\partial_y - \frac{3}{4}\rho'^2 + \frac{1}{2}\rho''\right. \\ &\left. + \frac{\rho'}{\exp\left(-3\rho/2\right)\Phi'}\left(\partial_y^2 - \rho'\partial_y\right)\left(\frac{\exp\left(-3\rho/2\right)\Phi'}{\rho'}\right)\right) \end{aligned}$$

The spectrum of the branon:

$$\tilde{\psi}\left(X\right) = \sum_m \psi^{(m)}\left(x\right)\tilde{\psi}_m\left(y\right), \qquad \hat{D}_y\tilde{\psi}_m = m^2\tilde{\psi}_m, \qquad \int dz \tilde{\psi}_m\tilde{\psi}_{m'} = \delta_{m,m}$$

the replacement $\exp(-\rho)\tilde{\psi}_m = \Psi_m$

$$\left(-\partial_y^2 + V\left(y\right) - m^2 \exp(2\rho)\right) \Psi_m = 0,$$

$$V = \frac{1}{4}\rho'^2 - \frac{1}{2}\rho'' + \frac{\rho'}{\exp\left(-3\rho/2\right)\Phi'}\left(\partial_y^2 - \rho'\partial_y\right)\left(\frac{\exp\left(-3\rho/2\right)\Phi'}{\rho'}\right)$$

zero-mode in the potential $\tilde{V} = V - m^2 \exp(2\rho)$

These formulas allow to calculate the spectrum of quadratic fluctuations of the boson field minimally interacting to gravity!

Branon mass spectrum in the theory with potential ϕ^4

induced by five-dimensional fermions

$$S_{eff}(\Phi, g) = \frac{1}{2} M_*^3 \int d^5X \sqrt{|g|} \left\{ -R + 2\lambda + \frac{3\kappa}{M^2} \left(\partial_A \Phi \partial^A \Phi + 2M^2 \Phi^2 - \Phi^4 \right) \right\}$$

we assume that κ *is a small parameter, which characterize the interaction of gravity and matter fields*

use the warped metric in gaussian normal coordinates:

$$ds^2 = \exp\left(-2\rho\left(y\right)\right) dx_\mu dx^\mu - dy^2$$

system of three equations

$$\Phi'' = -2M^2\Phi + 4\rho'\Phi' + 2\Phi^3,$$

$$\rho'' = \frac{\kappa}{M^2}\,\Phi'^2, \qquad \lambda + 6\rho'^2 = \frac{3\kappa}{2M^2}\left\{\Phi'^2 + 2M^2\Phi^2 - \Phi^4\right\}$$

$$2M^2\frac{\lambda}{3\kappa} \equiv 2M^2\lambda_{eff} = 2M^2\Phi^2 - \Phi^4 + \Phi'^2 - \frac{4M^2\rho'^2}{\kappa}$$

five-dimensional λ
cosmological constant !

$$\frac{|\rho'(y)|}{M} = O(\kappa) = \frac{|\rho''(y)|}{M^2}$$

$$\Phi'' = -2M^2\Phi + 2\Phi^3 + O(\kappa) \qquad \frac{\rho''}{M^2} = \frac{\kappa}{M^4}\Phi'^2 + O(\kappa^2)$$

the metric is completely determined by matter !

$\Phi_0 = M\tanh(My) + O(\kappa)$ *and the conformal factor*

$$\rho_0(y) = \frac{2\kappa}{3}\left\{\ln\cosh(My) + \frac{1}{4}\tanh^2(My) + tMy\right\} + O(\kappa^2)$$

solutions to the asymmetric brane are possible, that corresponds to $t \neq 0$

In the case of a symmetric $t = 0$

$$V(y, t=0, \kappa=0) = M^2\left(4 + \frac{2}{\sinh^2(My)} + 8\frac{1 - 4\cosh^2(My)}{\left(1 + 2\cosh^2(My)\right)^2}\right)\Bigg|_{y\to 0} \sim \frac{2}{y^2}$$

a centrifugal barrier !

Numerical calculations show that at the leading order in the gravitational constant there are neither zero-modes, no resonances at $m^2 > 0$
And localized scalar states don't exist near a symmetric brane with potential !!!

case $t \sim 1$

In the main approximation in κ the potential with asymmetric brane :

$$V\,(u = My, t, \kappa = 0) = 2M^2\,\left(2 + \cdot \frac{-12(1+t^2)\cosh^4 u + 12t\cosh u \sinh u(1 - 2\cosh^2 u) + 24\cosh^2 u - 3}{4(1+t^2)\cosh^6 u + 4t\cosh^3 u \sinh u(1 + 2\cosh^2 u) - 3\cosh^2 u - 1}\right)$$

Numerical calculations show that at zero mass normalizable localized states don't arise, but localized states with nonzero mass arise when t> t min, t min = 0.21 !!!

They are resonances, since $V - m^2\exp(2\rho)$ exponentially decreases at infinity and the barrier is penetrable, although the probability of its penetration is very small.

Asymmetric background solutions and defect of cosmological constant

the exact asymptotics of the metric and the scalar field with $y \to \pm\infty$

$$\Phi_0 \to const = \Phi_\pm,$$

$$\rho \to k_\pm y, \quad k_\pm = \frac{2}{3} M\kappa(1 \pm t)$$

In the limit $y \to \pm\infty$ we obtain $\rho' \to k_\pm, \quad \rho'' \to 0, \quad \Phi_0' \to 0$

dimensionless parameters $\Phi_\pm = \varphi_\pm M, \quad k_\pm = M\bar{k}_\pm, \quad \lambda_\pm = M^2\bar{\lambda}_\pm$

$$2\bar{k}^2 + \frac{1}{3}\bar{\lambda} = \frac{\kappa}{2}\left(2\varphi^2 - \varphi^4\right), \qquad \varphi^2 = 1, \qquad |\varphi_+| = |\varphi_-|$$

For the existence of asymmetric geometries on both sides of the brane one must require that the parameter of curvature of $\bar{k}$ has a different modulo value at ∞ and $-\infty$, which is impossible with a constant $\bar{\lambda}$ and the asymptotics of the background scalar field, similar in magnitude. Thus, for potentials that are symmetric under the reflection $\Phi \to -\Phi$, an alignment a solution with an asymmetric geometry is impossible.

For different asymptotics one must introduce an asymmetry in the cosmological constant or break the symmetry under the reflection : $\Phi \to -\Phi$

$\mathcal{L}_{def} = 6\kappa M_*^3 M \eta(y)\Phi(X)$ *dimensionless function* $\eta \longrightarrow \eta_\pm \; y \to \pm\infty$

EOM with defect :

$$\Phi_0{}'' = -2M^2\Phi_0 + 4\rho'\Phi_0' + 2\Phi_0^3 + M^3\eta,$$

$$\rho'' = \frac{\kappa}{M^2}\,\Phi_0'^{\,2}, \qquad \lambda + 6\rho'^2 = \frac{3\kappa}{2M^2}\left\{\Phi_0'^{\,2} + 2M^2\Phi_0^2 - \Phi_0^4 - 2M^3\eta\Phi_0\right\}$$

the cosmological "constant" should depend on "y " so that the relation were satisfied on the solutions of the equations of motion

$$\frac{2M^2}{3\kappa}\lambda' + 2M^3\eta'\Phi_0 = \left((\Phi_0')^2 + 2M^2\Phi_0^2 - \Phi_0^4 - \frac{4M^2(\rho')^2}{\kappa}\right)' - 2M^3\eta\Phi' = 0$$

This is possible only if its (fixed) functional dependence of "y" coincides exactly with the solution $\Phi_0(y)$

$$\lambda(y) = \lambda_0 + 3\kappa M\int_0^y dy'\eta'(y')\Phi_0(y'), \qquad \lambda_0 = const$$

$$\lambda(y) \to \lambda_\pm$$

the defect is not a constant, $\eta'(y) \neq 0$ $\lambda(y) \to \lambda_\pm$ *easy to obtain:*

$$2\bar{k}^2 + \frac{1}{3}\bar{\lambda} = \frac{\kappa}{2}\left\{2\varphi^2 - \varphi^4 - 2\eta\varphi\right\}, \quad 0 = -2\varphi + 2\varphi^3 + \eta$$

The equation has three solutions, one of them $\varphi = 0$ *realizes an unstable state, it is the maximum. To calculate two other solutions, we assume* $\eta \ll 1$ *and then obtain :*

$$\varphi_\pm = \pm 1 - \frac{\eta_\pm}{4}$$

$$2\bar{k}_\pm^2 = \frac{\kappa}{2} - \frac{1}{3}\bar{\lambda}_\pm \mp \kappa\eta_\pm$$

It should be compared with: $\bar{k}_\pm = \frac{2}{3}\kappa(1 \pm t)$

The relations between the asymmetry parameter "t", the asymptotics of the defect and the cosmological function :

$$\begin{aligned} \frac{32}{9} t\kappa^2 &= \frac{1}{3}(\lambda_- - \lambda_+) - \kappa(\eta_+ + \eta_-), \\ \frac{16}{9}\kappa^2(1 + t^2) &= \kappa - \frac{1}{3}(\lambda_- + \lambda_+) + \kappa(\eta_- - \eta_+) > 0 \end{aligned}$$

The asymptotics of the defect of scalar matter and of the cosmological constant completely determine the asymmetry of the conformal factor and of the cosmological function

Conclusions

1) A model of domain wall ("thick brane") in the noncompact five-dimensional space-time with asymmetric geometries on both sides of the brane is generated by self-interacting fermions in the presence of gravity;

2)The asymmetric geometry in the bulk is provided by the asymmetry of scalar field potential and a corresponding defect of the cosmological constant;

3)The defect of matter fields is accompanied by a defect of the cosmological "constant" in order to ensure consistency of the equations of motion;

4) In the model with a minimal interaction of gravity and scalar fields for the symmetric anti-de Sitter geometry there are no localized states in the vicinity of the brane;

5) In the case of anti-de Sitter geometries asymmetric against reflection of the fifth coordinate such states occur;

6) There is only slowly decaying resonance when a conformal factor for anti-de Sitter spaces on both sides of the brane have different signs. This case is of a physical interest because the lifetime of the resonance is longer than the expected lifetime of the proton. It is expected that the tunneling probability is of order $\exp\{-3\ln\left(\frac{2M}{m}\right)/\kappa\}\ \kappa < 10^{-8}.$ *However, if the conformal factor starts to grow on one side of the brane, bound states may appear as zero-modes, but in this case there is a problem with localization of a massless graviton on the brane.*

THANK YOU !

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Multicomponent scalar field model with spontaneously broken translational symmetry

(V.A.,A.A. and O.O.Novikov)

$$\mathcal{L} = Z_s\Big(\frac{1}{2}\sum_{j=1}^{N}\partial_A\Phi_j\partial^A\Phi_j - V\left(\Phi_j\right)\Big), \quad V\left(\Phi_j\right) = -\sum_{j=1}^{N}\Delta_j\Phi_j^2 + \frac{1}{2}\Big(\sum_{j=1}^{N}\Phi_j^2\Big)^2$$

$$\partial_A\partial^A\Phi_j = 2\Big(\Delta_j - \sum_{l=1}^{N}\Phi_l^2\Big)\Phi_j$$

reduced to a Schrödinger-type equations,

$$\begin{aligned} H_N\Phi_{N,j} &\equiv \big(-\partial_y^2 + \mathcal{V}_N(y)\big)\Phi_{N,j} = 2(\Delta_{N,j} - \Delta_{N,1})\Phi_{N,j} \\ \mathcal{V}_N(y) &\equiv 2\sum_{l=1}^{N}\Phi_{N,l}^2(y) - 2\Delta_{N,1}, \end{aligned}$$

Integrability for multicomponent kink solution!

N = 2

(A.&V.Andrianovs, P.Giacconi, R.Soldati)

In order to provide the full basis of scalar fields as bounded and localized states in concordance with (4) one must correct the argument of hyperbolic functions $My \to \beta y, \beta = \sqrt{M^2 - \mu^2}$. It exists only if $\Delta_{2,1} = M^2 \geqslant \Delta_{2,2} = \frac{1}{2}(\mu^2 + M^2) \geqslant \frac{1}{2}M^2$. Accordingly,

$$\Phi_{2,1}(y) = M \tanh(\beta y); \quad \Phi_{2,2}(y) = \mu \frac{1}{\cosh(\beta y)}; \quad \mathcal{V}_2 = -2\beta^2 / \cosh^2(\beta y).$$

Localization of massive fermions on a brane

Consider two types of bispinors in order to localize light massive fermions on an asymmetric thick brane

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$

$$\mathcal{W}_f = \int d^5X\,\bar{\Psi}[(i\gamma^\mu\partial_\mu + \gamma_5\partial_y)\mathbf{I} - \hat{\Phi}(y))]\Psi,$$

$$\hat{\Phi} \equiv \sum_{a=1,\ldots,4} \Phi_a(y)T_a, \qquad (T_a) \equiv (\tau_3, \tau_2, \mathbf{I}, \tau_1),$$

Nontrivial configurations of Φ_2 and Φ_4

eventually lead to CP breaking in the Yukawa vertices $\overline{\Psi}\,\hat{\Phi}(x)\,\Psi$ **on a brane**

expansions of left- and right-handed components of fermions,

$$\Psi(x,y) = \sum_n \left(\psi_{\mathrm{L}}^{(n)}(x)\mathbf{F}_{\mathrm{L}}^{(n)}(y) + \psi_{\mathrm{R}}^{(n)}(x)\mathbf{F}_{\mathrm{R}}^{(n)}(y)\right)$$

$$\mathbf{F}_{\mathrm{L(R)}}^{(n)}(y) \equiv \begin{pmatrix} F_{1,\mathrm{L(R)}}^{(n)}(y) \\ F_{2,\mathrm{L(R)}}^{(n)}(y) \end{pmatrix} \qquad i\gamma^\mu\partial_\mu\psi_{\mathrm{L}}^{(n)} = m_n^*\psi_{\mathrm{R}}^{(n)}, \qquad i\gamma^\mu\partial_\mu\psi_{\mathrm{R}}^{(n)} = m_n\psi_{\mathrm{L}}^{(n)},$$

Mass spectrum equations

$$\partial_y \mathbf{F}_{\mathrm{L}}^{(n)} + \hat{\Phi}(y)\mathbf{F}_{\mathrm{L}}^{(n)} = m_n \mathbf{F}_{\mathrm{R}}^{(n)},$$

$$-\partial_y \mathbf{F}_{\mathrm{R}}^{(n)} + \hat{\Phi}(y)\mathbf{F}_{\mathrm{R}}^{(n)} = m_n^* \mathbf{F}_{\mathrm{L}}^{(n)},$$

Two sets of solutions corresponding $\pm$ m

$$F_{2L}^* = \pm F_{1R}; \quad F_{2R}^* = \mp F_{1L};$$

For upper signs

$$(\partial_y + \Phi_1)F_{1L} + \Phi_c F_{1R}^* = mF_{1R};$$
$$(-\partial_y + \Phi_1)F_{1R} + \Phi_c F_{1L}^* = mF_{1L}.$$

$$\Phi_c \equiv \Phi_4 + i\Phi_2$$

Zero approximation m = 0, massless Dirac fermion

$$\Phi_4(y) = \Phi_2(y) = 0; \quad F_{1L} \longrightarrow f_0(y) = C\exp\left\{-\int^y dy'\Phi_1(y')\right\} = f_0(-y); \quad F_{1R} \longrightarrow 0,$$

No CP breaking!

Next approximation

$$F_{1R}(y) = -\exp\left\{\int_0^y dy'\Phi_1(y')\right\}\int_0^y dy'\exp\left\{-\int_0^{y'} dy''\Phi_1(y'')\right\}\Big(mF_{1L}(y') - \Phi_c(y')F_{1L}^*(y')\Big).$$

To provide a normalizable right-handed component one has to impose

$$\int_0^\infty dy'\exp\left\{-\int_0^{y'} dy''\Phi_1(y'')\right\}\Big(mF_{1L}(y') - \Phi_c(y')F_{1L}^*(y')\Big) = 0.$$

This gives the equation for complex mass spectrum if $\Phi_2, \Phi_4 \neq 0$

$$m = \frac{\int_0^\infty dy'\Phi_c(y')\ f_0^2(y')}{\int_0^\infty dy'\ f_0^2(y')}.$$

CP breaking!

Generation of asymmetric brane

Two scalar doublets

$$\Phi_{11} = M_1 \tanh(\beta y - a), \Phi_{12} = \mu_1 / \cosh(\beta y - a),$$
$$\Phi_{21} = M_2 \tanh(\beta y + a), \Phi_{22} = \mu_2 / \cosh(\beta y + a)$$

Mass spectrum equation

$$(\gamma_5 \partial_y + (g_{11}\Phi_{11} + g_{21}\Phi_{21})\tau_3 + g_{22}\Phi_{22}\tau_2 + g_{12}\Phi_{12}\tau_1)\Psi = \gamma^\mu \partial_\mu \Psi$$

Solution

$$F_L^{(0)} = C \exp - \int^y \Phi_A^{(0)} = \frac{C}{\cosh^{(g_{11}M_1 + g_{12}M_2)/\beta} \beta z}, F_R^{(0)} = 0$$ **even function**

$$F_R^{(1)} = sgn(y) \frac{C}{2}(g_{12}\mu_1 - i g_{22}\mu_2) \cosh^\gamma \beta y \left(B(\frac{1}{2}, \gamma + \frac{1}{2}) \frac{B(\tanh^2 \beta y; \frac{1}{2}, \gamma)}{B(\frac{1}{2}, \gamma)} - B(\tanh^2 \beta y; \frac{1}{2}, \gamma + \frac{1}{2}) \right)$$

$$F_L^{(1)} = -\frac{Ca(g_{21}M_2 - g_{11}M_1)}{2\cosh^\gamma \beta y}\left(-\frac{\pi}{2\beta} + \frac{2}{\beta}\arctan e^{\beta y}\right)$$ **odd function**

$F_L^{(0)} + F_L^{(1)}$ **provides an asymmetric brane localization for fermions**