Relativistic description of the double P-wave charmonium production in e^+e^- annihilation

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Double charmonium production

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General formalism

Two sources of the enhancement of the nonrelativistic cross section for the double charmonium production are revealed to the present: the radiative corrections of order $O(\alpha_s)$ and relative motion of c-quarks forming the bound states.

In the following we investigate second relative order relativistic corrections to the double P-wave charmonium production amplitudes in LO α_s as well as to the quarks bound state wave functions.

Two stages of the production process:

1. The virtual photon γ^* produces two heavy c-quarks and two heavy $\bar{c}$-antiquarks. Pertubative QCD.

$$p_{1,2} = \frac{1}{2}P \pm p, \quad (p \cdot P) = 0, \qquad q_{1,2} = \frac{1}{2}Q \pm q, \quad (q \cdot Q) = 0, \quad (1)$$

P, Q — the total four-momenta,
$p = L_P(0, \mathbf{p})$, $q = L_Q(0, \mathbf{q})$ — the relative four-momenta.

General formalism

2. In the second nonpertubative stage quarks and antiquarks form the final P-wave mesons h_c and χ_{cJ}, $J = 0, 1, 2$. Quasipotential approach to the relativistic quark model. The color-singlet mechanism is considered as a basic one for the pair charmonium production.

The production amplitude [8]:

$$\mathcal{M}(p_-, p_+, P, Q) = \frac{8\pi^2\alpha\,\alpha_s(4m^2)\mathcal{Q}_c}{3s}\bar{v}(p_+)\gamma_\beta u(p_-)\int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{d\mathbf{q}}{(2\pi)^3}\times$$
$$\times\,\mathrm{Tr}\Big\{\bar{\Psi}^{h_c}(p, P)\Gamma_1^{\beta\nu}(p, q, P, Q)\bar{\Psi}^{\chi_c}(q, Q)\gamma_\nu +$$
$$+\,\bar{\Psi}^{\chi_c}(q, Q)\Gamma_2^{\beta\nu}(p, q, P, Q)\bar{\Psi}^{h_c}(p, P)\gamma_\nu\Big\}, \quad (2)$$

$\mathcal{Q}_c$ — c-quark electric charge,

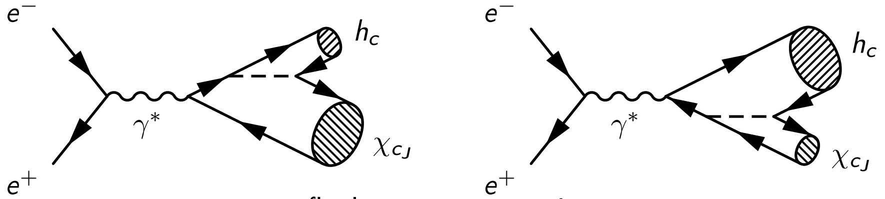
$s = l^2 = (p_+ + p_-)^2$,

$\Gamma_{1,2}$ — vertex functions.

[8] D. Ebert and A.P. Martynenko, Phys. Rev. D **74**, 054008 (2006)

Production diagrams

The leading order in α_s:



+ final states permutations.

$$
\begin{aligned}
\Gamma_1^{\beta\nu} &= \gamma_\mu \frac{(\hat{l} - \hat{q}_1 + m)}{(l - q_1)^2 - m^2} \gamma_\beta D^{\mu\nu}(k_2) + \gamma_\beta \frac{(\hat{p}_1 - \hat{l} + m)}{(l - p_1)^2 - m^2} \gamma_\mu D^{\mu\nu}(k_2), \\
\Gamma_2^{\beta\nu} &= \gamma_\beta \frac{(\hat{q}_2 - \hat{l} + m)}{(l - q_2)^2 - m^2} \gamma_\mu D^{\mu\nu}(k_1) + \gamma_\mu \frac{(\hat{l} - \hat{p}_2 + m)}{(l - p_2)^2 - m^2} \gamma_\beta D^{\mu\nu}(k_1),
\end{aligned} \tag{3}
$$

$k_{1,2} = p_{1,2} + q_{1,2}$ — gluon four-momenta,
$l = p_+ + p_- = P + Q$.

Bound quarks wave functions

The relativistic wave functions was transformed from the rest frame (CM) to the moving one with four-momenta P, Q:

$$\begin{aligned}
\bar{\Psi}^{h_c}(p,P) &= \frac{\bar{\Psi}_0^{h_c}(\mathbf{p})}{\left[\frac{\epsilon(p)}{m}\frac{(\epsilon(p)+m)}{2m}\right]}\left[\frac{\hat{v}_1-1}{2}+\hat{v}_1\frac{\mathbf{p}^2}{2m(\epsilon(p)+m)}-\frac{\hat{p}}{2m}\right]\times \\
&\times\gamma_5(1+\hat{v}_1)\left[\frac{\hat{v}_1+1}{2}+\hat{v}_1\frac{\mathbf{p}^2}{2m(\epsilon(p)+m)}+\frac{\hat{p}}{2m}\right], \\
\bar{\Psi}^{\chi_c}(q,Q) &= \frac{\bar{\Psi}_0^{\chi_c}(\mathbf{q})}{\left[\frac{\epsilon(q)}{m}\frac{(\epsilon(q)+m)}{2m}\right]}\left[\frac{\hat{v}_2-1}{2}+\hat{v}_2\frac{\mathbf{q}^2}{2m(\epsilon(q)+m)}+\frac{\hat{q}}{2m}\right]\times \\
&\times\hat{\varepsilon}^*_{\chi_c}(Q,S_z)(1+\hat{v}_2)\left[\frac{\hat{v}_2+1}{2}+\hat{v}_2\frac{\mathbf{q}^2}{2m(\epsilon(q)+m)}-\frac{\hat{q}}{2m}\right],
\end{aligned} \tag{4}$$

$v_1=\frac{P}{M_h}$, $v_2=\frac{Q}{M_\chi}$,

$\epsilon(p)=\sqrt{m^2+\mathbf{p}^2}$,

$\varepsilon^*_{\chi_c}(Q,S_z)$ — the polarization vector of the spin-triplet state χ_{cJ}.

Expansions of quark and gluon propagators

$$\frac{1}{k_{1,2}^2} = \frac{4}{s}\left(1 \mp \frac{4(lp+lq)}{s} - \frac{4(p+q)^2}{s} + \frac{16(lp+lq)^2}{s^2} + \dots\right),$$
$$\frac{1}{(l-p_{1,2})^2 - m^2} = \frac{1}{w}\left(1 \pm 2\frac{lp}{w} - \frac{p^2}{w} + \frac{4(lp)^2}{w^2} + \dots\right), \quad (5)$$
$$\frac{1}{(l-q_{1,2})^2 - m^2} = \frac{1}{x}\left(1 \pm 2\frac{lq}{x} - \frac{q^2}{x} + \frac{4(lq)^2}{x^2} + \dots\right),$$

$$w = \frac{s}{2} + \frac{1}{4}\left(2M_{\chi_{cJ}}^2 - M_{h_c}^2 - 4m^2\right) \approx \frac{s}{2},$$
$$x = \frac{s}{2} + \frac{1}{4}\left(2M_{h_c}^2 - M_{\chi_{cJ}}^2 - 4m^2\right) \approx \frac{s}{2}. \quad (6)$$

Relativistic corrections was preserved up to the second relative order in $|\mathbf{p}|/s$, $|\mathbf{q}|/s$.

The angular integration

$$\int q_\alpha \Psi_0^{\mathcal{P}}(\mathbf{q}) \frac{d\mathbf{q}}{(2\pi)^3} = -\frac{i}{\sqrt{6}\pi} \varepsilon_\alpha(Q, L_z) \int\limits_0^\infty \mathrm{q}^3 R_{\mathcal{P}}(\mathrm{q}) d\mathrm{q},$$

$$\int q_\alpha q_\beta q_\mu \Psi_0^{\mathcal{P}}(\mathbf{q}) \frac{d\mathbf{q}}{(2\pi)^3} = \frac{i}{5\sqrt{6}\pi} \big(\varepsilon_\mu(Q, L_z) P_{\alpha\beta} + + \varepsilon_\alpha(Q, L_z) P_{\beta\mu} + \varepsilon_\beta(Q, L_z) P_{\alpha\mu}\big) \int\limits_0^\infty \mathrm{q}^5 R_{\mathcal{P}}(\mathrm{q}) d\mathrm{q}, \quad (7)$$

$P_{\alpha\beta} = (g_{\alpha\beta} - v_\alpha v_\beta)$.

Summing over S_z and L_z with Clebsch-Gordon coefficients:

$$\sum_{S_z, L_z} \langle 1, L_z; 1, S_z | J, J_z \rangle \varepsilon_\alpha^*(Q, L_z) \varepsilon_\beta^*(Q, S_z) = \begin{cases} \frac{1}{\sqrt{3}} (g_{\alpha\beta} - v_{2\alpha} v_{2\beta}), & J = 0, \\ \frac{i}{\sqrt{2}} e_{\alpha\beta\sigma}{}^{\rho} v_2^\sigma \varepsilon_\rho^*(Q, J_z), & J = 1, \\ \varepsilon_{\alpha\beta}^*(Q, J_z), & J = 2, \end{cases} \quad (8)$$

Effective relativistic Hamiltonian

$$H = H_0 + \Delta U_1 + \Delta U_2 + \Delta U_3,$$
$$H_0 = 2\sqrt{\mathbf{p}^2 + m^2} - 2m - \frac{C_F \tilde{\alpha}_s}{r} + Ar + B, \tag{9}$$

$$\Delta U_1(r) = -\frac{C_F \alpha_s^2}{4\pi r}\big(2\beta_0 \ln(\zeta r) + a_1 + 2\gamma_E \beta_0\big), \tag{10}$$

$$\Delta U_2(r) = -\frac{C_F \alpha_s}{2m^2 r}\left(\mathbf{p}^2 + \frac{\mathbf{r}(\mathbf{r}\mathbf{p})\mathbf{p}}{r^2}\right) + \frac{3 C_F \alpha_s}{2m^2 r^3}(\mathbf{S}\mathbf{L}) -$$
$$-\frac{C_F \alpha_s}{2m^2}\left(\frac{\mathbf{S}^2}{r^3} - 3\frac{(\mathbf{S}\mathbf{r})^2}{r^5}\right) - \frac{C_A C_F \alpha_s^2}{2mr^2}, \tag{11}$$

$$\Delta U_3(r) = f_V \frac{A}{2m^2 r}\left(\frac{4}{3}\mathbf{S}^2 - 1 + 3(\mathbf{S}\mathbf{L}) - \frac{2}{3}\Big(\mathbf{S}^2 - 3\frac{(\mathbf{S}\mathbf{r})^2}{r^2}\Big)\right) -$$
$$-(1 - f_V)\frac{A}{2m^2 r}\mathbf{S}\mathbf{L}, \tag{12}$$

$$\alpha_s(m^2) \approx 0.314, \quad \tilde{\alpha}_s(m^2) \approx 0.242. \tag{13}$$

Effective relativistic Hamiltonian

$$A = 0.18\ GeV^2, \quad B = -0.16\ GeV,\\ m = 1.55\ GeV. \tag{14}$$

Kinetic energy operator:

$$T = 2\sqrt{\mathbf{p}^2 + m^2} = 2\frac{\mathbf{p}^2 + m^2}{\sqrt{\mathbf{p}^2 + m^2}} \approx \frac{\mathbf{p}^2}{\tilde{m}} + \frac{2m^2}{\tilde{E}}, \tag{15}$$

$$\tilde{m} = \frac{\tilde{E}}{2} = \frac{1}{2}\sqrt{\mathbf{p}_{eff}^2 + m^2}. \tag{16}$$

Table I: The parameters of the effective relativistic Hamiltonian and masses of P-wave charmonium states.

$(c\bar{c})$	$n^{2S+1}L_J$	J^{PC}	${\mathbf{p}_{eff}}^2$, GeV^2	$\tilde{m}$, GeV	M^{exp}, GeV	M^{theor}, GeV
χ_{c_0}	1^3P_0	0^{++}	0.54	0.857	3.415	3.418
χ_{c_1}	1^3P_1	1^{++}	0.54	0.857	3.511	3.493
χ_{c_2}	1^3P_2	2^{++}	0.54	0.857	3.556	3.557
h_c	1^1P_1	1^{+-}	0.54	0.857	3.526	3.499

The structure of the production amplitudes

$$\mathcal{M}(p_-, p_+, P, Q) = \frac{16\,\alpha\,\alpha_s(4m^2)\mathcal{Q}\,r^6}{45 M_0^6 \kappa^2\, u^{\frac{5}{2}}(1-u)^{\frac{5}{2}}}\,\bar{v}(p_+)\gamma^\beta u(p_-)\times$$

$$\times \int_0^\infty \mathrm{p}^3 R_{h_c}(\mathrm{p})\,\frac{\epsilon(\mathrm{p})+m}{2\epsilon(\mathrm{p})}d\mathrm{p}\int_0^\infty \left[\mathcal{K}_\beta(\mathrm{p},\mathrm{q})\right]\mathrm{q}^3 R_{\chi_{cJ}}(\mathrm{q})\,\frac{\epsilon(\mathrm{q})+m}{2\epsilon(\mathrm{q})}d\mathrm{q}, \quad (17)$$

$$\mathcal{K}_\beta(h_c+\chi_{c_0}) = A_1\, e_{\alpha\lambda\mu\beta} v_1^\alpha v_2^\lambda \varepsilon_{h_c}^{*\mu},$$

$$\mathcal{K}_\beta(h_c+\chi_{c_1}) = B_1\left(v_1\varepsilon^*_{\chi_{c_1}}\right)\varepsilon^*_{h_c\,\beta} + B_2\left(v_2\varepsilon^*_{h_c}\right)\varepsilon^*_{\chi_{c_1}\,\beta} + B_3\left(v_1\varepsilon^*_{\chi_{c_1}}\right)\left(v_2\varepsilon^*_{h_c}\right)v_{1\,\beta}+$$

$$+B_4\left(\varepsilon^*_{h_c}\varepsilon^*_{\chi_{c_1}}\right)v_{1\,\beta} + B_5\left(v_1\varepsilon^*_{\chi_{c_1}}\right)\left(v_2\varepsilon^*_{h_c}\right)v_{2\beta} + B_6\left(\varepsilon^*_{h_c}\varepsilon^*_{\chi_{c_1}}\right)v_{2\beta},$$

$$\mathcal{K}_\beta(h_c+\chi_{c_2}) = C_1\, e_{\alpha\mu\beta\nu}\varepsilon^{*\delta\nu}_{\chi_{c_2}} v_{1\delta} v_2^\alpha \varepsilon^{*\mu}_{h_c}+ \quad (18)$$

$$+\Big(C_2 {\varepsilon^*_{\chi_{c_2}\,\beta}}^{\rho} + C_3\varepsilon^{*\delta\rho}_{\chi_{c_2}} v_{1\,\beta} v_{1\delta} + C_4 \varepsilon^{*\delta\rho}_{\chi_{c_2}} v_{2\beta} v_{1\delta}\Big) e_{\alpha\mu\nu\rho} v_1^\alpha v_2^\mu \varepsilon^{*\nu}_{h_c}+$$

$$+C_5 \varepsilon^{*\delta\rho}_{\chi_{c_2}} v_{1\delta} v_{1\,\rho} e_{\alpha\mu\nu\beta} v_1^\alpha v_2^\mu \varepsilon^{*\nu}_{h_c} + \Big(C_6 \varepsilon^*_{h_c\,\delta} + C_7\left(v_2\varepsilon^*_{h_c}\right)v_{1\delta}\Big)\varepsilon^{*\delta\rho}_{\chi_{c_2}} e_{\alpha\mu\beta\rho} v_1^\alpha v_2^\mu +$$

$$+C_8 \varepsilon^{*\delta\rho}_{\chi_{c_2}} v_{1\delta} e_{\alpha\mu\beta\rho} v_1^\alpha \varepsilon^{*\mu}_{h_c}.$$

The structure of the production amplitudes

$$A_i = A_i(\mathrm{p}, \mathrm{q}; \kappa, u; r), \quad B_i = B_i(\mathrm{p}, \mathrm{q}; \kappa, u; r), \quad C_i = C_i(\mathrm{p}, \mathrm{q}; \kappa, u; r)$$

$$\kappa = \frac{m}{M_0}, \quad u = \frac{M_{\chi_{cJ}}}{M_0}, \qquad (19)$$
$$M_0 = M_{h_c} + M_{\chi_{cJ}},$$

$$r^2 = \frac{M_0^2}{s}. \qquad (20)$$

Dependence on p, q can be described in terms of the following functions:

$$c_{ij}(\mathrm{p}, \mathrm{q}) = \left(\frac{m - \epsilon(\mathrm{p})}{m + \epsilon(\mathrm{p})}\right)^i \left(\frac{m - \epsilon(\mathrm{q})}{m + \epsilon(\mathrm{q})}\right)^j, \qquad (21)$$
$$i = 0 \ldots 2, \quad j = 0 \ldots 2.$$

The structure of the cross sections

$$\sigma(h_c + \chi_{c_J}) = \frac{2\alpha^2\alpha_s^2(4m^2)\mathcal{Q}_c^2\pi r^6\sqrt{1-r^2}\sqrt{1-r^2(2u-1)^2}}{9\kappa^4 u^{11}(1-u)^{11}} \times \\ \times \frac{|\tilde{R}'_{h_c}(0)|^2|\tilde{R}'_{\chi_{c_J}}(0)|^2}{s(M_{\chi_{c_J}}+M_{h_c})^{10}} \sum_{i=0}^{7} F_i^{(J)}(r^2)\omega_i, \tag{22}$$

$$J_n = \int_0^\infty q^3 R_{\mathcal{P}}(q)\frac{\epsilon(q)+m}{2\epsilon(q)}\left(\frac{m-\epsilon(q)}{m+\epsilon(q)}\right)^n dq, \\ \tilde{R}'_{\mathcal{P}}(0) = \frac{1}{3}\sqrt{\frac{2}{\pi}}J_0, \tag{23}$$

$$\omega_0 = 1, \quad \omega_1 = \frac{J_1(h_c)}{J_0(h_c)}, \quad \omega_2 = \frac{J_2(h_c)}{J_0(h_c)}, \quad \omega_3 = \omega_1^2, \\ \omega_4 = \frac{J_1(\chi_{cJ})}{J_0(\chi_{cJ})}, \quad \omega_5 = \frac{J_2(\chi_{cJ})}{J_0(\chi_{cJ})}, \quad \omega_6 = \omega_4^2, \quad \omega_7 = \omega_1\omega_4. \tag{24}$$

Numerical results

Table II: Numerical values of the charmonium wave functions integrals

$(c\bar{c})$	$n^{2S+1}L_J$	J^{PC}	$\tilde{R}'_{\mathcal{P}}(0),\ GeV^{\frac{5}{2}}$	ω_1 or ω_4	ω_2 or ω_5
χ_{c_0}	1^3P_0	0^{++}	0.33	-0.28	0.13
χ_{c_1}	1^3P_1	1^{++}	0.20	-0.18	0.07
χ_{c_2}	1^3P_2	2^{++}	0.13	-0.08	0.01
h_c	1^1P_1	1^{+-}	0.17	-0.14	0.04

Numerical results

Table III: Comparison of the obtained results with previous theoretical predictions and experimental data

State $H_1 H_2$	$\sigma_{Belle} \times$ $\times \mathcal{B}_{>2}$ [1], *fb*	$\sigma_{\text{BaBar}} \times$ $\times \mathcal{B}_{>2}$ [2], *fb*	σ [3], *fb*	σ [4], *fb*	σ [5], *fb*	σ_{LO} [6], *fb*	Our result (22), *fb*
$J/\psi + \chi_{c_0}$	$6.4 \pm 1.7 \pm 1.0$	$10.3 \pm 2.5^{+1.4}_{-1.8}$	6.7	14.4	17.9 (6.35)	2.4 ± 1.02	14.47 ± 5.64
$J/\psi + \chi_{c_1}$	—	—	1.1	—	—	0.38 ± 0.12	1.78 ± 0.69
$J/\psi + \chi_{c_2}$	—	—	1.6	—	—	0.69 ± 0.13	0.44 ± 0.17
$\eta_c + h_c$	—	—	1.6	—	—	0.308 ± 0.017	0.25 ± 0.10
$h_c + \chi_{c_0}$	—	—	—	—	—	0.053 ± 0.019	0.075 ± 0.029
$h_c + \chi_{c_1}$	—	—	—	—	—	0.258 ± 0.064	0.132 ± 0.051
$h_c + \chi_{c_2}$	—	—	—	—	—	0.017 ± 0.002	0.004 ± 0.002

Table IV: The role of the relativistic corrections (r.c.) to σ (*fb*)

State $H_1 H_2$	w/o r.c.	r.c. to ampl. only	r.c. to w.f. only	r.c. to w.f. and ampl.	r.c. to w.f. and ampl. $u \neq \frac{1}{2}$, $\kappa \neq \frac{1}{4}$
$h_c + \chi_{c_0}$	0.14	0.10	0.097	0.13	0.075
$h_c + \chi_{c_1}$	0.60	0.83	0.15	0.37	0.13
$h_c + \chi_{c_2}$	0.035	0.11	0.0039	0.015	0.0036

Conclusion

- We presented a systematic treatment of relativistic effects in the P-wave double charmonium production in e^+e^- annihilation. We separated two different types of relativistic contributions to the production cross sections: v/c corrections to the wave functions and $p/\sqrt{s}$ corrections appearing from the expansion of the quark and gluon propagators in amplitude.
- Relativistic corrections to the quark bound state wave functions in the rest frame was considered by means of the Breit-like potential. Mass spectrum of P-wave charmonium was obtained.
- It turns out that the examined effects change essentially the nonrelativistic results of the cross section for the reaction $e^+e^- \to h_c + \chi_{c_J}$ at the center-of-mass energy $\sqrt{s} = 10.6\ GeV$, e.g. relativistic corrections to the amplitude increase $\sigma(h_c + \chi_{c_1})$ and $\sigma(h_c + \chi_{c_2})$ in 1.38 and 3.14 times respectively. Nevertheless, all types of corrections taken together significantly decrease double P-wave production cross sections.