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Perturbative and nonperturbative QCD to the Bjorken sum rule up to four loops

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Outline

- Introduction
- Standard PT description vs Jlab data
- Higher twist (HT) contributions
- Results of applying Analytic Perturbation Theory *)
- Summary and discussion

*) The Analytic Perturbation Theory (APT) is the ghost-free modification of pQCD proposed by D.V. Shirkov and I. Solovtsov.

Introduction

At low Q^2 scales, Q^2 ~ *a few GeV*^{2}, the QCD running coupling is not small, therefore higher-order pQCD corrections are important. Until very recently, the pQCD contribution to the Bjorken sum rule has been known up to the **third order** in α_S. The corresponding expression *[Larin, Vermaseren, 1991]* have been used in many studies, in particular, to extract α_S values at low momentum scales.

Now there are only a few physical quantities which have been calculated up to four-loop (N^{3}LO) level. In low Q^2 domain:

Hadronic τ decay [Baikov-Chetyrkin-Kuhn, 2008]. (M_τ=1.78 GeV)

$$R_\tau = \frac{\Gamma(\tau^- \to \text{hadrons}\ \nu_\tau)}{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} = 3.640 \pm 0.010$$

The precise determination of α_S from hadronic τ decay is very important for testing Q^2-evolution of α_S. (The subject has been treated by many authors.)

	$\alpha_s^{FO}(M_\tau)$	$\alpha_s^{CI}(M_\tau)$
no α_s^4	$0.337 \pm 0.004 \pm 0.03$	$0.354 \pm 0.006 \pm 0.02$
$d_4 = 25$	$0.325 \pm 0.004 \pm 0.02$	$0.347 \pm 0.006 \pm 0.009$
$d_4 = 49.08$	$0.322 \pm 0.004 \pm 0.02$	$0.342 \pm 0.005 \pm 0.01$

Difference in α_S

0.017

0.025

$$\alpha_s(M_\tau^2) = 0.332 \pm 0.005_{\text{exp}} \pm 0.015_{\text{theo}}$$

Convergence properties of PT expansion

FOPT	1 (%)	2 (%)	3 (%)	4 (%)
3-loop	**54.0**	**30.0**	**16.0**	**-**
4-loop	**51.4**	**27.4**	**14.2**	**7.0**

The aim of the talk is to present the 4-loop QCD analysis for the polarized Bjorken sum rule in both the PT and APT approaches.

The basis for such analysis gives

- **the four-loop expression** for the pQCD contribution to the Bjorken sum rule which became recently available *[Baikov, Chetyrkin, Kühn (2010)];*
- **high precision data** on the polarized Bjorken sum rule at low Q^2 in the wide range $0.05 < Q^2 < 3.2\ GeV^2$ obtained at the Jefferson Lab *[Dharmawardane et al. (2006); Bosted et al. (2007); Prok et al. (2009)]* .

We study the higher loop stability of PT /APT expansion,

how far can the perturbative theory penetrate to nonperturbative region,

the interplay between the higher twists (HT) and higher order pQCD.

Definitions

$$\Gamma_1^{p-n}\left(Q^2\right)=\int_0^1\left[g_1^p\left(x,Q^2\right)-g_1^n\left(x,Q^2\right)\right]dx \;=\; \underbrace{\frac{g_A}{6}C_{Bj}\left(Q^2\right)}_{\text{leading twist}} + \underbrace{\sum_{i=2}^{\infty}\frac{\mu_{2i}^{p-n}}{Q^{2i-2}}}_{\text{higher twists}}$$

$$\boxed{\Gamma_1^{p-n}\left(Q^2\to\infty\right)=\frac{g_A}{6}}$$

leading twist **higher twists**

nonsinglet coefficient function

the nucleon axial charge defined from the neutron β-decay data

$$g_A = 1.267 \pm 0.004$$

[PDG (2010)]

$$C_{Bj}\left(Q^2\right)=1-\Delta_{Bj}\left(Q^2\right)$$

The pQCD correction Δ_{Bj} has a form of the power series in α_s and at the four-loop level (in the massless case) reads as

$$\Delta_{Bj}\left(Q^2\right)=0.31831\alpha_s + 0.36307\alpha_s^2 + 0.65197\alpha_s^3 + 1.8042\alpha_s^4$$

Baikov, Chetyrkin, Kühn (2010)

The numerical values are given in the modified minimal subtraction scheme and for three active flavors.

Note. **At low Q^2 a value of the strong coupling is quite large, $\alpha_S \sim 1/3$, providing a similar magnitude of three- and four- loop contributions.**

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QCD running coupling α_s

The running coupling is defined as a solution of the RG equation. In the analysis we use the exact numerical solutions of the RG equation (in different orders).

NOT use the PDG expression
as a result is sensitive to a form

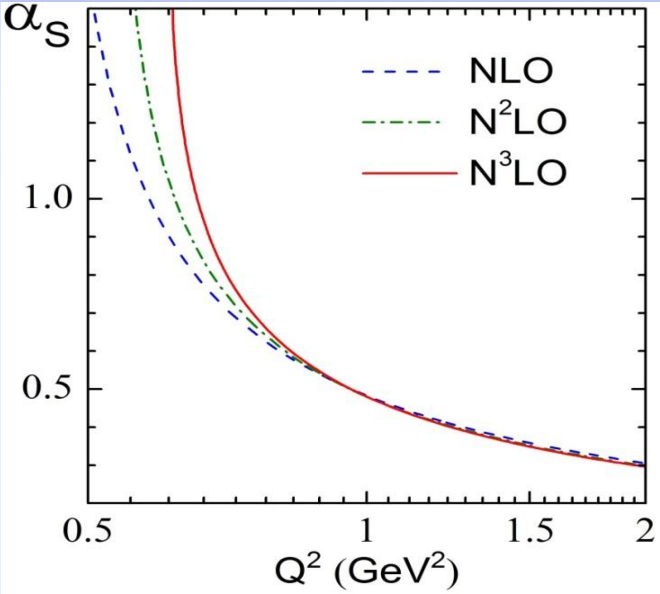
$$\alpha_s^{(4)}(L) = \frac{1}{\beta_0 L} - \frac{1}{\beta_0^3 L^2}\beta_1 \ln L + \frac{1}{\beta_0^3 L^3}\left(\frac{\beta_1^2}{\beta_0^2}(\ln^2 L - \ln L - 1) + \frac{\beta_2}{\beta_0}\right) + \frac{1}{\beta_0^4 L^4}\left[\frac{\beta_1^3}{\beta_0^3}\left(-\ln^3 L + \frac{5}{2}\ln^2 L + 2\ln L - \frac{1}{2}\right) - 3\frac{\beta_1\beta_2}{\beta_0^2}\ln L - \frac{\beta_3}{2\beta_0}\right];$$

As a normalization point, we use the most accurate α_S–**value:**

$$\alpha_S(M_Z) = 0.1184 \pm 0.0007$$

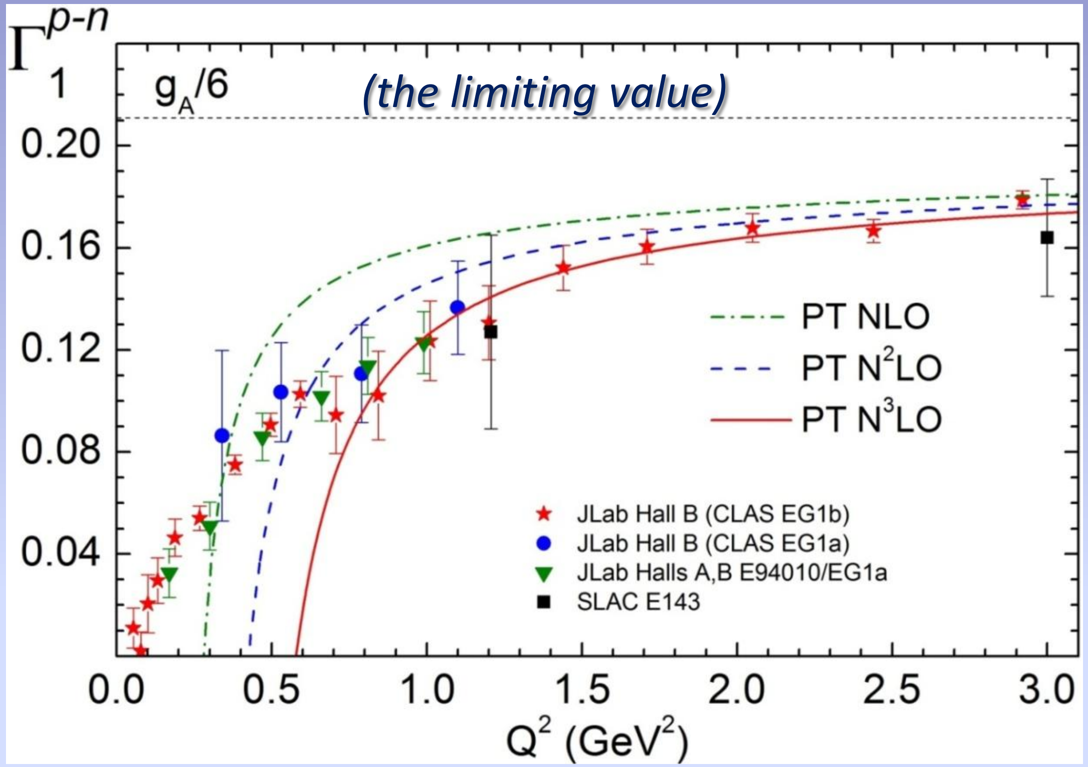
[Bethke (2009), PDG (2010)]

In order to take into account flavor thresholds, we apply the matching conditions for the values of α_S which are rather nontrivial in higher PT orders.
[Chetyrkin (1997), Schröder (2006), Kniehl (2006)]



$$\Lambda^{\mathrm{NLO}} = (394 \pm 13)\,\mathrm{MeV} \quad \Lambda^{\mathrm{N^2LO}} = (348 \pm 11)\,\mathrm{MeV} \quad \Lambda^{\mathrm{N^3LO}} = (336 \pm 10)\,\mathrm{MeV}$$

Q^2 -dependence



Perturbative part of the BSR vs Q^2 in different orders in the standard PT approach against the combined set of the Jefferson Lab and SLAC data.

✓ *At $Q^2 < 0.8$ GeV^2 t*he **four-loop** approximation describes the data equally bad as **the three** and **two-loop** ones.

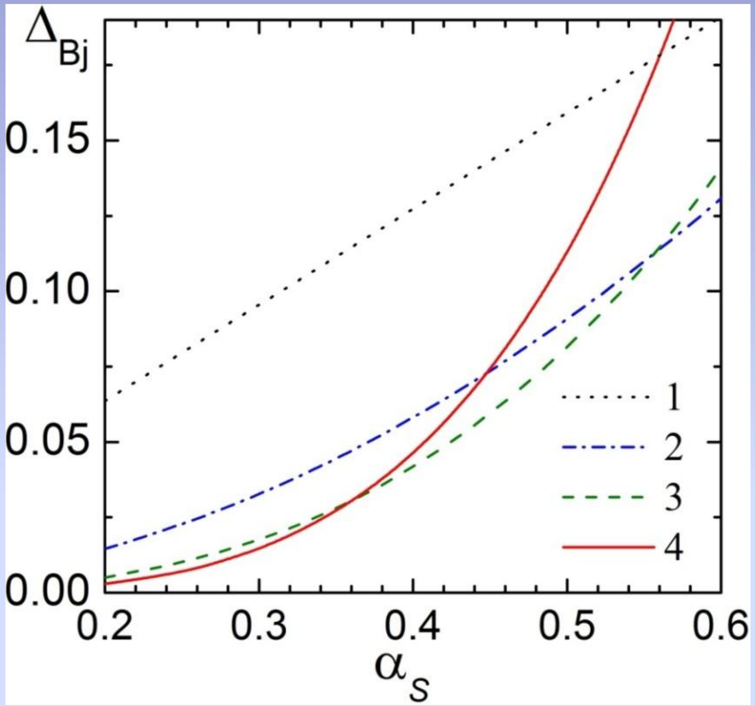
This is a signal of the necessity to account for the HT contributions.

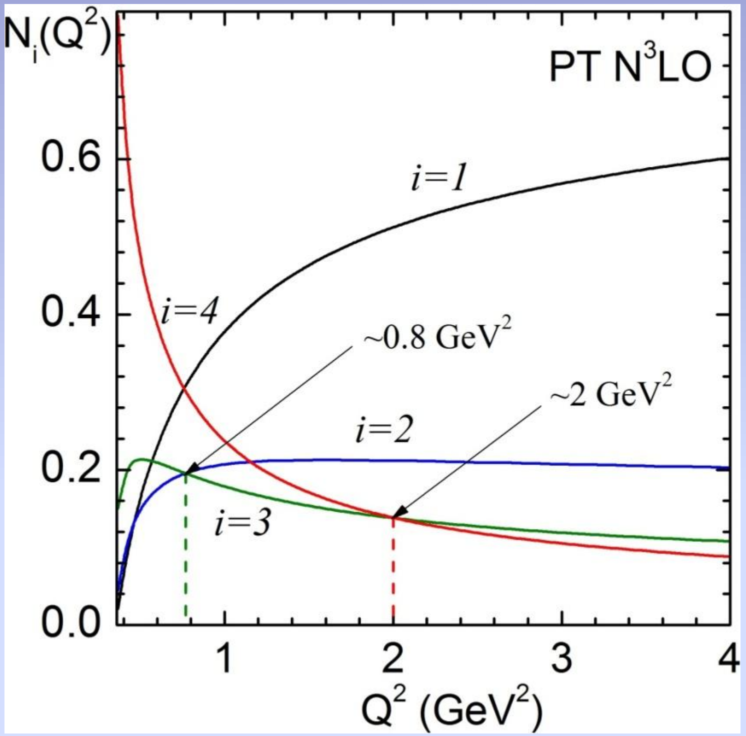
✓ *At $Q^2 \geq 0.8$ GeV* - quite well. However, the experimental accuracy (which is of the same order as both the three- and four-loop contributions) does not allow one to make a definite choice between four- and three-loop approximations.

Convergence properties of the PT expansion

$$\Delta_{Bj}\left(\alpha_S\right)=c_1\alpha_S+c_2\alpha_S^2+c_3\alpha_S^3+c_4\alpha_S^4$$

$$\mathrm{N}_i\left(Q^2\right)=\frac{c_i\alpha_S^i\left(Q^2\right)}{\Delta_{Bj}\left(Q^2\right)}$$





✓ *The fourth term becomes greater than the third at α_S ~ 0.35 and than the second at α_S ~ 0.45*

✓ *The third term is greater than the second at Q^2 ~0.8 GeV^2. The fourth term becomes greater than the third at Q^2 ~2 GeV^2*

This situation may be considered as an indication of the transition of PT series to the asymptotic regime.

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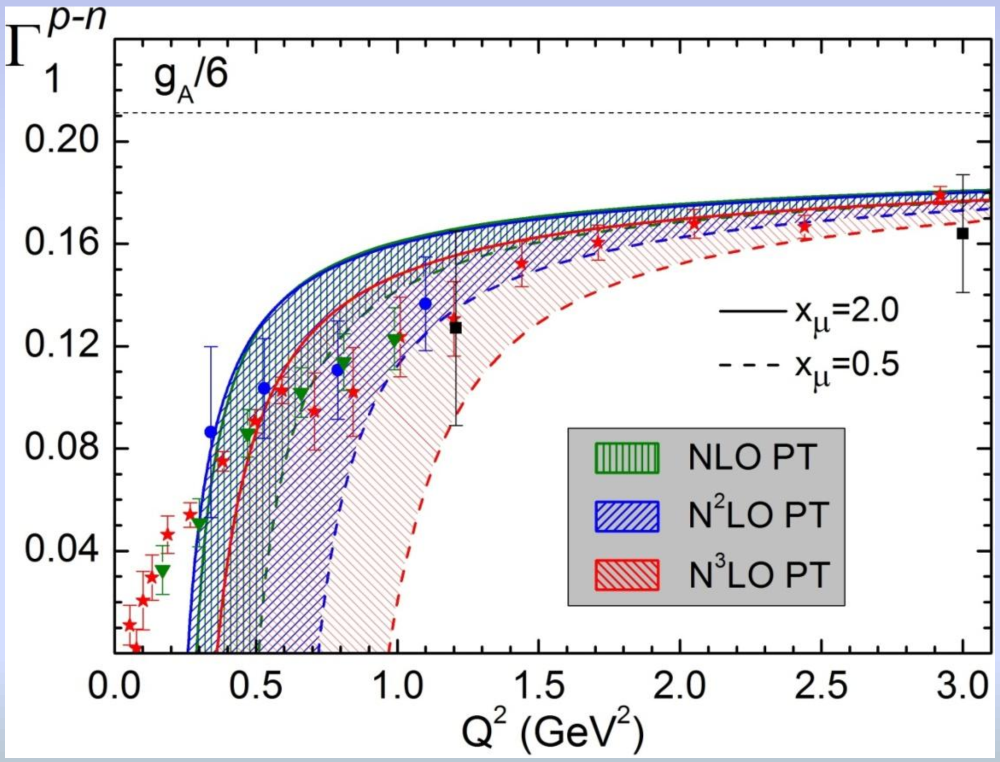
The dependence on the renormalization scale μ

$$C_{Bj}\left(x_\mu,Q^2\right)=1-0.318\alpha_S+\left[-0.363-0.228\ln\left(x_\mu\right)\right]\alpha_S^2$$
$$+\left[-0.652-0.649\ln\left(x_\mu\right)-0.163\ln^2\left(x_\mu\right)\right]\alpha_S^3$$
$$+\left[-1.804-1.798\ln\left(x_\mu\right)-0.7897\ln^2\left(x_\mu\right)-0.1169\ln^3\left(x_\mu\right)\right]\alpha_S^4$$

[Baikov, Chetyrkin, Kühn (2010)]

$$x_\mu=\frac{\mu^2}{Q^2}$$

$$x_\mu=0.5\;\div 2.0$$



The width of the arising strip for the four-loop expression is close to the one for the three-loop and more than two-loop approximation

The four-loop result does not improve noticeably the data description in the low-energy domain.

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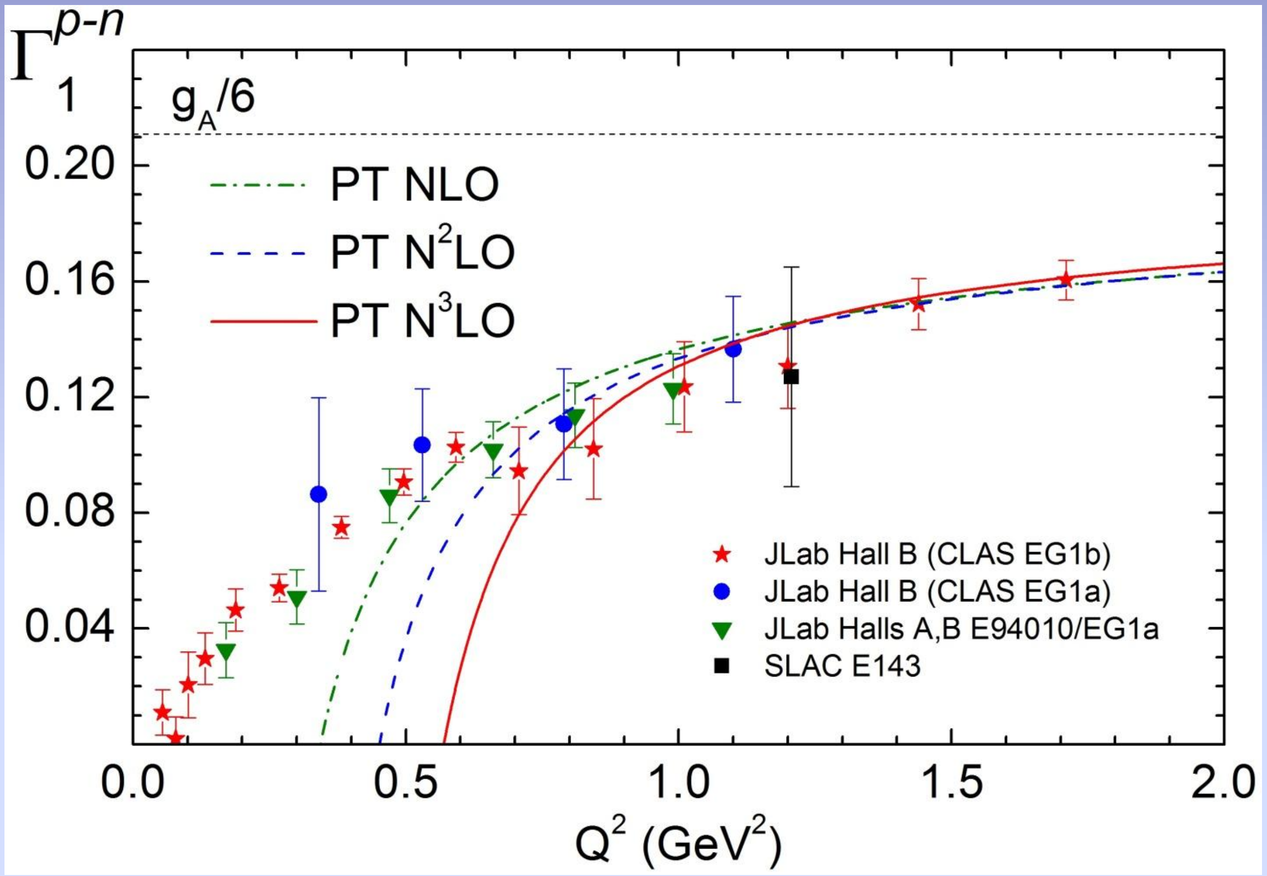
μ_4 -fit results



The one-parametric fits of the BSR JLab data in orders of the PT.

The four-loop result does not improve the data description compared to the three-loop one.

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Results of μ_4 extraction in various orders of PT with the left border Q_{min}^2 [in GeV^2] of fitting domain

68% C.L.

	Q^2_{min}	μ_4, GeV^2	χ^2/D.o.f
NLO PT	0.5	-0.025±0.004	0.80
N^{2}LO PT	0.66	-0.012±0.006	0.59
N^{3}LO PT	0.707	0.005±0.008	0.51

Coefficient of the twist -4

μ_4^{p-n}	Ref.
-0.06±0.03	Balitsky, Braun, Kolesnichenko (1990)
-0.22±0.12	Ross, Roberts (1994)
-0.03±0.01	Stein (1995)
-0.03±0.06	Ioffe (1997)
-0.09±0.03	Balla, Polyakov, Weiss (1998)
-0.10±0.07	Sidorov, Weiss (2006)

- *The lower border shifts up* to higher Q^2 scales with increasing of the PT expansion order.
- The absolute value of μ_4 decreases with the order of PT and just at four-loop order becomes compatible to zero.
- This may be considered as a manifestation of *duality between higher orders of PT and HT* [Zakharov (1990+), Kataev, Kotikov, Parente, Sidorov (1998); Narison&Zakharov (2009)]

Sensitivity of the HT to Λ_{QCD} variations

In the direct QCD analysis of the experimental data on the moments of the structure functions may be obtained different values of the QCD scale parameter Λ .
Having this in mind, we investigate additionally the sensitivity of the extracted values of the higher twist term to the Λ in various orders of PT.

	$\alpha_s(M_Z^2)$
BBG	$0.1134\ ^{+0.0019}_{-0.0021}$
GRS	0.112
ABKM	0.1135 ± 0.0014
ABKM	0.1129 ± 0.0014
JR	0.1124 ± 0.0020
JR	0.1158 ± 0.0035
MSTW	0.1171 ± 0.0014
ABM	0.1147 ± 0.0012
Gehrmann et al.	$0.1153 \pm 0.0017 \pm 0.0023$
Abbate et al.	$0.1135 \pm 0.0011 \pm 0.0006$
BBG	$0.1141\ ^{+0.0020}_{-0.0022}$
world average	0.1184 ± 0.00

S. Alekhin[1,2], J. Blümlein

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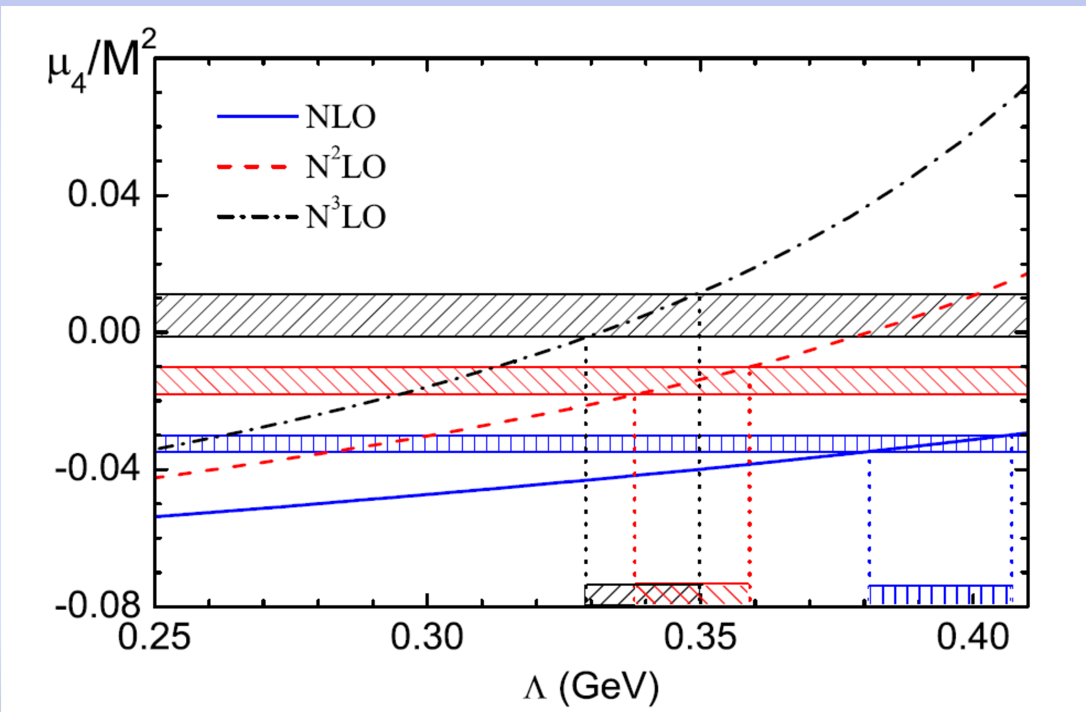

✓ The PT does not lead to a stable results:
the extracted coefficient μ_4 changes quite strongly between different orders of the PT expansion.

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New step: APT approach

The moments of the structure functions are analytic functions in the complex Q^2 -plane with a cut along the negative part of the real axis (see, e.g., [Solovtsov (2000)]).
The pQCD representation $\Delta_{Bj}(Q^2)$ violates these analytic properties due to the unphysical singularities of $\alpha_S(Q^2)$. To resolve the issue, we apply the *APT method*, which allows one to combine the RG-invariance with proper analytical properties of the RG-invariant .

Historical overview

First application of APT to the BSR

1998

"The Bjorken sum rule in the analytic approach to perturbative QCD"
Milton, Solovtsov, Solovtsova

Next application of the APT to the BSR

2008

"Bjorken sum rule and perturbative QCD frontier on the move"
Pasechnik, Shirkov, Teryaev

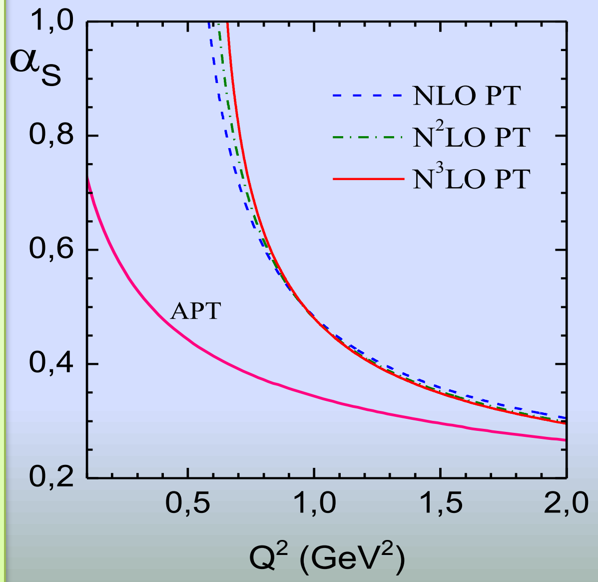
2010

"Nucleon spin structure and perturbative QCD frontier on the move"
Pasechnik, Shirkov, Teryaev, Solovtsova, Khandramai

2011

"Four-loop QCD analysis of the Bjorken sum rule vs data"

For an overview on the APT concept and results see [Shirkov, Solovtsov (2007)]



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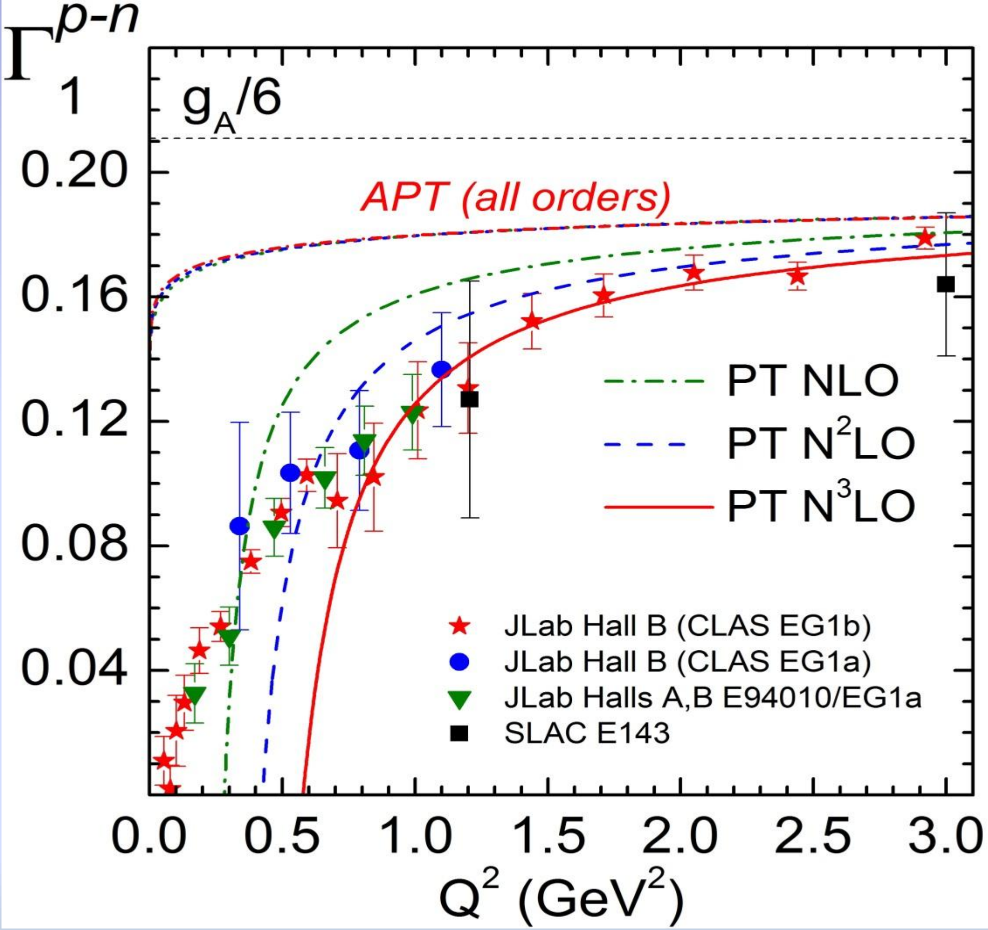
APT & BSR

$$\Delta_{Bj}^{APT}\left(Q^2\right)=0.31831\mathcal{A}_1^{(4)}+0.36307\mathcal{A}_2^{(4)}+0.65197\mathcal{A}_3^{(4)}+1.8042\mathcal{A}_4^{(4)}$$

$$\mathcal{A}_k^{(l)}\left(Q^2\right)=\frac{1}{\pi}\int_0^{+\infty}\frac{\rho_k(\sigma)\,d\sigma}{\sigma+Q^2}$$

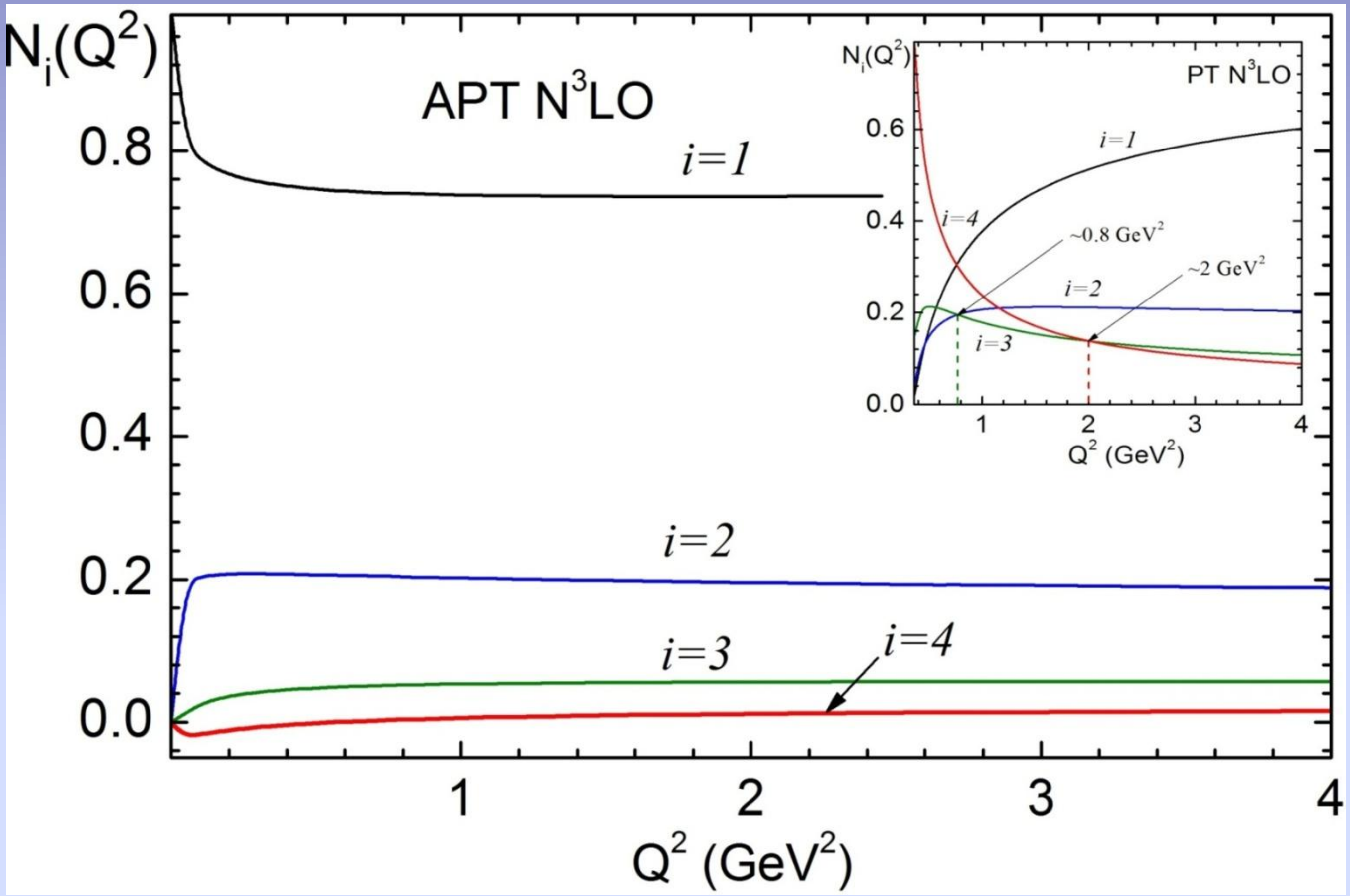
$$\rho_k(\sigma)=\mathrm{Im}\left(\left[\alpha_S^{(l)}\left(-\sigma,n_f\right)\right]^k\right)$$

At large momentum transfers, the APT expansion reduces to the power PT series. However, at small enough Q^2 the properties of the non-power APT expansion become considerably different from the PT power series.



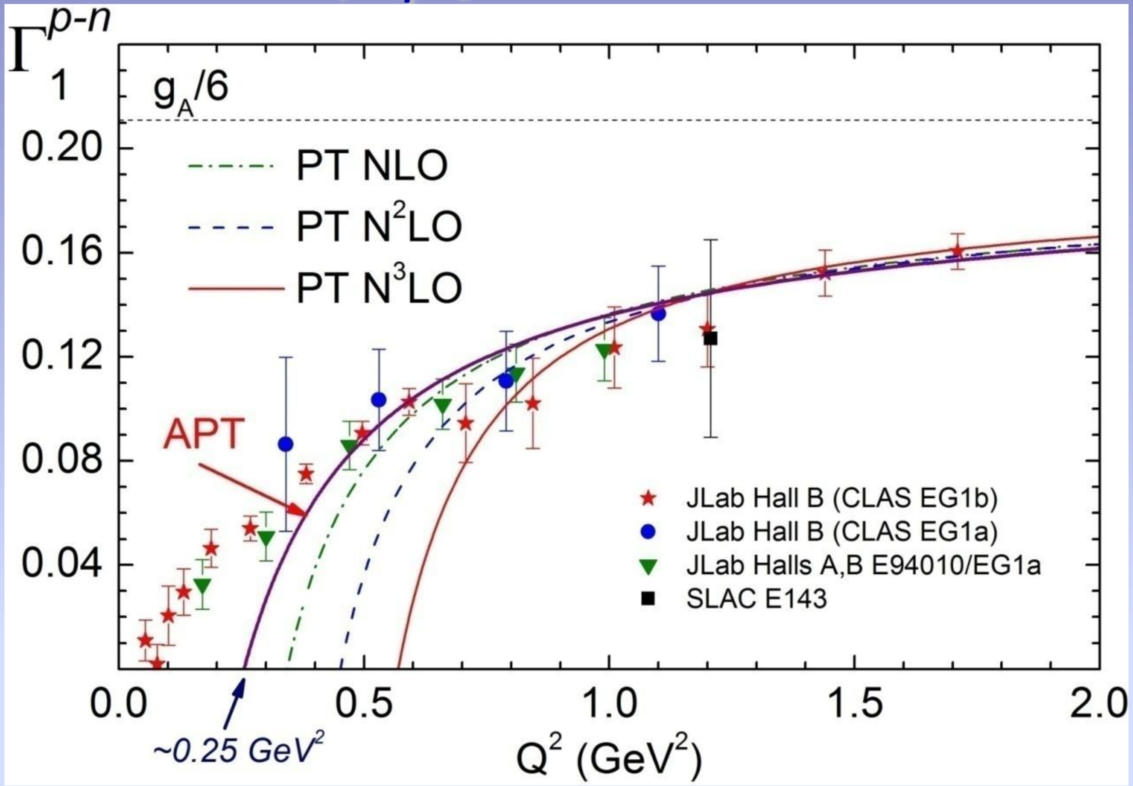
➢ Higher-loop stability is achieved. This is a well-known feature of the APT free from unphysical singularities.

The relative contributions in APT



In the APT case, the higher order contributions are stable at all Q^2 values, and one-loop contribution gives about 70 %, two-loop – 20 %, three-loop – not exceeds 5 %, and four-loop -- up to 1 %.

μ_4-fit results



APT allows to describe data down to low values of Q^2 ~0.25GeV^2 and gives stable extracted values of HT coefficient.

	Q^2_{min}	μ_4, GeV^2	χ^2/D.o.f
NLO PT	0.5	-0.025±0.004	0.80
N^{2}LO PT	0.66	-0.012±0.006	0.59
N^{3}LO PT	0.707	0.005±0.008	0.51
APT	0.47	-0.043±0.002	0.82

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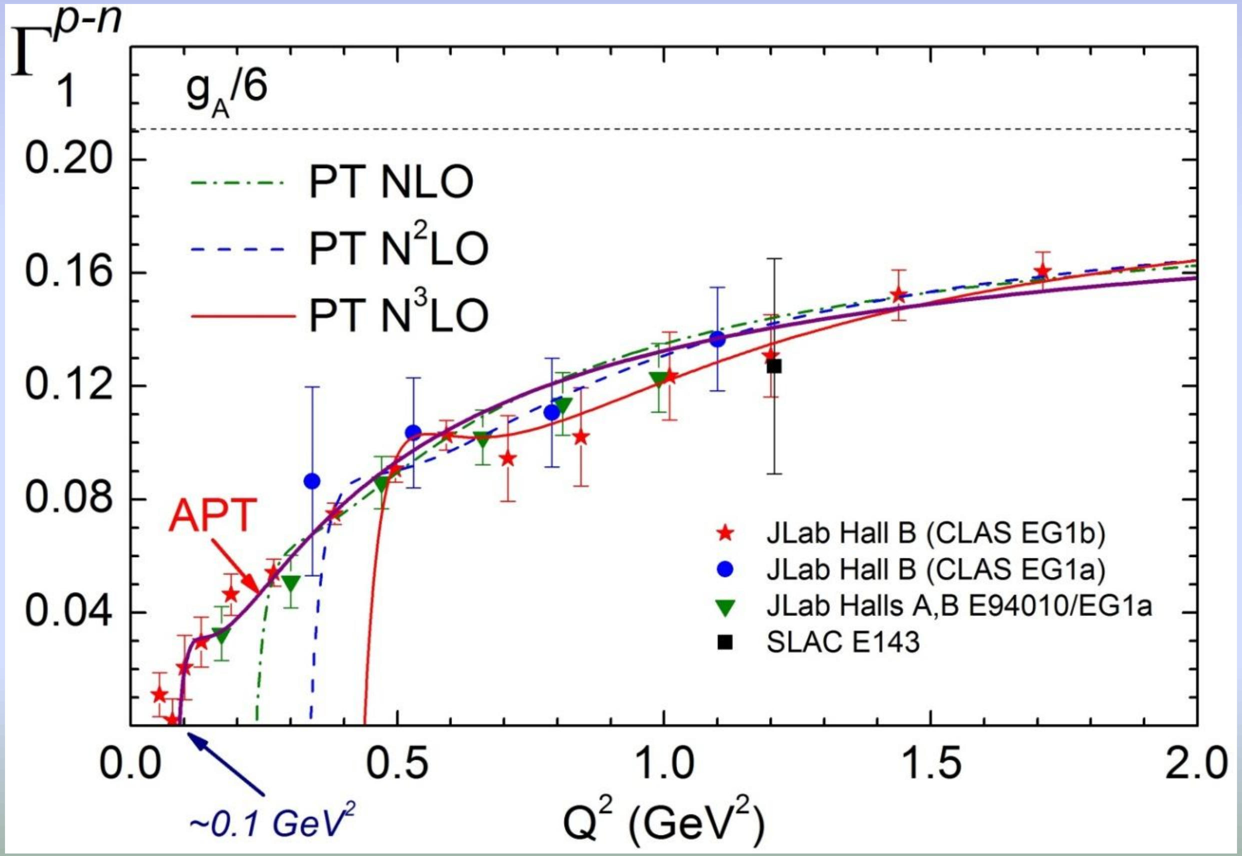
$\mu_{4,6,8}$-fit results

Results of $\mu_{4,6,8}$-extraction with left border Q_{min}^2 [in GeV²]

	Q^2_{min}	μ_4/M^2	μ_6/M^4	μ_8/M^6	χ^2/D.o.f
NLO	0.268	-0.03(1)	-0.01(1)	0.008(4)	0.69
N^2LO	0.34	0.01(2)	-0.06(4)	0.04(2)	0.67
N^3LO	0.47	0.05(4)	-0.2(1)	0.12(6)	0.46
APT	0.08	-0.061(2)	0.009(1)	-0.0004(1)	0.91

PT:

The coefficients of higher-twists strongly changes from order to order



APT

Higher-twist series converges and is stable in all orders

APT allows to describe data down to low values of $Q^2 \sim 0.1$ GeV²

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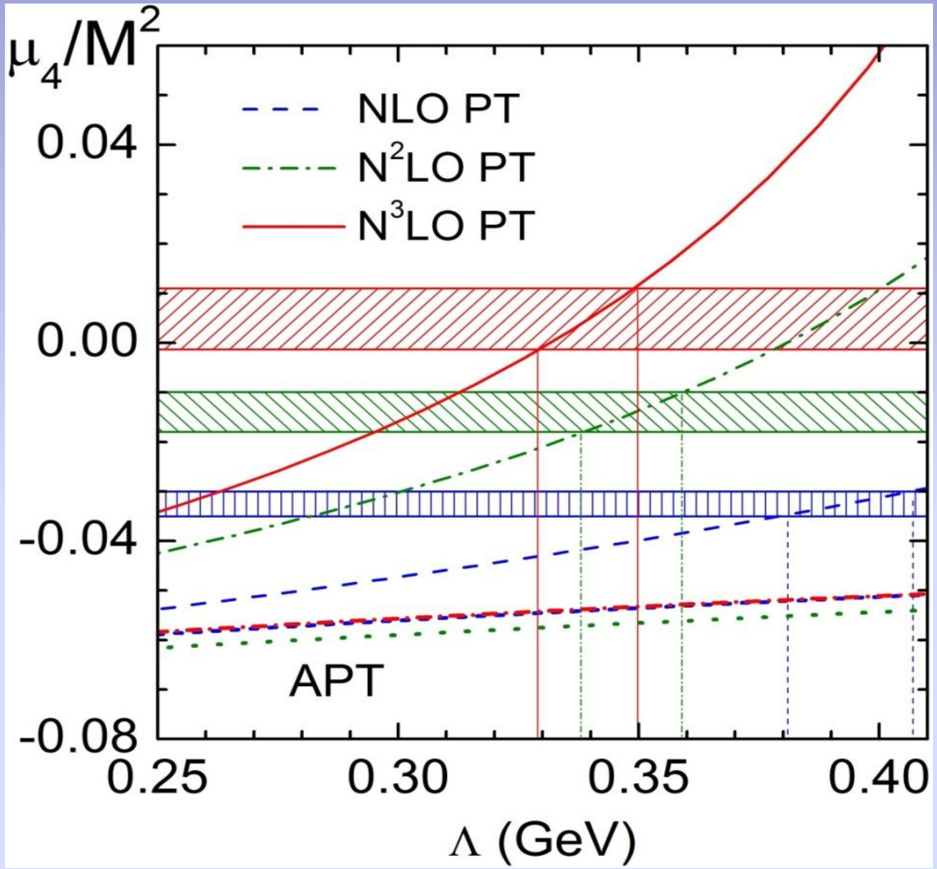
Sensitivity of the HT to Λ_{QCD} variations



✓ **The PT does not lead to a stable results:**
the extracted coefficient μ_4 changes quite strongly between different orders of the PT expansion.

✓ **In the framework of APT, the sensitivity of μ_4 to the Λ is weak, and it does not depend on the order of the loop expansion. Correspondingly, the values of the HT coefficients turn out to be considerably more precise than those extracted in the PT approach.**

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Summary and discussion

We performed the QCD analysis of the precise low energy JLab data on the BSR in the N^3LO PT order and extracted the OPE higher twist terms using the four-loop expression for the QCD correction to the Bjorken integral published recently.

- ✓ The four-loop approximation provides good description of the data for the highest JLab $Q^2 \sim 3$ GeV^2. For several data points there is an impression that the four-loop approximation is better than the three-loop one. At the same time, the order of magnitude of both these contributions is the same as an experimental error, so a more precise statement can hardly be made.
- ✓ For lower $Q^2 \leq 0.7$ GeV^2 the four-loop PT contribution does not help to describe the data. Meanwhile, the APT application leads to higher loops stability of the HT extraction. In turn, this results in accurate data description down to $Q^2 \sim 0.1$ GeV^2 always at the two-loop APT level.
- ✓ The magnitude of HT decreases with an order of PT and becomes compatible to zero at the four-loop level.

The above features may indicate that the asymptotic nature of the QCD PT series is revealed at the four-loop level at $Q^2 \sim 1$ -2 GeV^2.


Thanks for your
attention !


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