Nonlocal cosmological models

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To specify different types of cosmic fluids one uses a relation between the pressure p and the energy density ϱ

$$p = w\varrho, \qquad p = E_k - V, \qquad \varrho = E_k + V$$

where w is the state parameter.

Contemporary experiments give strong support that
$w > 0$ — **Atoms. (4%)**
$w = 0$ — **the Cold Dark Matter. (23%)**
$w < 0$ — **the Dark Energy. (73%)**
the dark energy state parameter is close to -1:

$$w_{DE} = -1 \pm 0.2.$$

The spatially flat Friedmann–Robertson–Walker metric:

$$ds^2 = -\,dt^2 + a^2(t)\left(dx_1^2 + dx_2^2 + dx_3^2\right),$$

where $a(t)$ **is the scale factor, the Hubble parameter** $H \equiv \dot{a}/a$**.**

$$w(t) = -\,1 - \frac{2}{3}\frac{\dot{H}}{H^2} = -\,1 + \frac{2E_k}{\varrho}. \qquad (1)$$

We consider the case $w_{DE} < -1$**. In this case the Null energy condition (NEC) is violated and there are problems of instability at classical and quantum levels.**

A possible way to evade the instability problem for models with $w < -1$ **is to yield a phantom model as an effective one, arising from a more fundamental theory. Such a possibility does appear in the string field theory framework (*I.Ya. Aref'eva, astro-ph/0410443, 2004*).**

Models with nonlocal scalar fields

The SFT inspired nonlocal gravitation models are introduced as a sum of the SFT action of the tachyon field ϕ plus the gravity part of the action. One cannot deduce this form of the action from the SFT.

Let us consider the $f(R)$ gravity, which is a straightforward modification of the general relativity, and the following action:

$$S_f = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N} + \frac{1}{\alpha' g_o^2}\left(\frac{1}{2}\phi\,\mathcal{F}\left(\alpha'\Box_g\right)\phi - V(\phi)\right) - \Lambda\right), \quad (2)$$

where g_o is the open string coupling constant, α' is the string length squared.

From the SFT after some approximations we obtained:

$$\mathcal{F}_{SFT}(\alpha'\Box_g) = (\xi^2\alpha'\Box_g + 1)e^{-2\alpha'\Box_g} - c,$$

where c and ξ^2 are constants.

$\mathcal{F}_{SFT}(\Box_g)$ has only simple and (for some values of c and ξ^2) double roots.

The system of the Einstein equations is
a system of nonlocal nonlinear equations !!!

In terms of dimensionless coordinates $\bar{x}_\mu = x_\mu/\sqrt{\alpha'}$ **and constants the equations look as follow:**

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} - \Lambda g_{\mu\nu}\right), \tag{3}$$

$$\mathcal{F}(\Box_g)\phi = \frac{dV}{d\phi}, \tag{4}$$

where $G_{\mu\nu}$ **is the Einstein tensor, the energy–momentum tensor**

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}.$$

HOW CAN WE FIND A SOLUTION?

There are two different cases:

- **The potential $V(\phi) = C_2\phi^2 + C_1\phi + C_0$, where C_2, C_1 and C_0 are arbitrary constants. In this case one can construct the equivalent action with local fields and quadratic potentials. Number of local fields is equal to number of roots of $\mathcal{F}(\Box)$, with a glance of order of them. It has been proved for an arbitrary analytic function $\mathcal{F}$ with simple and double roots.**

  I.Ya. Aref'eva, L.V. Joukovskaya, S.Yu.V., *J. Phys. A: Math. Theor.* 41 (2008) 304003, `arXiv:0711.1364`;

  D.J. Mulryne, N.J. Nunes, *Phys. Rev.* D 78 (2008) 063519, `arXiv:0805.0449`

  S.Yu.V., *Class. Quant. Grav.* 27 (2010) 035006, `arXiv:0907.0468`

  S.Yu.V., *Phys. Part. Nucl. Lett.* 8 (2011) 310–320, `arXiv:1005.0372`

  A.S. Koshelev, S.Yu.V., Class. Quant. Grav. 28 (2011) 085019, `arXiv:1009.0746`

A straightforward modification of the General Relativity is $f(R)$ metric gravity:

$$S_1 = \int d^4x\sqrt{-g}\left\{\frac{1}{16\pi G_N}\left(R+\frac{1}{L^2}f(L^2R)\right)+\mathcal{L}_{\mathrm{matter}}\right\}\ , \qquad (5)$$

The equations of $f(R)$ metric gravity are

$$f'(R)R_{\mu\nu}-\frac{f(R)}{2}g_{\mu\nu}-D_\mu\partial_\nu f'(R)+g_{\mu\nu}\Box_g f'(R)=8\pi G_N T_{\mu\nu}. \qquad (6)$$

For $f(R)$ metric gravity models with minimally coupling a nonlocal scalar field with a quadratic potential the way of its localization has been proposed in

S. Yu. V., Proc. of QFTHEP2010.

- **The potential $V(\phi) \neq C_2\phi^2 + C_1\phi + C_0$. In this case situation is more difficult and exact solutions is possible to find only adding some scalar field, for example, a k-essence field.**

  Numerical Solution:

  L. Joukovskaya, *JHEP* 0902 (2009) 045, `arXiv:0807.2065`

  Approximate solutions for field equation:

  G. Calcagni and G. Nardelli, *Int. J. Mod. Phys.* D 19 (2010) 329–338, `arXiv:0904.4245`

  Exact solutions for field equation:

  S.Yu.V., *Theor. Math. Phys.* 166 (2011) 392–402, `arXiv:1005.5007`

NONLOCAL GRAVITY

There are another type of modifications that explicitly includes a function of $\Box_g$ operator, in particular, $\Box_g^{-1}$ and defines a non-local modification of gravity.

A modification that assumes the existence of a new dimensional parameter M_* can be of the form

$$S = \int d^4x \sqrt{-g} \left(\frac{M_P^2}{2} R + \frac{1}{2} R \mathcal{F}(\Box / M_*^2) R - \Lambda \right) \tag{7}$$

where M_* is the mass scale at which the higher derivative terms in the action become important.

M_P is the Planck mass: $8\pi G_N = 1/M_P^2$.

An analytic function $\mathcal{F}(\Box/M_*^2) = \sum\limits_{n \geqslant 0} f_n \Box^n$.

Biswas T., Mazumdar A., and Siegel W. 2006, *JCAP* 0603 009 (hep-th/0508194)

Biswas T., Koivisto T., and Mazumdar T. 2010, *JCAP* 1011 008 (arXiv:1005.0590)

By virtue of the field redefinition one can transform the non-local gravity action (7) as follows:

$$S=\int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}(1+\Phi)R+\frac{1}{2}\tau\mathcal{F}(\Box)\tau-\frac{M_P^2}{2}\Phi\tau-\Lambda\right) \qquad (8)$$

with two new scalar fields Φ and τ.

Variation w.r.t. Φ gives $\tau=R$ and, therefore, the connection (8) with action (7) is obvious.

From action (8) one gets the following equations of motion:

$$M_P^2(1+\Phi)\left(R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}\right)=\frac{1}{2}\sum_{n=1}^{\infty}f_n\sum_{l=0}^{n-1}\Big(\partial_\mu\Box^l\tau\partial_\nu\Box^{n-1-l}\tau+$$

$$+\partial_\nu\Box^l\tau\partial_\mu\Box^{n-1-l}\tau-g_{\mu\nu}\left(g^{\rho\sigma}\partial_\rho\Box^l\tau\partial_\sigma\Box^{n-1-l}\tau+\Box^l\tau\Box^{n-l}\tau\right)\Big)+$$

$$+\frac{1}{2}g_{\mu\nu}\left(\tau\mathcal{F}(\Box)\tau-M_P^2\Phi\tau\right)+M_P^2(D_\mu\partial_\nu\Phi-g_{\mu\nu}\Box\Phi)-\Lambda g_{\mu\nu}\,,$$

$$\mathcal{F}(\Box)\tau=\frac{M_P^2}{2}\Phi\,,$$

$$\tau=R\,.$$

A modification that does not assume the existence of a new dimensional parameter in the action

$$S_2 = \int d^4x \sqrt{-g} \left\{ \frac{1}{16\pi G_N} \left[R \left(1 + f(\Box^{-1} R) \right) - 2\Lambda \right] + \mathcal{L}_{\mathrm{m}} \right\} , \qquad (9)$$

The reason to consider (9) with corrections involving $\Box^{-1}R$ as an origin for dark energy is the following. This term is dimensionless and it can appear as a prefactor for the Newtonian gravitational constant, and explain weakening of gravity at cosmological scales. Therefore, no new scale needs to be introduced, in particular the hierarchy between the observed magnitude of dark energy density and the Planck scale does not to have to prescribed into the action.

Deser S., Woodard R.P., **2007,** ***Phys. Rev. Lett.*** **99, 111301**

Koivisto T.S., **2008, Phys. Rev. D 77, 123513**

Nojiri Sh., Odintsov S.D., **2008,** ***Phys. Lett.*** **B 659, 821–826**

Capozziello S., Elizalde E., Nojiri Sh., Odintsov S.D., **2009, Phys. Lett. B 671 193–198**

K. Bamba, Sh. Nojiri, S.D. Odintsov, M. Sasaki, **arXiv:1104.2692**

The action (9) can be rewritten by introducing two scalar fields ϕ and ξ in the following form:

$$\tilde{S}_2 = \int d^4x\sqrt{-g}\left\{\frac{1}{16\pi G_N}\left[R\left(1+f(\eta)\right)+\xi\left(\Box\eta - R\right) - 2\Lambda\right] + \mathcal{L}_{\rm m}\right\} . \tag{10}$$

By the variation over ξ, we obtain $\Box\phi = R$.

Substituting $\phi = \Box^{-1}R$ into (10), one reobtains (9). All of these generalizations can be written in the local form due to incorporation of local scalar fields to the Einstein action.

We take the spatially flat FRW metric

$$ds^2 = -dt^2 + a^2(t)\left(dx_1^2 + dx_2^2 + dx_3^2\right) . \tag{11}$$

The scalar fields η and ξ depend only on time.

In the FRW metric gravity equations are

$$-3H^2\left(1+f(\eta)-\xi\right)+\frac{1}{2}\dot{\xi}\dot{\eta}-3H\frac{d}{dt}\left(f(\eta)-\xi\right)+\Lambda+\kappa^2\rho_{\rm m}=0\,, \tag{12}$$

$$\left(2\dot{H}+3H^2\right)\left(1+f(\eta)-\xi\right)+\frac{1}{2}\dot{\xi}\dot{\eta}+\left(\frac{d^2}{dt^2}+2H\frac{d}{dt}\right)\left(f(\eta)-\xi\right)-\Lambda+\kappa^2P_{\rm m}=0\,, \tag{13}$$

where $H=\dot{a}/a$ **is the Hubble parameter.**

Summing equations (12) and (13), we get

$$2\dot{H}\left(1+f(\eta)-\xi\right)+\dot{\xi}\dot{\eta}+\left(\frac{d^2}{dt^2}-H\frac{d}{dt}\right)\left(f(\eta)-\xi\right)+\kappa^2(P_{\rm m}+\rho_{\rm m})=0\,, \tag{14}$$

The state equation is

$$\dot{\rho}_{\rm m}=-3H(P_{\rm m}+\rho_{\rm m}). \tag{15}$$

The equations of motion for the scalar fields η **and** ξ **are**

$$\ddot{\eta}+3H\dot{\eta}=-6\left(\dot{H}+2H^2\right)\,, \tag{16}$$

$$\ddot{\xi}+3H\dot{\xi}=6\left(\dot{H}+2H^2\right)f'(\eta). \tag{17}$$

Let us consider the system of equations (14)–(17).

Together with (12) those equations are equivalent to the full system of the Einstein equations.

Differentiating (12) over t, we get that (12) is an integral of motion for the system of equations (14)–(17).

Therefore, to find a solution of the Einstein equation one can solve system (14)–(17), which do not include the cosmological constant Λ. After this, substituting into (12) the initial values of the obtained solution one gets the corresponding value of Λ.

The considering system of equations does not include the function η, only $f(\eta)$ and the time derivatives of η. This property can be used to analyse the stability of the de Sitter solutions.

de Sitter solutions

We consider

$$f(\eta) = f_0 e^{\frac{\eta}{\beta}}\,, \tag{18}$$

where f_0 and β are constants.

If $H = H_0$, then equation (16) has the following solution:

$$\eta(t) = -4H_0(t - t_0) - \eta_0 e^{-3H_0(t-t_0)}, \tag{19}$$

with constants of integration, t_0 and η_0.

Equation (17) has the following general solution at $\eta_0 \neq 0$:

$$\xi = \frac{12H_0^2 f_0}{\beta}\int\limits_0^t \left\{\left(C_1 + \int\limits_0^{t_1} e^{(-\eta_0\exp[-3H_0(\tilde{t}-t_0)]-4H_0(\tilde{t}-t_0))/\beta+3H_0 t_2} d\tilde{t}\right) e^{-3H_0 t_1} dt_1\right\} - \xi_0,$$

where C_1 and ξ_0 are arbitrary constants. If $\beta = 2/3$, then $\xi(t)$ can be found explicitly:

$$\xi(t) = \frac{8f_0}{9\eta_0^2} e^{-(3/2)\eta_0\exp(-3H_0(t-t_0))} - C_1 e^{-3H_0(t-t_0)} - \xi_0.$$

The obtained solutions include four arbitrary parameters: η_0, ξ_0, C_1 and t_0.

We assume that the state parameter $w_{\mathrm{m}} \equiv P_{\mathrm{m}}/\rho_{\mathrm{m}}$ is a constant, so, equation (15) has the following general solution

$$\rho_{\mathrm{m}} = \rho_0\, e^{-3(1+w_{\mathrm{m}})H_0(t-t_0)}, \tag{20}$$

where ρ_0 is an arbitrary constant.

At $H(t) = H_0$, equation (14) has the following form:

$$\dot{\xi}\dot{\eta} + \frac{1}{\beta}f(\eta)\left(\frac{1}{\beta}\dot{\eta}^2 + \ddot{\eta}\right) - \frac{1}{\beta}H_0 f(\eta)\dot{\eta} - \ddot{\xi} + H_0\dot{\xi} + \kappa^2(1+w_{\mathrm{m}})\rho_{\mathrm{m}} = 0\,. \tag{21}$$

Using (17) we get:

$$(\dot{\eta} + 4H_0)\dot{\xi} + \frac{f(\eta)}{\beta}\left(\frac{1}{\beta}\dot{\eta}^2 + \ddot{\eta} - H_0\dot{\eta} - 12H_0^2\right) + \kappa^2(1+w_{\mathrm{m}})\rho_{\mathrm{m}} = 0\,. \tag{22}$$

The straight forward calculations show that equation (22) has no solutions at any values of parameters such that $f_0 \neq 0$, $\eta_0 \neq 0$, and $H_0 \neq 0$.

Thus, without loss of the generality we can put $\eta_0 = 0$. In this case, at $\beta \neq 4/3$ from (16) and (17) the following solution can be obtain:

$$\xi = -\frac{3f_0\beta}{3\beta-4}e^{-\frac{4H_0(t-t_0)}{\beta}} + \frac{c_0}{3H_0}e^{-3H_0(t-t_0)} - \xi_0, \qquad \eta = -4H_0(t-t_0),$$

where c_0 is an arbitrary constant,

$$\Lambda = 3H_0^2(1+\xi_0), \qquad \rho_{\mathrm{m}0} = \frac{6\,(\beta-2)\,H_0^2 f_0}{\kappa^2\beta}\ , \qquad w_{\mathrm{m}} = -1 + \frac{4}{3\beta}.$$

The case $\beta = 2$ corresponds to $\rho_{\mathrm{m}0} = 0$.

At $\beta = 4/3$ we get

$$\xi(t) = -f_0(c_0 + 3H_0(t-t_0))e^{-3H_0(t-t_0)} - \xi_0,$$

$$\Lambda = 3H_0^2(1+\xi_0), \qquad P_{\mathrm{m}} = 0, \qquad \rho_{\mathrm{m}} = -3H_0^2 f_0 e^{-3H_0(t-t_0)}.$$

This solution corresponds to the dark matter, because, $w_{\mathrm{m}} = 0$.

The obtained solutions generalize de Sitter solutions obtained in arXiv:1104.2692. We have proved that the obtained solutions are the most general de Sitter solutions.

Stability of the de Sitter solutions

Let us introduce new variables

$$\phi = f(\eta) = f_0 e^{\frac{\eta}{\beta}}, \qquad \psi = \dot{\eta}. \tag{23}$$

The functions $\phi(t)$ **and** $\psi(t)$ **are connected by the equation:**

$$\dot{\phi} = \frac{1}{\beta}\phi\psi. \tag{24}$$

Let us present the system of equations (14)–(17) as a system of the first order differential equations in terms of new variables. We rewrite equations (16) and (17) as follows:

$$\dot{\psi} = \, -3H\psi - \frac{6}{\beta}\left(\dot{H} + 2H^2\right)\phi\,, \tag{25}$$

$$\vartheta = \dot{\xi}, \tag{26}$$

$$\vartheta = \dot{\xi}, \qquad \dot{\vartheta} = \, -3H\vartheta + \frac{6}{\beta}\left(\dot{H} + 2H^2\right)\phi\,. \tag{27}$$

Equation (14) **is equivalent to**

$$2\dot{H}\left(1+\frac{\beta+12}{\beta}\phi-\xi\right)=4H\left(\frac{\phi\psi}{\beta}-\xi\right)-\frac{1}{\beta^2}\phi\psi^2+\frac{24}{\beta}H^2\phi-\vartheta\psi-\kappa^2(1+\omega_m)\rho_{\mathrm{m}}\,.$$

Let us consider the de Sitter solution:

$$\rho_m=\rho_{0m}e^{-3(\omega_m+1)H_0(t-t_0)},\qquad P_m=\omega_m\rho_m,\qquad \Lambda=3H_0^2(1+\xi_0), \quad (28)$$

$$\beta=\frac{4}{3(1+\omega_m)},\qquad \psi=\,-4H_0,\qquad \phi=f_0e^{\frac{-4H_0t}{\beta}}. \quad (29)$$

At $\beta\neq 4/3$ **the function**

$$\xi=\,-\frac{3f_0\beta}{3\beta-4}e^{-\frac{4H_0(t-t_0)}{\beta}}+\frac{c_0}{3H_0}e^{-3H_0(t-t_0)}-\xi_0,$$

at $\beta=4/3$

$$\xi=\,-f_0(c_0+3H_0(t-t_0))e^{-3H_0(t-t_0)}-\xi_0,$$

At t **tends to** $+\infty$, **we get**

$$\rho_m\to 0,\qquad \phi\to 0,\qquad \psi=-4H_0,\qquad \xi\to-\xi_0,$$

at $H_0 > 0$ and $\beta > 0$. System has a fixed point $\phi = 0$, $\xi = -\xi_0$, $\psi = 0$, $\rho_m = 0$.

To analyse the stability of this fixed point we use the Lyapunov theorem. The Lyapunov theorem states that to prove the stability of fixed point of nonlinear system it is sufficient to prove the stability of this fixed point for the corresponding linearized system.

There are two different cases: $\Lambda = 0$ and $\Lambda \neq 0$.

At $\Lambda \neq 0$ one get $\xi_0 \neq -1$, in the neighbourhood of the fixed point

$$\zeta \equiv \left(1 + \frac{\beta + 12}{\beta}\phi - \xi\right) \approx 1 + \xi_0 \neq 0$$

and we can divide equation () on this expression to get equation in the standard form:

$$\dot{H} = \frac{1}{2\zeta}\left(4H\left(\frac{\phi\psi}{\beta} - \vartheta\right) - \frac{1}{\beta^2}\phi\psi^2 + \frac{24}{\beta}H^2\phi - \vartheta\psi - \kappa^2(1+\omega_m)\rho_{\rm m}\right).$$

In the neighborhood of y_f

$$\begin{aligned} H(t) &= H_0 + \varepsilon h_1(t) + \mathcal{O}(\varepsilon^2), \\ \phi(t) &= \varepsilon\phi_1(t) + \mathcal{O}(\varepsilon^2), \qquad \psi(t) = -4H_0 + \varepsilon\psi_0(t) + \mathcal{O}(\varepsilon^2), \\ \xi(t) &= -\xi_0 + \varepsilon\xi_1(t) + \mathcal{O}(\varepsilon^2), \qquad \vartheta(t) = \varepsilon\vartheta_1(t) + \mathcal{O}(\varepsilon^2), \\ \rho_m(t) &= \varepsilon\rho_{m1}(t) + \mathcal{O}(\varepsilon^2), \end{aligned}$$

where ε is a small parameter.

To first order in ε we obtain

$$\dot{\rho}_{m1} = -\frac{4}{\beta}H_0\rho_{m1}, \tag{30}$$

$$\dot{\phi}_1 = -\frac{4}{\beta}H_0\phi_1, \tag{31}$$

$$\dot{\psi}_1 = -3H_0\psi_1 + 12H_0h_1 - \frac{12}{\beta}H_0^2\phi_1, \tag{32}$$

$$\dot{\vartheta}_1 = -3H_0\vartheta_1 + \frac{12}{\beta}H_0^2\phi_1, \tag{33}$$

$$\dot{h}_1 = \frac{2}{(1+\xi_0)}\left(-\frac{2}{\beta}H_0\phi_1 - \xi_0h_1 - \frac{\kappa^2}{3\beta}\rho_{m1}\right), \tag{34}$$

Note that the function ξ_1 is not included into this system. It can be defined from equation (12).

The sufficient stability conditions:

$$\beta > 0, \qquad H_0 > 0, \qquad \frac{\xi_0}{1+\xi_0} > 0. \tag{35}$$

The fixed point is stable if either $\xi_0 > 0$, or $\xi_0 < -1$. Note that the first case corresponds to $\Lambda > 0$, whereas the second case corresponds to $\Lambda < 0$.

The observational consistency requires a positive value of the cosmological constant. A positive effective cosmological constant (vacuum energy density) is required for inflation models.

On the other hand, the string theory, the leading candidate for a consistent theory of quantum gravity, predicts the existence of negative energy vacuum.

The possibility of the changing of the cosmological constant sign in nonlocal gravity models is actively discussed:

T. Prokopec, arXiv:1105.0078

T. Biswas, T. Koivisto, A. Mazumdar, arXiv:1105.2636

To analyse the stability of the obtained solutions at $\Lambda = 0$, we transform the system of equation using new depended variables:

$$X = -\frac{\dot{\eta}}{4H}, \quad W = \frac{\dot{\xi}}{6Hf}, \quad Y = \frac{1-\xi}{3f(\eta)}, \quad Z = \frac{\kappa^2\rho_m}{3H^2 f} \tag{36}$$

and the independent variable N: $\frac{d}{dN} \equiv a\frac{d}{da} = \frac{1}{H}\frac{d}{dt}$**.**

Equations (15)–(17) are equivalent to the following equations in terms of new variables:

$$\begin{aligned}
\frac{dZ}{dN} &= \frac{4}{\beta}(X-1)Z - 2\frac{Z}{H}\frac{dH}{dN}\,, \\
\frac{dX}{dN} &= 3(1-X) + \frac{1}{H}\left(\frac{3}{2} - X\right)\frac{dH}{dN}\,, \\
\frac{dW}{dN} &= \frac{2}{\beta}(1+2WX) - 3W + \frac{1}{H}\left(\frac{1}{\beta} - W\right)\frac{dH}{dN}\,.
\end{aligned}$$

To get the full system of equations add equation for $\frac{dH}{dN}$:

$$\left(8WX - 12W - \frac{8}{\beta}X + \frac{12}{\beta} - 2Z\right)\frac{1}{H}\frac{dH}{dN} -$$
$$- \frac{12}{\beta}XY - 24W + 24\left(1 + \frac{1}{\beta}\right)XW - \frac{20}{\beta}X + \frac{24}{\beta} - \frac{16}{\beta}WX^2 + \frac{4}{\beta}XZ - \frac{4}{\beta}Z = 0.$$

The de Sitter solution in terms of new variables corresponds to the following fixed point:

$$W_0 = \frac{2}{3\beta - 4}, \quad X_0 = 1, \quad Y_0 = 1, \quad Z_0 = \frac{2(\beta - 2)}{\beta}, \quad H = H_0.$$

We can say that the de Sitter solutions are stable with respect to perturbations in the FRW metric at $4/3 < \beta \leqslant 2$.

Note that stability at $\beta = 2$, when $P_{\mathrm{m}} = 0$ and $\rho_{\mathrm{m}} = 0$ has been proved in the paper *Nojiri Sh., Odintsov S.D.*, 2008, *Phys. Lett.* B 659, 821–826.

Conclusion

- **Nonlocal cosmological models are very popular.**
- **There are a lot of possibilities to include nonlocality in cosmological models.**
- **We hope that nonlocal cosmological models can be constructed as an effective action inspired by the string field theory.**
- **In the nonlocal cosmological model, proposed by Nojiri and Odintsov, the de Sitter solutions have been obtained in the most general form and their stability in the FRW metric has been analysed.**
- **The de Sitter solution is stable both for $\Lambda > 0$, and for $\Lambda < 0$. So, it is possible that the cosmological constant is negative, but due to nonlocality we get stable de Sitter solution.**