# Bifurcation sets in extensions of *Higgs Potential*

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## *Outline*

- One-dimensional picture of T evolution
- Two-dimensional picture ( v1(T), v2(T) )
- Bifurcation sets in the catastrophe theory

M. Dolgopolov, M. Dubinin, E. Rykova Threshold corrections to the MSSM finite-temperature Higgs potential. *Journal of Modern Physics, 2011, 2, 301-322* ; Electroweak phase transition beyond the Standard Model. *QFTHEP2010 Proc*.; *Physics of Atomic Nuclei, 2010, Vol. 73, No. 6, pp. 1032–1036*

*The absence of antimatter in the Universe, a small ratio of the observed number of baryons to the observed number of photons and the absence of light CP-even Higgs boson signal at LEP2 and Tevatron energies lay a specific claims to models of particle physics*

- **Two problems in the Standard Model**
  - **First order phase transition requires $m_h < 50$ GeV**
  - **Need new sources of CP violation**
- **Supersymmetric Models**
  - **1st order phase transition is possible**
  - **New CP violating phases**

# One-dimensional picture.
# The effective high temperature MSSM potential

$\varphi^2 = v^2_1 + v^2_2$, $\quad$ $\mathrm{tg}\,\beta = v_2/v_1$

$$V_{\rm eff}(\phi, T) \simeq \left[a(\theta)T^2 - b(\theta)\right]\varphi^2 - ET\varphi^3 + \frac{1}{4}\lambda_T(\theta)\varphi^4 \,,$$

where

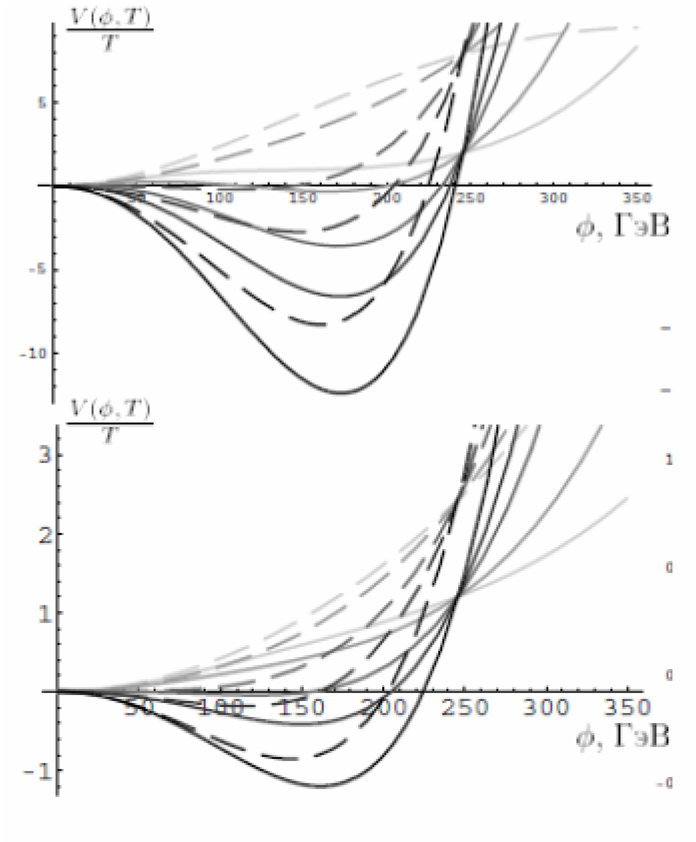
$$a(\theta) = \frac{3g^2 + g'^2}{16} + \frac{h_t^2}{4}\sin^2\theta\,,$$

$$b(\theta) = \frac{m_Z^2}{2}\cos 2\beta \cos 2\theta - m_A^2 \sin^2(\beta - \theta) + \frac{3h_t^2}{8\pi^2}\sin^2\theta\, m_t^2\left(1 + \log\frac{\tilde{m}^2}{m_t^2}\right)\,,$$

$$E = \frac{2}{3}\frac{\sqrt{2}}{16\pi}\left[2g^3 + (g^2 + g'^2)^{3/2}\right]\,,$$

$$\lambda_T(\theta) = \frac{1}{2}(g^2 + g'^2)\cos^2 2\theta + \frac{3h_t^4}{4\pi^2}\sin^4\theta\left(\log\frac{\tilde{m}^2}{T^2} - 1.14\right)\,.$$

A.Brignole, J.Espinosa, M.Quiros, F.Zwirner, PL B324 (1994) 181




From D.Gorbunov, V.Rubakov, solid lines — numerical calculation, dashed — high T expansion, for different T and mH

# THDM: Fields

Georgi: A Model Of Soft CP Violation. 1978
Lee: A Theory Of Spontaneous T Violation. 1973

$$\Phi_1 = \begin{pmatrix} \phi_1^+(x) \\ \phi_1^0(x) \end{pmatrix} = \begin{pmatrix} -i\omega_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) \end{pmatrix},$$

$$\Phi_2 = e^{i\xi} \begin{pmatrix} \phi_2^+(x) \\ \phi_2^0(x) \end{pmatrix} = e^{i\xi} \begin{pmatrix} -i\omega_2^+ \\ \frac{1}{\sqrt{2}}(v_2 e^{i\zeta} + \eta_2 + i\chi_2) \end{pmatrix}$$

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle\Phi_2\rangle = \frac{e^{i\xi}}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i\zeta} \end{pmatrix} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i\theta} \end{pmatrix}.$$

$$\mathrm{tg}\,\beta = \frac{v_2}{v_1}, \qquad v^2 \equiv v_1^2 + v_2^2 = (246\ \mathrm{GeV})^2.$$

**Ilya F. Ginzburg, M. Krawczyk,**
**Symmetries of two Higgs doublet model and CP violation. Phys.Rev.D72,2005.**

**Akhmetzyanova E.N., *D M.V.*, Dubinin M.N.**
**Higgs Bosons in the Two-Doublet Model with CP Violation Phys.Rev.D71.2005.**
**Violation of CP invariance in the two-doublet Higgs sector of the MSSM. Phys.Part.Nucl.37,2006.**

# Scalar sector for MSSM

**The main contribution to self-couplings due to Yukawa 3rd generation couplings.**

**The corresponding potential with CPV sources**

$$\mathcal{V}^0 = \mathcal{V}_M + \mathcal{V}_\Gamma + \mathcal{V}_\Lambda + \mathcal{V}_{\widetilde{Q}}\,,$$

$$\mathcal{V}_M = (-1)^{i+j} m_{ij}^2 \Phi_i^\dagger \Phi_j + M_{\widetilde{Q}}^2 \left(\widetilde{Q}^\dagger \widetilde{Q}\right) + M_{\widetilde{U}}^2 \widetilde{U}^* \widetilde{U} + M_{\widetilde{D}}^2 \widetilde{D}^* \widetilde{D}\,,$$

$$\mathcal{V}_\Gamma = \Gamma_i^D \left(\Phi_i^\dagger \widetilde{Q}\right) \widetilde{D} + \Gamma_i^U \left(i\Phi_i^T \sigma_2 \widetilde{Q}\right) \widetilde{U} + \overset{*}{\Gamma}{}_i^D \left(\widetilde{Q}^\dagger \Phi_i\right) \widetilde{D}^* - \overset{*}{\Gamma}{}_i^U \left(i\widetilde{Q}^\dagger \sigma_2 \Phi_i^*\right) \widetilde{U}^*$$

$$\mathcal{V}_\Lambda = \Lambda_{ik}^{jl} \left(\Phi_i^\dagger \Phi_j\right)\left(\Phi_k^\dagger \Phi_l\right) + \left(\Phi_i^\dagger \Phi_j\right)\left[\Lambda_{ij}^Q \left(\widetilde{Q}^\dagger \widetilde{Q}\right) + \Lambda_{ij}^U \widetilde{U}^* \widetilde{U} + \Lambda_{ij}^D \widetilde{D}^* \widetilde{D}\right] +$$

$$+\overline{\Lambda}_{ij}^Q \left(\Phi_i^\dagger \widetilde{Q}\right)\left(\widetilde{Q}^\dagger \Phi_j\right) + \frac{1}{2}\left[\Lambda \epsilon_{ij} \left(i\Phi_i^T \sigma_2 \Phi_j\right) \widetilde{D}^* \widetilde{U} + \text{э.с.}\right]\,, \quad i, j, k, l = 1, 2$$

$$\Gamma_{\{1;\,2\}}^U = h_U \{-\mu^*; A_U\}, \qquad \Gamma_{\{1;\,2\}}^D = h_D \{A_D; -\mu^*\}$$

# Effective THDM potential with explicit CP violation

**General hermitian renormalized *SU(2)*x*U(1)* invariant potential:**

$$U(\Phi_1,\Phi_2) = -\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) - \mu_{12}^2(\Phi_1^\dagger\Phi_2) - \mu_{12}^{*2}(\Phi_2^\dagger\Phi_1) +$$

$$+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) +$$

$$+\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \frac{\lambda_5^*}{2}(\Phi_2^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) +$$

$$+\lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_6^*(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) +$$

$$+\lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \lambda_7^*(\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$

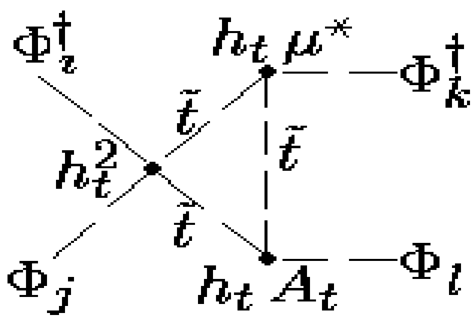
$$\Phi_1^\dagger\Phi_2 \xrightarrow{CP} \Phi_2^\dagger\Phi_1$$

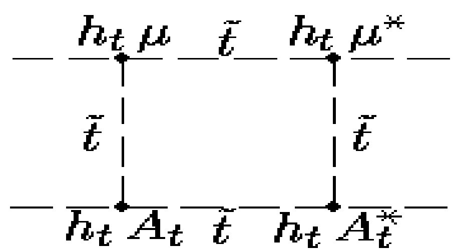
$$\boxed{\lambda_{5,6,7} \xrightarrow{CP} \lambda_{5,6,7}}$$

**$\mu_{12}^2$, $\lambda_5, \lambda_6, \lambda_7$ complex**

**$\varphi = \arg(\lambda_{6,7})$**

**$= \arg(\lambda_5)/2$**




***U* is *CP*-invariant**

**at the $M_{SUSY}$ scale, because $\lambda_{5,6,7}=0$**

**One-loop (t, b) contributions** ~~*CP*~~ $U_{eff}$

$m_{top}$ $M_{SUSY}$ *μ - mass-energy scale*

Eff. potential method
or Feynman diags (temperature T)

# Transformation of SU(2) eigenstates to mass eigenstates

$$U_{eff}(\Phi_1,\Phi_2) = -\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) - \mu_{12}^2(\Phi_1^\dagger\Phi_2) - \overset{*}{\mu_{12}^2}(\Phi_2^\dagger\Phi_1) + \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2$$

$$+\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \frac{\lambda_5^*}{2}(\Phi_2^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) +$$

$$+\lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \overset{*}{\lambda_6}(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \overset{*}{\lambda_7}(\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$

▼

$$U_{eff}(\Phi_1,\Phi_2) \Longrightarrow \frac{m_h^2}{2}(hh) + \frac{m_H^2}{2}(HH) + \frac{m_A^2}{2}(AA) + m_{H^\pm}^2(H^+H^-) + h, H, A, H^\pm \quad \text{interaction terms}$$

$(s_\alpha = \sin\alpha,\ c_\beta = \cos\beta$ etc.$)$

$$\varphi_1 = \begin{pmatrix} -i*(-H^+ s_\beta + G^+ c_\beta) \\ \frac{1}{\sqrt{2}}[v_1 + Hc_\alpha - hs_\alpha + i*(A^0 c_\beta + G' s_\beta)] \end{pmatrix}$$

$$\varphi_2 = e^{i\xi}\begin{pmatrix} -i*(H^+ c_\beta + G^+ s_\beta) \\ \frac{1}{\sqrt{2}}[v_2 e^{i\zeta} + Hs_\alpha + hc_\alpha + i*(-A^0 s_\beta + G' c_\beta)] \end{pmatrix}$$

$$\mathrm{tg}2\alpha = \frac{s_{2\beta}(m_A^2 + m_Z^2) + v^2((\Delta\bar\lambda_3 + \Delta\bar\lambda_4)s_{2\beta} + 2c_\beta^2\Delta\mathrm{Re}\bar\lambda_6 + 2s_\beta^2\mathrm{Re}\Delta\bar\lambda_7)}{c_{2\beta}(m_A^2 - m_Z^2) + v^2(\Delta\bar\lambda_1 c_\beta^2 - \Delta\bar\lambda_2 s_\beta^2 - \mathrm{Re}\Delta\bar\lambda_5 c_{2\beta} + (\mathrm{Re}\Delta\bar\lambda_6 - \mathrm{Re}\Delta\bar\lambda_7)s_{2\beta})}$$

## lead to the nonlinear equations for effective parameters lambda

$$\lambda_1 = \frac{1}{2v^2}\Big[\Big(\frac{s_\alpha}{c_\beta}\Big)^2 m_h^2 + \Big(\frac{c_\alpha}{c_\beta}\Big)^2 m_H^2 - \frac{s_\beta}{c_\beta^3}\mathrm{Re}\mu_{12}^2\Big] + \frac{1}{4}\big(\mathrm{Re}\lambda_7 \mathrm{tg}^3\beta - 3\mathrm{Re}\lambda_6 \mathrm{tg}\beta\big),$$

$$\lambda_2 = \frac{1}{2v^2}\Big[\Big(\frac{c_\alpha}{s_\beta}\Big)^2 m_h^2 + \Big(\frac{s_\alpha}{s_\beta}\Big)^2 m_H^2 - \frac{c_\beta}{s_\beta^3}\mathrm{Re}\mu_{12}^2\Big] + \frac{1}{4}\big(\mathrm{Re}\lambda_6 \mathrm{ctg}^3\beta - 3\mathrm{Re}\lambda_7 \mathrm{ctg}\beta\big),$$

$$\lambda_3 = \frac{1}{v^2}\Big[2m_{H\pm}^2 - \frac{\mathrm{Re}\mu_{12}^2}{s_\beta c_\beta} + \frac{s_{2\alpha}}{s_{2\beta}}(m_H^2 - m_h^2)\Big] - \frac{\mathrm{Re}\lambda_6}{2}\mathrm{ctg}\beta - \frac{\mathrm{Re}\lambda_7}{2}\mathrm{tg}\beta,$$

$$\lambda_4 = \frac{1}{v^2}\Big(\frac{\mathrm{Re}\mu_{12}^2}{s_\beta c_\beta} + m_A^2 - 2m_{H\pm}^2\Big) - \frac{\mathrm{Re}\lambda_6}{2}\mathrm{ctg}\beta - \frac{\mathrm{Re}\lambda_7}{2}\mathrm{tg}\beta,$$

$$\mathrm{Re}\lambda_5 = \frac{1}{v^2}\Big(\frac{\mathrm{Re}\mu_{12}^2}{s_\beta c_\beta} - m_A^2\Big) - \frac{\mathrm{Re}\lambda_6}{2}\mathrm{ctg}\beta - \frac{\mathrm{Re}\lambda_7}{2}\mathrm{tg}\beta,$$

M.Dubinin., A. Semenov, Eur.J.Phys. C28 (2003) 223
M.D., M.Dubinin.,E.Rykova, Phys.Rev. D71 (2005) 075008

**and the minimization conditions for dimension 2 parameters mu**

$$\mu_1^2 = \lambda_1 v_1^2 + (\lambda_3 + \lambda_4 + \mathrm{Re}\lambda_5)\frac{v_2^2}{2} - \mathrm{Re}\mu_{12}^2 \mathrm{tg}\beta + \frac{v^2 s_\beta^2}{2}\big(3\mathrm{Re}\lambda_6 \mathrm{ctg}\beta + \mathrm{Re}\lambda_7 \mathrm{tg}\beta\big),$$

$$\mu_2^2 = \lambda_2 v_2^2 + (\lambda_3 + \lambda_4 + \mathrm{Re}\lambda_5)\frac{v_1^2}{2} - \mathrm{Re}\mu_{12}^2 \mathrm{ctg}\beta + \frac{v^2 c_\beta^2}{2}\big(\mathrm{Re}\lambda_6 \mathrm{ctg}\beta + 3\mathrm{Re}\lambda_7 \mathrm{tg}\beta\big).$$

# Parameters of the effective potential

## (forms of contribitions)

$$\Delta\lambda_1^{thr} = 3h_t^4|\mu|^4 I_2[m_Q,m_U] + 3h_b^4|A_b|^4 I_2[m_Q,m_D]+$$
$$+h_t^2|\mu|^2(-\frac{g_1^2-3g_2^2}{2}I_1[m_Q,m_U] + 2g_1^2 I_1[m_U,m_Q])$$

$$\Delta\lambda_2^{thr} = 3h_t^4|A_t|^4 I_2[m_Q,m_U] + 3h_b^4|\mu|^4 I_2[m_Q,m_D]+$$
$$+h_b^2|\mu|^2(\frac{g_1^2+3g_2^2}{2}I_1[m_Q,m_D] + g_1^2 I_1[m_D,m_Q])+$$
$$+h_t^2|A_t|^2(\frac{12h_t^2+g_1^2-3g_2^2}{2}I_1[m_Q,m_U] + (6h_t^2-2g_1^2)I_1[m_U,m_Q])$$

$$\Delta\lambda_3^{thr} = h_t^2((|\mu|^2\frac{3g_2^2+g_1^2}{12} + |A_t|^2\frac{12h_t^2-g_1^2-3g_2^2}{12})I_1[m_Q,m_U]+$$
$$+(|\mu|^2\frac{3h_t^2-g_1^2}{3} + |A_t|^2\frac{g_1^2}{3})I_1[m_U,m_Q])+$$
$$+(h_b^2(|\mu|^2\frac{3g_2^2-g_1^2}{12} + |A_b|^2\frac{12h_t^2+g_1^2-3g_2^2}{4})I_1[m_Q,m_D]+$$
$$+(|\mu|^2\frac{6h_b^2-g_1^2}{6} + |A_b|^2\frac{g_1^2}{6})I_1[m_D,m_Q])+$$
$$+h_t^2|\mu|^2|A_t|^2 I_2[m_Q,m_U] + h_b^2|\mu|^2|A_b|^2 I_2[m_Q,m_D]+$$
$$+h_t^2h_b^2(2(A_tA_b-|\mu|^2)I_3[m_Q,m_U,m_D] + (|\mu|^4+|A_t|^2|A_b|^2-2A_tA_b|\mu|^2)I_4[m_Q$$

$$\Delta\lambda_4^{thr} = 6h_t^4|\mu|^2|A_t|^2 I_2[m_Q,m_U] + 6h_b^4|\mu|^2|A_b|^2 I_2[m_Q,m_D]+$$
$$+h_t^2((|\mu|^2\frac{12h_t^2+g_1^2-3g_2^2}{4} - |A_t|^2\frac{g_1^2-3g_2^2}{4})I_1[m_Q,m_U]+$$
$$+(|A_t|^2 g_1^2 - |\mu|^2(g_1^2-3h_t^2))I_1[m_U,m_Q])+$$
$$+h_b^2((|\mu|^2\frac{-12h_t^2+g_1^2+3g_2^2}{4} - |A_b|^2\frac{g_1^2+3g_2^2}{4})I_1[m_Q,m_D]+$$
$$+\frac{1}{2}(|A_b|^2 g_1^2 - |\mu|^2(g_1^2-6h_b^2))I_1[m_D,m_Q]) - \Delta\lambda_3^{th}$$

$$\Delta\lambda_5^{thr} = 3h_t^4\mu^2 A_t^2 I_2[m_Q,m_U] + 3h_b^4\mu^2 A_b^2 I_2[m_Q,m_D]$$

In the MSSM we calculate the 1-loop FT corrections from the squarks-Higgs bosons sector, reconstruct the effective two-Higgs-doublet potential and study possibilities of the EWPT in the full MSSM (**$m_{H\pm}$, tgβ, $A_{t,b}$, μ, $m_Q$, $m_U$, $m_D$**) parameter space.

$$I_1[M_a,M_b] = \frac{T}{2M_a}\frac{\partial}{\partial M_a}I_0 = -\frac{1}{64\pi^4T^2}\int_0^1 dx\, x\, \zeta(2,\frac{3}{2},M^2),$$

$$I_2[M_a,M_b] = -\frac{1}{2M_b}\frac{\partial}{\partial M_b}(-I_1) = \frac{3}{256\pi^6T^4}\int_0^1 dx\, x\,(1-x)\,\zeta(2,\frac{5}{2},M^2).$$

$$\zeta(u,s,t) = \sum_{n=1}^{\infty}\frac{1}{(n^u+t)^s}.$$

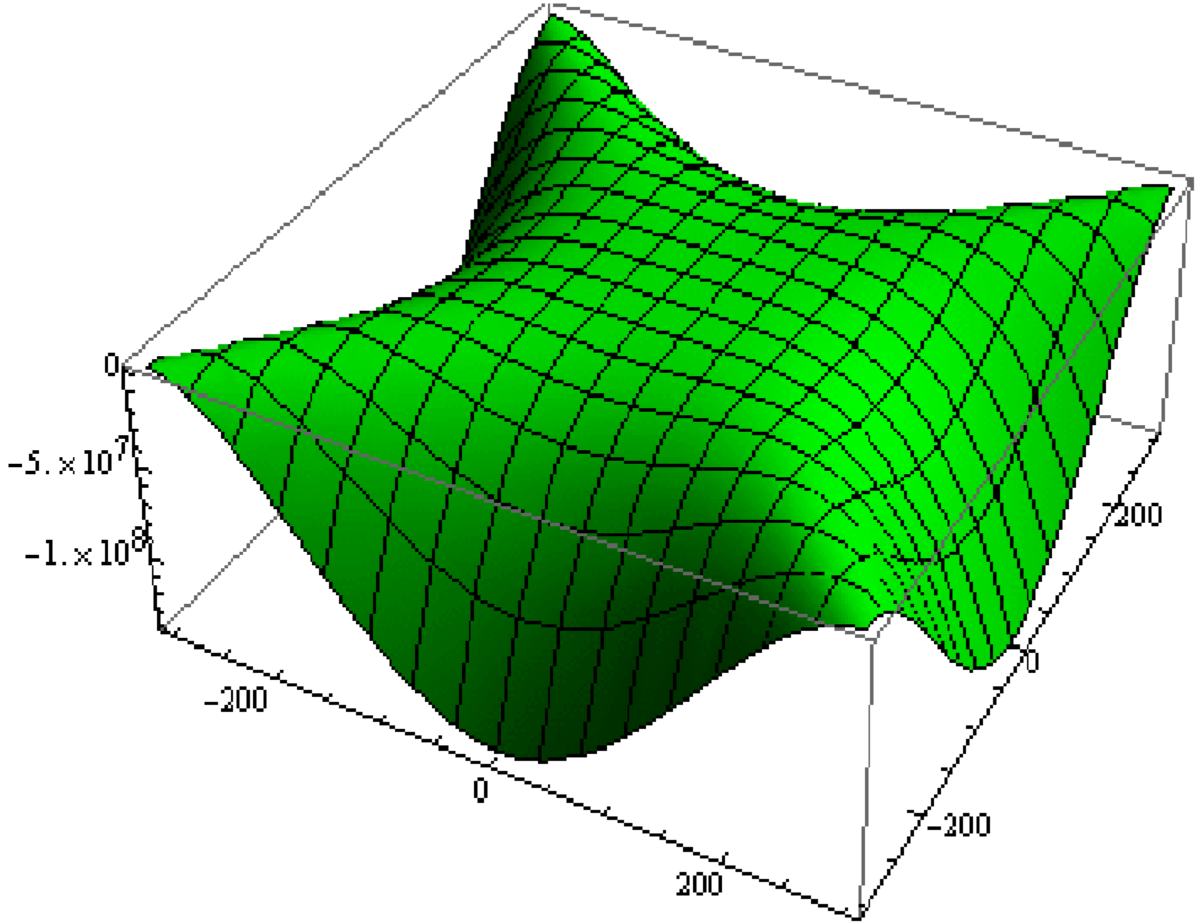
**Hurwitz zeta-function**

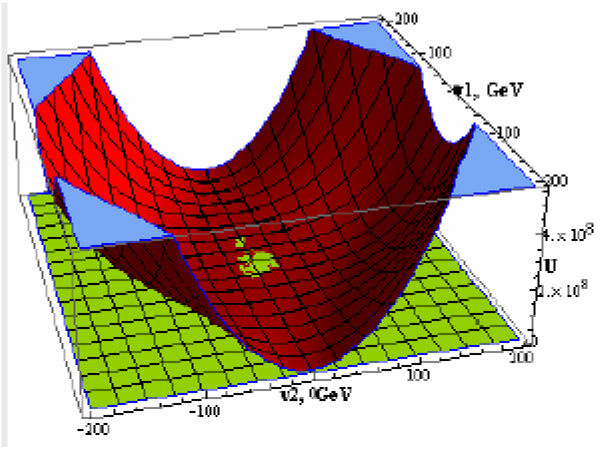
$$\Delta\lambda_5 = 3h_t^4\mu^2A^2I_2[m_Q,m_t] + 3h_b^4\mu^2A^2I_2[m_Q,m_b]$$

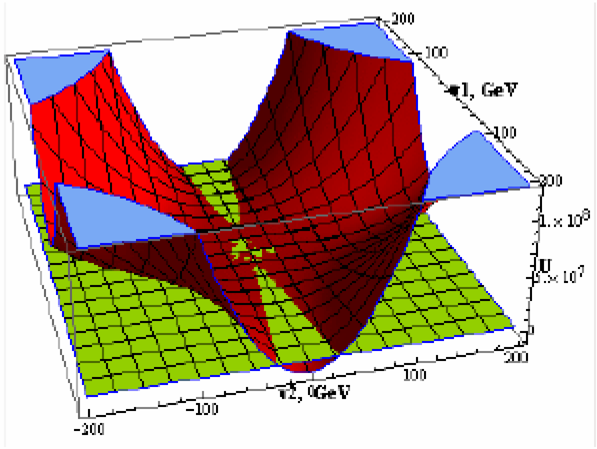
$$\Delta\lambda_6 = -3h_t^4\mu A|\mu|^2I_2[m_Q,m_t] - 3h_b^4\mu A|A|^2I_2[m_Q,m_b]+$$
$$+h_t^2\mu A(\frac{g_1^2-3g_2^2}{4}I_1[m_Q,m_t] - g_1^2I_1[m_t,m_Q])+$$
$$+h_b^2\mu A(\frac{-12h_b^2+g_1^2+3g_2^2}{4}I_1[m_Q,m_b] - \frac{6h_b^2-g_1^2}{2}I_1[m_b,m_Q])$$

$$\Delta\lambda_7 = -3h_t^4\mu A|A|^2I_2[m_Q,m_t] - 3h_b^4\mu A|\mu|^2I_2[m_Q,m_b]+$$
$$+h_b^2\mu A(-\frac{g_1^2+3g_2^2}{4}I_1[m_Q,m_b] - \frac{g_1^2}{2}I_1[m_b,m_Q])+$$
$$+h_t^2\mu A(\frac{12h_t^2+g_1^2-3g_2^2}{4}I_1[m_Q,m_t] - (3h_t^2-g_1^2)I_1[m_t,m_Q])$$

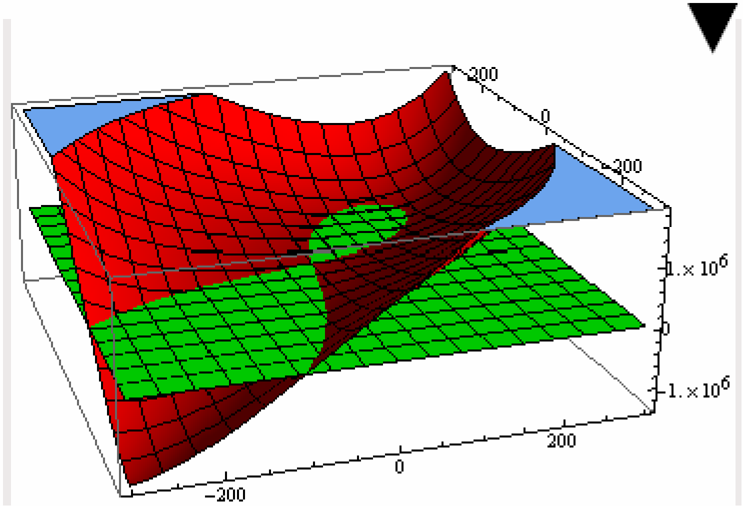
## The *surface of minima* for zero-temperature two-doublet Higgs potential at the scale $M_{SUSY}$






Two-dimensional effective potential in the v1, v2 plane
(in red, U=0 surface in green colour)



# Effective potential at finite temperature

$$v_1(T) = v(T)\cos\bar{\beta}(T), \quad v_2(T) = v(T)\sin\bar{\beta}(T)$$

Mass term

$$U_{mass}(v,\bar{\beta}) = -\frac{v^2}{2}(\mu_1^2\cos^2\bar{\beta} + \mu_2^2\sin^2\bar{\beta}) - \frac{v^2}{2}\mu_{12}^2\sin 2\bar{\beta}$$

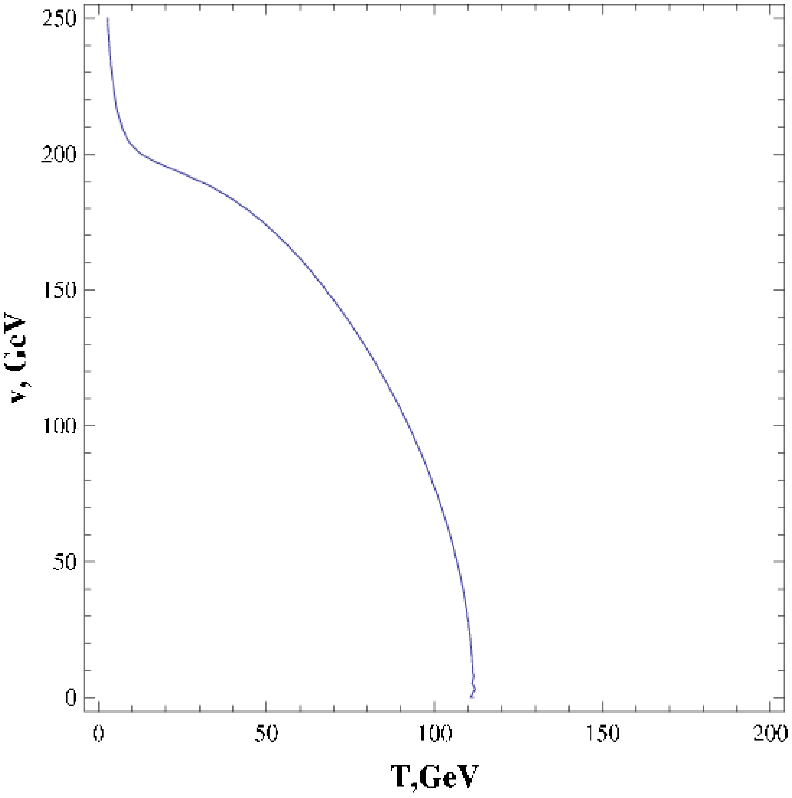
Critical temperature determination

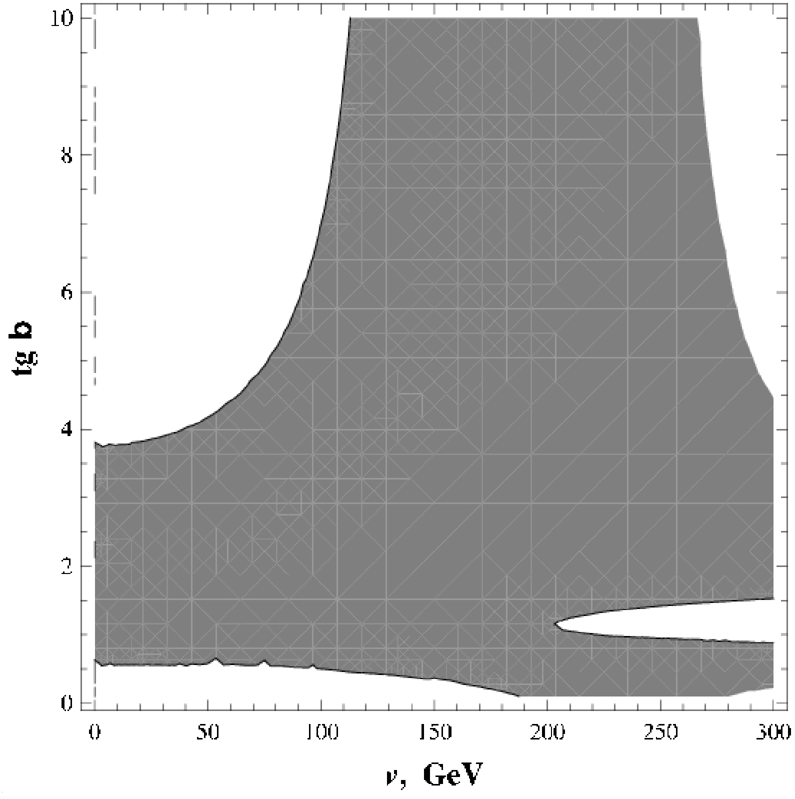
$$\partial U_{mass}/\partial v = 0 \qquad 1/v\ \partial U_{mass}/\partial\bar{\beta} = 0$$

$$\mathtt{tg}2\bar{\beta} = \frac{2\mu_{12}^2}{\mu_1^2 - \mu_2^2}, \qquad (\mu_1^2\mu_2^2 - \mu_{12}^4)[(\mu_1^2 - \mu_2^2)^2 + 4\mu_{12}^4] = 0$$

$$\mu_1^2\mu_2^2 = \mu_{12}^4$$

# Evolution of the critical parameters



# Effective potential U(v1,v2) at the critical temperature and nonzero $\lambda_6$ $\lambda_7$

$$\mathtt{tg}2\bar{\beta} = \mathtt{tg}2\beta \, \frac{1}{\left(\frac{v^2}{2m_A^2} - \alpha_1\right)} \, \frac{1}{\frac{2\lambda_1\cos^2\beta - 2\lambda_2\sin^2\beta}{\cos 2\beta} - \lambda_{345} + \frac{2m_A^2}{v^2} + \alpha_2}$$

$$\alpha_1 = \frac{\lambda_5}{2} + \frac{1}{4}(\lambda_6 \mathtt{ctg}\beta + \lambda_7 \mathtt{tg}\beta),$$

$$\alpha_2 = \lambda_6(\mathtt{tg}2\beta - \mathtt{ctg}\beta) - \lambda_7(\mathtt{tg}\beta + \mathtt{tg}2\beta).$$

$$-\frac{m_A^2}{v^2}(2\lambda_5 + \lambda_6 \, \mathtt{ctg}\beta + \lambda_7 \, \mathtt{tg}\beta) +$$

$$+ \frac{v^2}{m_A^2}\left[\frac{2\lambda_1 - 2\lambda_2 \mathtt{tg}^2\beta + \lambda_6(3\mathtt{tg}\beta - \mathtt{ctg}\beta) + \lambda_7(\mathtt{tg}^3\beta - 3\mathtt{tg}\beta)}{1 - \mathtt{tg}^2\beta} - \lambda_{345}\right] = 0.$$

$$\lambda_1 (2\lambda_2 - \lambda_{345})^2 + \lambda_2 (2\lambda_1 - \lambda_{345})^2 + \lambda_{345}(2\lambda_1 - \lambda_{345})(2\lambda_2 - \lambda_{345}) = 0$$

| *Evolution parameters* | *Control parameters* |
|---|---|

$U_{eff}$ ( $v_1(T)$ , $v_2(T)$ | $\lambda_1(T)$, $\lambda_2(T)$, $\lambda_3(T)$, $\lambda_4(T)$, $\lambda_5(T)$, $\lambda_6(T)$, $\lambda_7(T)$ )

Implicitly $\lambda_{1...7}$ ( T | $m_U$, $m_D$, $m_Q$, $A_t$, $A_b$, μ)

▼

**First order phase transition with Shaposhnikov criteria vc / Tc > 1**
**Where is does take place?**

▲

**Higgs boson masses mh(T), mH(T), mA(T) which are always positively defined and large enough at low T**

**Mixing angles α, β which respect some phenomenological constraintsat low T**

## General formalism is known as the theory of catastrophes

$$U(v_1, v_2) = -\frac{\mu_1^2}{2}v_1^2 - \frac{\mu_2^2}{2}v_2^2 - \mu_{12}^2 v_1 v_2 + \frac{\lambda_1}{4}v_1^4 + \frac{\lambda_2}{4}v_2^4 + \frac{\lambda_{345}}{4}v_1^2 v_2^2 + \frac{\lambda_6}{2}v_1^3 v_2 + +\frac{\lambda_7}{2}v_1 v_2^3$$

$$\nabla U(v_1, v_2) = 0$$

Isolated (nondegenerate) critical points

$$\det \partial U/\partial v_i \partial v_j = 0$$

Nonisolated (degenerate) critical points. Defines «bifurcation sets» as zero det of the equilibrium matrix (Hessian)

$$det \begin{Vmatrix} 2\lambda_1 v_1^2 + \mu_{12}^2 \frac{v_2}{v_1} & -\mu_{12}^2 + \lambda_{345} v_1 v_2 \\ -\mu_{12}^2 + \lambda_{345} v_1 v_2 & 2\lambda_2 v_2^2 + \mu_{12}^2 \frac{v_1}{v_2} \end{Vmatrix} = 0$$

**V.I. Arnold, Critical points of smooth functions and their canonical forms, Uspekhi Math. Nauk (USSR), 30 (1975) 3**

**R. Thom, Structural stability and morphogenesis, Reading, Benjamin, 1975**

**M. Morse, The critical points of a function of n variables, Trans. Am. Math. Soc., 33 (1931) 72**

The system of two nonlinear equations for $v_1, v_2$

$$\lambda_1 v_1^3 + \frac{\lambda_{345}}{2} v_1 v_2^2 - \mu_1^2 v_1 - \mu_{12}^2 v_2 = 0$$
$$\lambda_2 v_2^3 + \frac{\lambda_{345}}{2} v_1^2 v_2 - \mu_2^2 v_2 - \mu_{12}^2 v_1 = 0$$

can be factorized by the rotation in the $v_1, v_2$ plane

$$v_1 = \bar{v}_1 \mathtt{cos}\bar{\beta} - \bar{v}_2 \mathtt{sin}\bar{\beta}, \qquad v_2 = \bar{v}_1 \mathtt{sin}\bar{\beta} + \bar{v}_2 \mathtt{cos}\bar{\beta}$$

$$\bar{v}_1(\lambda_1 \bar{v}_1^2 + \frac{\lambda_{345}}{2} \bar{v}_2^2 - \bar{\mu}_1^2) = 0$$
$$\bar{v}_2(\lambda_2 \bar{v}_2^2 + \frac{\lambda_{345}}{2} \bar{v}_1^2 - \bar{\mu}_2^2) = 0$$

$$\bar{\mu}_{1,2}^2 = \frac{1}{2}(\mu_1^2 + \mu_2^2 \pm \sqrt{(\mu_1^2 - \mu_2^2)^2 + 4\mu_{12}^4})$$

## The four bifurcation sets for U(v1,v2 | λ1,λ2,λ3,λ4,λ5)

$$(1)\ \lambda_1\bar{v}_1^2 + \frac{\lambda_{345}}{2}\bar{v}_2^2 - \bar{\mu}_1^2 = 0 \text{ and } \lambda_2\bar{v}_2^2 + \frac{\lambda_{345}}{2}\bar{v}_1^2 - \bar{\mu}_2^2 = 0,\ U_{ij}(v_1,v_2) = \begin{Vmatrix} 2\lambda_1\bar{v}_1^2 & \lambda_{345}\bar{v}_1\bar{v}_2 \\ \lambda_{345}\bar{v}_1\bar{v}_2 & 2\lambda_2\bar{v}_2^2 \end{Vmatrix}$$

$$(2)\ \lambda_1\bar{v}_1^2 - \bar{\mu}_1^2 = 0 \text{ and } \bar{v}_2 = 0,\ U_{ij}(v_1,v_2) = \begin{Vmatrix} 2\lambda_1\bar{v}_1^2 & 0 \\ 0 & -\bar{\mu}_2^2 + \frac{\lambda_{345}}{2}\bar{v}_1^2 \end{Vmatrix}$$

$$(3)\ \bar{v}_1 = 0 \text{ and } \lambda_2\bar{v}_2^2 - \bar{\mu}_2^2 = 0,\ U_{ij}(v_1,v_2) = \begin{Vmatrix} -\mu_1^2 + \frac{\lambda_{345}}{2}\bar{v}_2^2 & 0 \\ 0 & 2\lambda_2\bar{v}_2^2 \end{Vmatrix}$$

$$(4)\ \bar{v}_1 = 0 \text{ and } \bar{v}_2 = 0,\ U_{ij}(v_1,v_2) = -\begin{Vmatrix} \bar{\mu_1}^2 & 0 \\ 0 & \bar{\mu_2}^2 \end{Vmatrix}$$

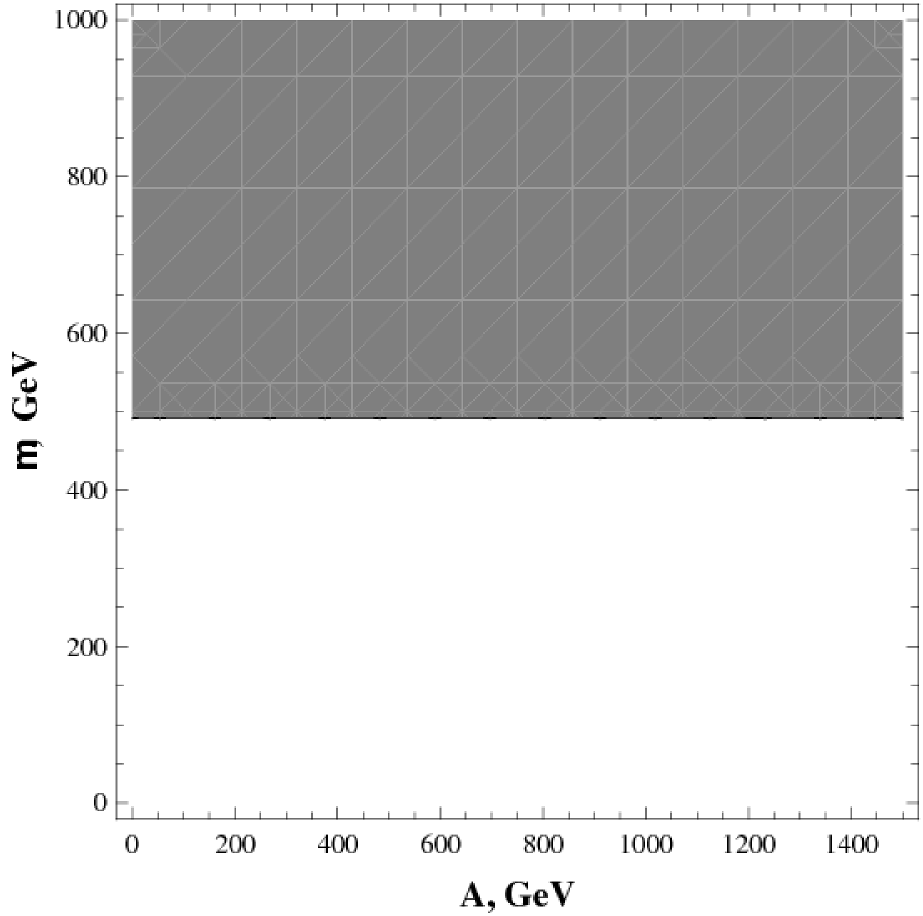
Set (1) is an elementary Sylvester's criteria $\lambda_1 < 0, \quad \lambda_2 < 0, \quad \lambda_1\lambda_2 - \frac{\lambda_{345}^2}{4} < 0$

Set (4) also elementary, sets (2) and (3) give

$$(4\lambda_1 + \lambda_{345})v_1^4 + (4\lambda_2 + \lambda_{345})v_2^4 + (6\lambda_{345} - 2\lambda_1 - 2\lambda_2)v_1^2v_2^2 = 0$$

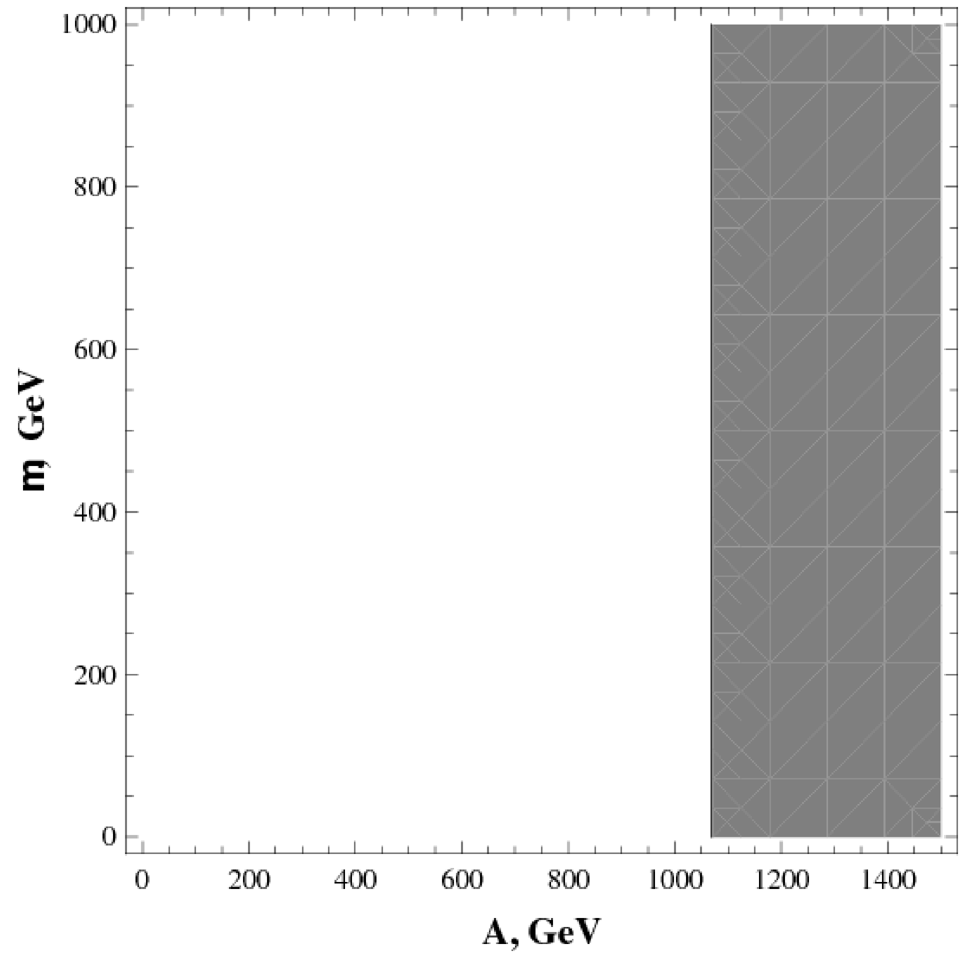
# Ограничения на параметры модели

$\lambda_1 < 0$



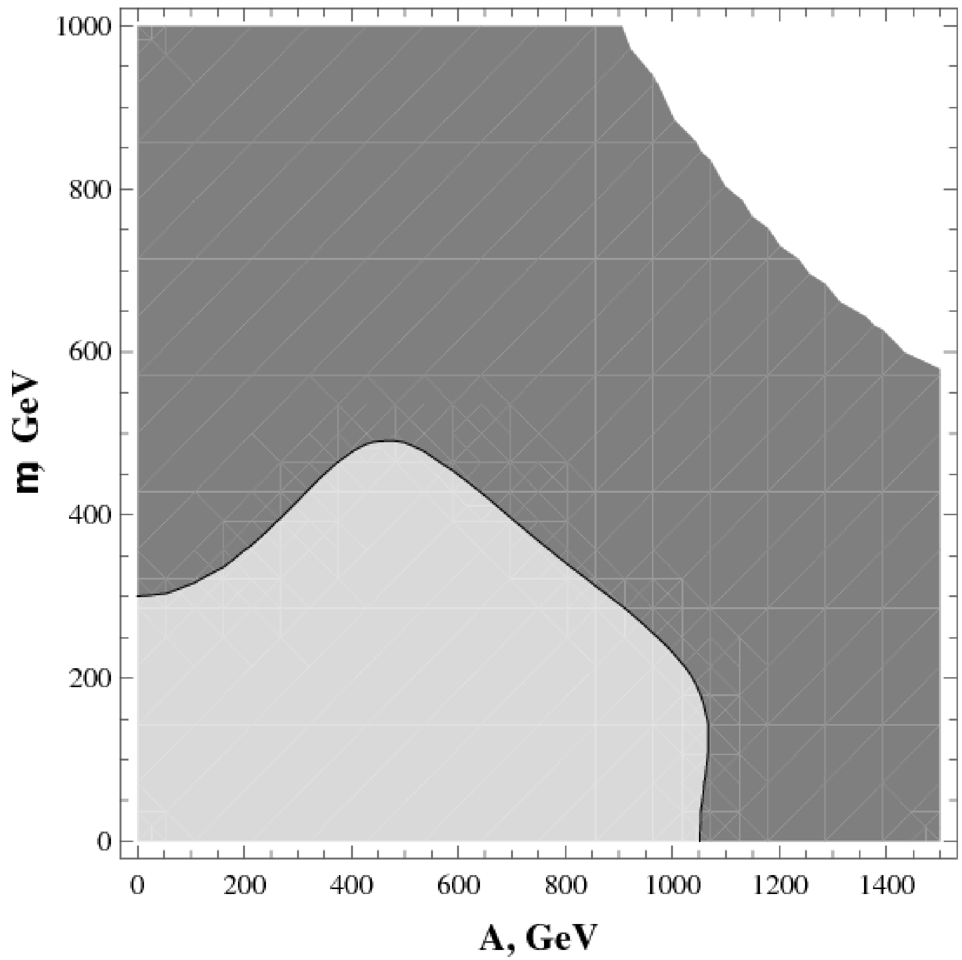
# Ограничения на параметры модели

$\lambda_2 < 0$



# Ограничения на параметры модели

$$\lambda_1 \lambda_2 - \frac{\lambda_{345}^2}{4} < 0$$



## Summary

Four bifurcation sets are found in the general THDM then projected onto the MSSM parameter space.

*On the base of*:

- Temperature MSSM one-loop effective Higgs potential with threshold corrections.
- Temperature evolution of masses and mixings from high T down to zero is explicitly obtained. The regions of MSSM parameter space where the mass(T) eigenstates exist are separated.

## Perspectives

- **The topology analysis of extended Higgs potentials, nonlinear transformations**
- **viable models:**

  **THDM, MSSM, split supersymmetry**
  **Singlet models: Next-to-MSSM many possibilities**
- **Electroweak baryogenesis is still viable in extended Higgs sectors**
- **It would offer  the possibiliy to compute the baryon asymmetry from parameters measured in collider experiments**
- **If  the result would match the observations, we could claim to understand the early universe up to electroweak temperature**
- **Strong constraints on CP phases from EDM's**