Threshold corrections to the MSSM finite-temperature Higgs potential

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Outline

- SM and phase transitions
- Minimal Supersymmetric Standard Model
- Effective THDM potential with explicit CP violation
- Integration and summation method
- Threshold corrections from the triangle and box diagrams
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- Higgs bosons masses
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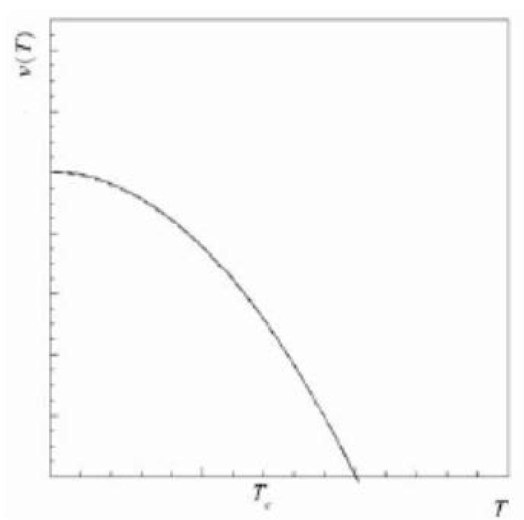
SM and phase transitions

$$U(\varphi) = -\frac{1}{2}\mu^2\varphi^2 + \frac{1}{4}\lambda\varphi^4$$

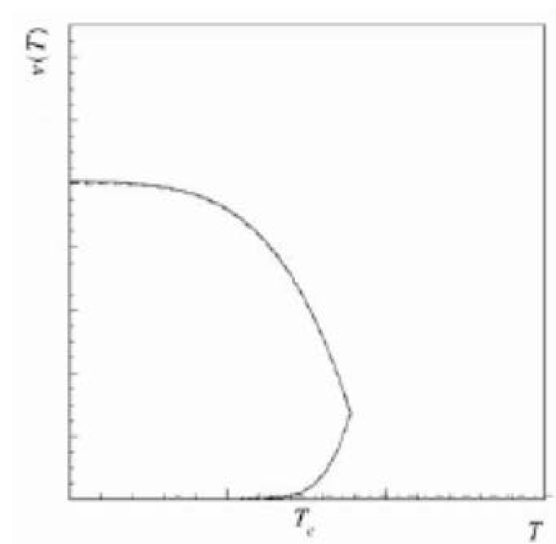
$$v(0) = 0 \quad \text{and} \quad v^2(T) = \mu^2/\lambda - T^2/4$$

$$T_c = 2\mu/\sqrt{\lambda} = 2v(0)$$

$$m_h^2 = -\mu^2 + \lambda T^2/4$$



(a)



(b)

Minimal Supersymmetric Standard Model

$$V_{eff}(v,T) = V_0(v_1, v_2, 0) + V_1(m(v), 0) + V_1(T) + V_{ring}(T).$$

$V_0(v_1, v_2, 0)$ - *the tree-level MSSM two-doublet potential at the SUSY scale*

$V_1(m(v), 0)$ - *the (non-temperature) one-loop resumed Coleman-Weinberg term, dominated by stop and sbottom contributions*

$V_1(T)$ - *the one-loop temperature term*

$V_{ring}(T)$ - *the correction of resummed leading infrared contribution from multi-loop ring (or daisy) diagrams*

g_2 and g_1, $\lambda_{1,2,3,4}$ $\tan\beta = v_2/v_1$ and $m_{H^\pm}$

$A_{t,b}$, μ, m_Q, m_U, m_D

Effective THDM potential with explicit CP violation

General hermitian renormalized *SU(2)*x*U(1)* invariant potential:

$$U(\Phi_1,\Phi_2) = -\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) - \mu_{12}^2(\Phi_1^\dagger\Phi_2) - \overset{*}{\mu}{}_{12}^2(\Phi_2^\dagger\Phi_1) +$$

$$+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) +$$

$$+\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \frac{\overset{*}{\lambda}_5}{2}(\Phi_2^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) +$$

$$+\lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \overset{*}{\lambda}_6(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) +$$

$$+\lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \overset{*}{\lambda}_7(\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

$$U_{eff}(\Phi_1,\Phi_2) \Longrightarrow \frac{m_h^2}{2}(hh) + \frac{m_H^2}{2}(HH) + \frac{m_A^2}{2}(AA) + m_{H^\pm}^2(H^+H^-) +$$

$$+h, H, A, H^\pm \quad \text{interaction terms}$$

Integration and summation method

In the finite temperature field theory Feynman diagrams with boson propagators, containing Matsubara frequencies, lead to structures of the form

$$I[m_1, m_2, ..., m_b] = T \sum_{n=-\infty}^{\infty} \int \frac{d\mathbf{k}}{(2\pi)^3} \prod_{j=1}^{b} \frac{(-1)^b}{(\mathbf{k}^2 + \omega_n^2 + m_j^2)},$$

$$\omega_n = 2\pi n T \; (n = 0, \pm 1, \pm 2, ...),$$

T – temperature

$n \neq 0$

$$I[m_1, m_2, ..., m_b] \; = \; 2T\,(2\pi T)^{3-2b} \, \frac{(-1)^b \pi^{3/2}}{(2\pi)^3} \, \frac{\Gamma(b-3/2)}{\Gamma(b)} \, S(M, b-3/2),$$

$$S(M, b-3/2) \; = \; \int \{\mathrm{d}x\} \sum_{n=1}^{\infty} \frac{1}{(n^2 + M^2)^{b-3/2}}, \qquad M^2 \equiv \left(\frac{m}{2\pi T}\right)^2.$$

Integration and summation method

A number of integrals can be easily calculated

$$J \equiv \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)(\mathbf{k}^2 + a_2^2)},$$

taking a residue in the spherical coordinate system:

$$J = \frac{1}{4\pi(a_1 + a_2)}$$

$a_{1;2}^2$ - the sums of squared frequency and squared mass.

Integration and summation method

Derivatives of first integral with respect to a_1 and a_2 can be used for calculation of integrals

$$J_1[a_1, a_2] \equiv \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)^2(\mathbf{k}^2 + a_2^2)} =$$

$$= -\frac{1}{2a_1}\frac{\partial I}{\partial a_1} = \frac{1}{8\pi a_1(a_1 + a_2)^2},$$

$$J_2[a_1, a_2] \equiv \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)^2(\mathbf{k}^2 + a_2^2)^2} =$$

$$= \frac{1}{4a_1 a_2}\frac{\partial^2 I}{\partial a_1 \partial a_2} = \frac{1}{8\pi a_1 a_2(a_1 + a_2)^3}.$$

Integration and summation method

and

$$J_3[a_1, a_2, a_3] \equiv \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)(\mathbf{k}^2 + a_2^2)(\mathbf{k}^2 + a_3^2)} =$$

$$= \frac{1}{4\pi(a_1 + a_2)(a_1 + a_3)(a_2 + a_3)},$$

$$J_4[a_1, a_2, a_3] \equiv \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + a_1^2)^2(\mathbf{k}^2 + a_2^2)(\mathbf{k}^2 + a_3^2)} =$$

$$= \frac{2a_1 + a_2 + a_3}{8\pi a_1(a_1 + a_2)^2(a_1 + a_3)^2(a_2 + a_3)}.$$

Integration and summation method

Substituting

$$a_1 \to \sqrt{4\pi^2 n^2 T^2 + m_1^2} \ \text{и} \ a_2 \to \sqrt{4\pi^2 n^2 T^2 + m_2^2},$$

for

$$J^n = \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{(\mathbf{k}^2 + \omega_n^2 + m_1^2)(\mathbf{k}^2 + \omega_n^2 + m_2^2)},$$

taking the sum over Matsubara frequencies after the integration we get:

$$\sum_{n=-\infty, n\neq 0}^{\infty} J^n = \sum_{n=-\infty, n\neq 0}^{\infty} \frac{1}{4\pi(\sqrt{4\pi^2 n^2 T^2 + m_1^2} + \sqrt{4\pi^2 n^2 T^2 + m_2^2})}.$$

Thus the temperature corrections to effective potential are expressed by summed integrals,

Integration and summation method

after redefinition of mass parameters

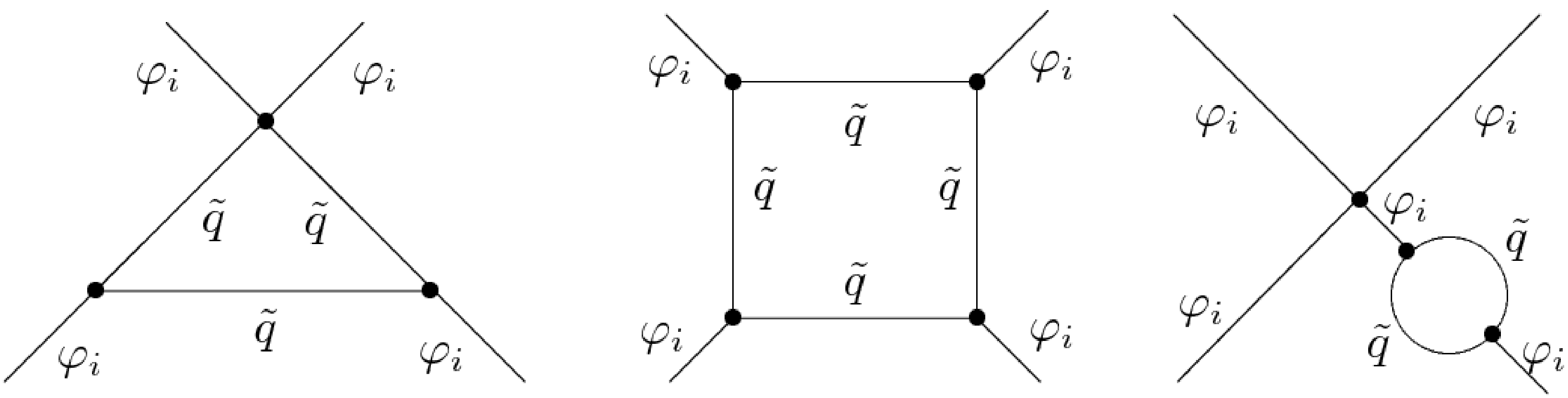
$$m_{1;2} \longrightarrow m'_{1;2} = 2\pi T\sqrt{M^2_{1;2} + n^2}, \quad \text{где } M_{1;2} = \frac{m_{1;2}}{2\pi T},$$

$$I_1 = \frac{-T}{8\pi}\frac{1}{(2\pi T)^3}\sum_{n=-\infty, n\neq 0}^{\infty}\frac{1}{\sqrt{M_1^2+n^2}(\sqrt{M_1^2+n^2}+\sqrt{M_2^2+n^2})^2},$$

$$I_2 = \frac{T}{8\pi}\frac{1}{(2\pi T)^5}\sum_{n=-\infty, n\neq 0}^{\infty}\frac{1}{\sqrt{M_1^2+n^2}\sqrt{M_2^2+n^2}(\sqrt{M_1^2+n^2}+\sqrt{M_2^2+n^2})^3}.$$

Threshold corrections
from the triangle and box diagrams

Calculation of the one-loop threshold corrections in the framework of the finite temperature field theory (imaginary time formalism, Matsubara series) gives the result for efeective parameters lambda



Threshold corrections from the triangle and box diagrams

$$\Delta\lambda_1^{thr} = 3h_t^4|\mu|^4 I_2[m_Q, m_U] + 3h_b^4|A_b|^4 I_2[m_Q, m_D] +$$
$$+h_t^2|\mu|^2\left(-\frac{g_1^2 - 3g_2^2}{2} I_1[m_Q, m_U] + 2g_1^2 I_1[m_U, m_Q]\right)$$
$$+h_b^2|A_b|^2\left(\frac{12h_b^2 - g_1^2 - 3g_2^2}{2} I_1[m_Q, m_D] + (6h_b^2 - g_1^2) I_1[m_D, m_Q]\right)$$

$$\Delta\lambda_2^{thr} = 3h_t^4|A_t|^4 I_2[m_Q, m_U] + 3h_b^4|\mu|^4 I_2[m_Q, m_D] +$$
$$+h_b^2|\mu|^2\left(\frac{g_1^2 + 3g_2^2}{2} I_1[m_Q, m_D] + g_1^2 I_1[m_D, m_Q]\right) +$$
$$+h_t^2|A_t|^2\left(\frac{12h_t^2 + g_1^2 - 3g_2^2}{2} I_1[m_Q, m_U] + (6h_t^2 - 2g_1^2) I_1[m_U, m_Q]\right)$$

$$\Delta\lambda_3^{thr} = h_t^2\left(\left(|\mu|^2 \frac{3g_2^2 + g_1^2}{12} + |A_t|^2 \frac{12h_t^2 - g_1^2 - 3g_2^2}{12}\right) I_1[m_Q, m_U] +\right.$$
$$+\left(|\mu|^2 \frac{3h_t^2 - g_1^2}{3} + |A_t|^2 \frac{g_1^2}{3}\right) I_1[m_U, m_Q]) + \left(h_b^2\left(|\mu|^2 \frac{3g_2^2 - g_1^2}{12} + |A_b|^2 \frac{12h_t^2 + g_1^2 - 3g_2^2}{4}\right) I_1[m_Q, m_D] +\right.$$
$$+\left(|\mu|^2 \frac{6h_b^2 - g_1^2}{6} + |A_b|^2 \frac{g_1^2}{6}\right) I_1[m_D, m_Q]) + h_t^2|\mu|^2|A_t|^2 I_2[m_Q, m_U] + h_b^2|\mu|^2|A_b|^2 I_2[m_Q, m_D] +$$
$$+h_t^2 h_b^2 (2(A_t A_b - |\mu|^2) I_3[m_Q, m_U, m_D] + (|\mu|^4 + |A_t|^2|A_b|^2 - 2A_t A_b|\mu|^2) I_4[m_Q, m_U, m_D]$$

$$\Delta\lambda_4^{thr} = 6h_t^4|\mu|^2|A_t|^2 I_2[m_Q, m_U] + 6h_b^4|\mu|^2|A_b|^2 I_2[m_Q, m_D] +$$

$$+h_t^2((|\mu|^2\frac{12h_t^2 + g_1^2 - 3g_2^2}{4} - |A_t|^2\frac{g_1^2 - 3g_2^2}{4})I_1[m_Q, m_U] +$$

$$+(|A_t|^2 g_1^2 - |\mu|^2(g_1^2 - 3h_t^2))I_1[m_U, m_Q]) +$$

$$+h_b^2((|\mu|^2\frac{-12h_t^2 + g_1^2 + 3g_2^2}{4} - |A_b|^2\frac{g_1^2 + 3g_2^2}{4})I_1[m_Q, m_D] +$$

$$+\frac{1}{2}(|A_b|^2 g_1^2 - |\mu|^2(g_1^2 - 6h_b^2))I_1[m_D, m_Q]) - \Delta\lambda_3^{th}$$

$$\Delta\lambda_5^{thr} = 3h_t^4\mu^2 A_t^2 I_2[m_Q, m_U] + 3h_b^4\mu^2 A_b^2 I_2[m_Q, m_D]$$

$$\Delta\lambda_6^{thr} = -3h_t^4\mu A_t|\mu|^2 I_2[m_Q, m_U] - 3h_b^4\mu A_b|A_b|^2 I_2[m_Q, m_D] +$$

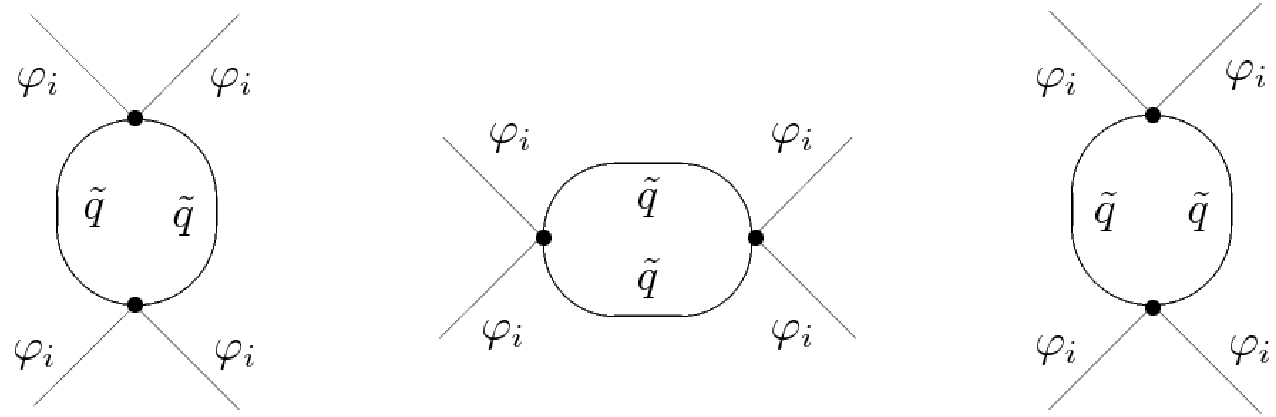
$$+h_t^2\mu A_t(\frac{g_1^2 - 3g_2^2}{4}I_1[m_Q, m_U] - g_1^2 I_1[m_U, m_Q]) +$$

$$+h_b^2\mu A_b(\frac{-12h_b^2 + g_1^2 + 3g_2^2}{4}I_1[m_Q, m_D] - \frac{6h_b^2 - g_1^2}{2}I_1[m_D, m_Q])$$

$$\Delta\lambda_7^{thr} = -3h_t^4\mu A_t|A_t|^2 I_2[m_Q, m_U] - 3h_b^4\mu A_b|\mu|^2 I_2[m_Q, m_D]$$

$$+h_b^2\mu A_b(-\frac{g_1^2 + 3g_2^2}{4}I_1[m_Q, m_D] - \frac{g_1^2}{2}I_1[m_D, m_Q]) +$$

$$+h_t^2\mu A_t(\frac{12h_t^2 + g_1^2 - 3g_2^2}{4}I_1[m_Q, m_U] - (3h_t^2 - g_1^2)I_1[m_U, m_Q])$$



$$-\Delta\lambda_1^f = \left[h_b^2 - \frac{g_1^2}{6}\right]^2 (I(m_Q) + I(m_D)) + \frac{g_1^4}{9} I(m_U),$$

$$-\Delta\lambda_2^f = \left[h_t^2 + \frac{g_1^2}{6}\right]^2 I(m_Q) + [h_t^2 - \frac{g_1^2}{3}]^2 I(m_U) + \frac{g_1^4}{36} I(m_D),$$

$$-(\Delta\lambda_3 + \Delta\lambda_4)^f = \frac{1}{72}\left(-g_1^4 + 6(h_b^2 - h_t^2)g_1^2 - 9(g_2^4 - 2(h_b^2 + h_t^2)g_2^2)\right) I(m_Q) +$$

$$+\frac{g_1^2}{3}(h_t^2 - \frac{g_1^2}{3}) I(m_U) + \frac{g_1^2}{6}(h_b^2 - \frac{g_1^2}{6}) I(m_D),$$

$$-\Delta\lambda_3^f = \frac{1}{72}\left(-g_1^4 + 6(h_b^2 - h_t^2)g_1^2 + 9\left(g_2^4 - 2\left(h_b^2 + h_t^2\right)g_2^2 + 8h_b^2 h_t^2\right)\right) I(m_Q) +$$

$$+\frac{g_1^2}{3}(h_t^2 - \frac{g_1^2}{3}) I(m_U) + \frac{g_1^2}{6}(h_b^2 - \frac{g_1^2}{6}) I(m_D) + h_t^2 h_b^2 I(m_U, m_D).$$

$$-\Delta\lambda_4^f = (h_b^2 - \frac{g_2^2}{2})(\frac{g_2^2}{2} - h_t^2) I(m_Q) - h_t^2 h_b^2 I(m_U, m_D).$$

$$J(m_I) = \frac{1}{8\pi m_I}, \qquad J(m_U, m_D) = \frac{1}{4\pi(m_U + m_D)}.$$

The logarithmic corrections

$$\Delta\lambda_1^{log} = -\frac{1}{384\pi^2}\left(11g_1^4 - 36h_b^2g_1^2 + 9\left(g_2^4 - 4h_b^2g_2^2 + 16h_b^4\right)\right)\ln\left(\frac{m_Q m_U}{m_t^2}\right),$$

$$\Delta\lambda_2^{log} = -\frac{1}{1536\pi^2}\left(44g_1^4 - 144h_t^2g_1^2 + 36g_2^4 + 576h_t^4 - 144g_2^2h_t^2\right)\ln\left(\frac{m_Q m_U}{m_t^2}\right),$$

$$\Delta\lambda_3^{log} = -\frac{1}{384\pi^2}\left(-11g_1^4 + 18\left(h_b^2 + h_t^2\right)g_1^2 + \right.$$

$$\left. +9\left(g_2^4 - 2\left(h_b^2 + h_t^2\right)g_2^2 + 16h_b^2h_t^2\right)\right)\ln\left(\frac{m_Q m_U}{m_t^2}\right),$$

$$\Delta\lambda_4^{log} = \frac{3}{64\pi^2}\left(g_2^4 - 2\left(h_b^2 + h_t^2\right)g_2^2 + 8h_b^2h_t^2\right)\ln\left(\frac{m_Q m_U}{m_t^2}\right).$$

The wave-function renormalization correction

$$\Delta\lambda_1^{\rm wfr} = \frac{1}{2}(g_1^2+g_2^2)A'_{11}, \qquad \Delta\lambda_2^{\rm wfr} = \frac{1}{2}(g_1^2+g_2^2)A'_{22},$$

$$\Delta\lambda_3^{\rm wfr} = -\frac{1}{4}(g_1^2-g_2^2)(A'_{11}+A'_{22}), \quad \Delta\lambda_4^{\rm wfr} = -\frac{1}{2}g_2^2(A'_{11}+A'_{22}), \quad \Delta\lambda_5^{\rm wfr} = 0,$$

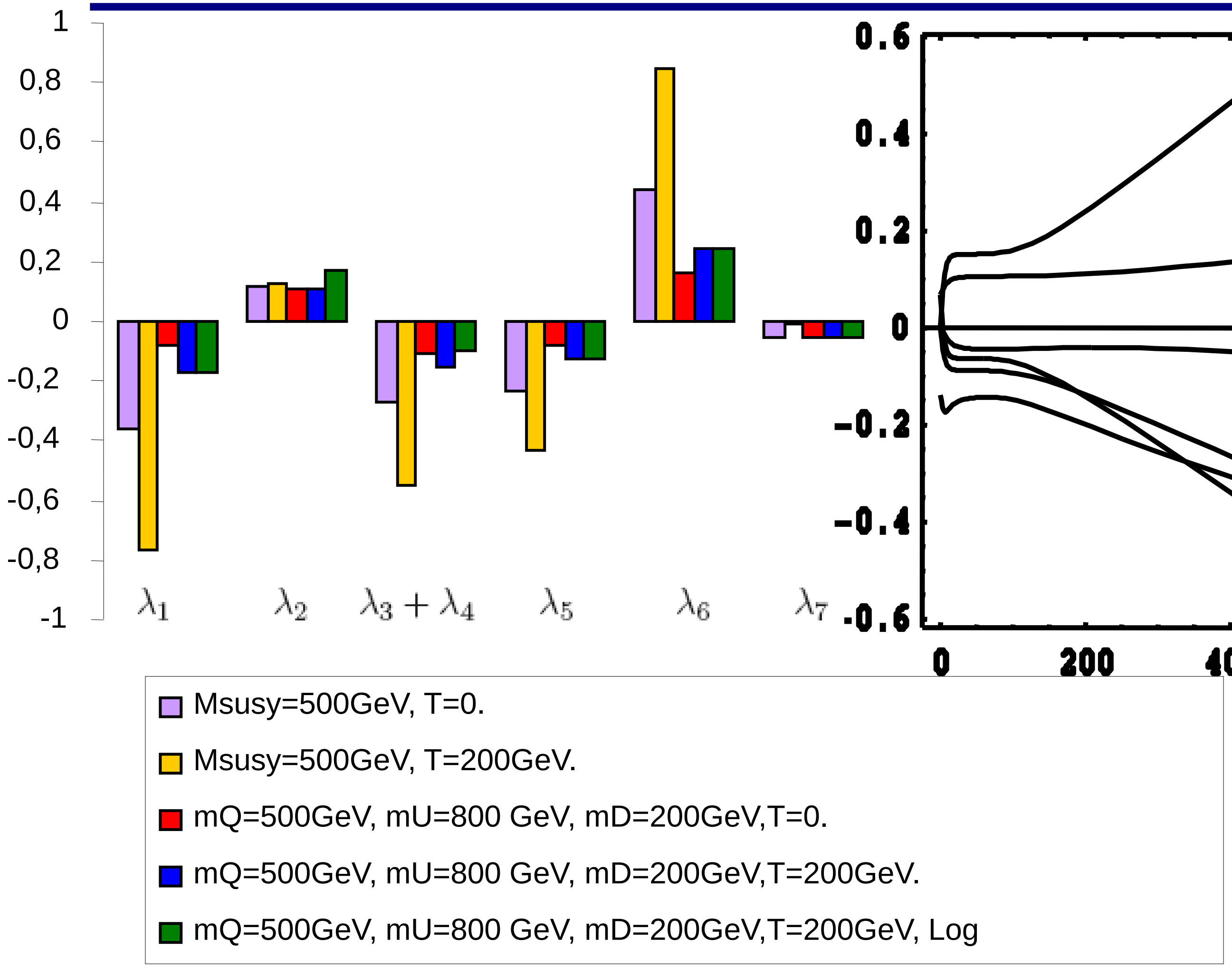
$$\Delta\lambda_6^{\rm wfr} = \frac{1}{8}(g_1^2+g_2^2)(A'_{12}-{A'_{21}}^*) = 0, \quad \Delta\lambda_7^{\rm wfr} = \frac{1}{8}(g_1^2+g_2^2)(A'_{21}-{A'_{12}}^*) = 0.$$

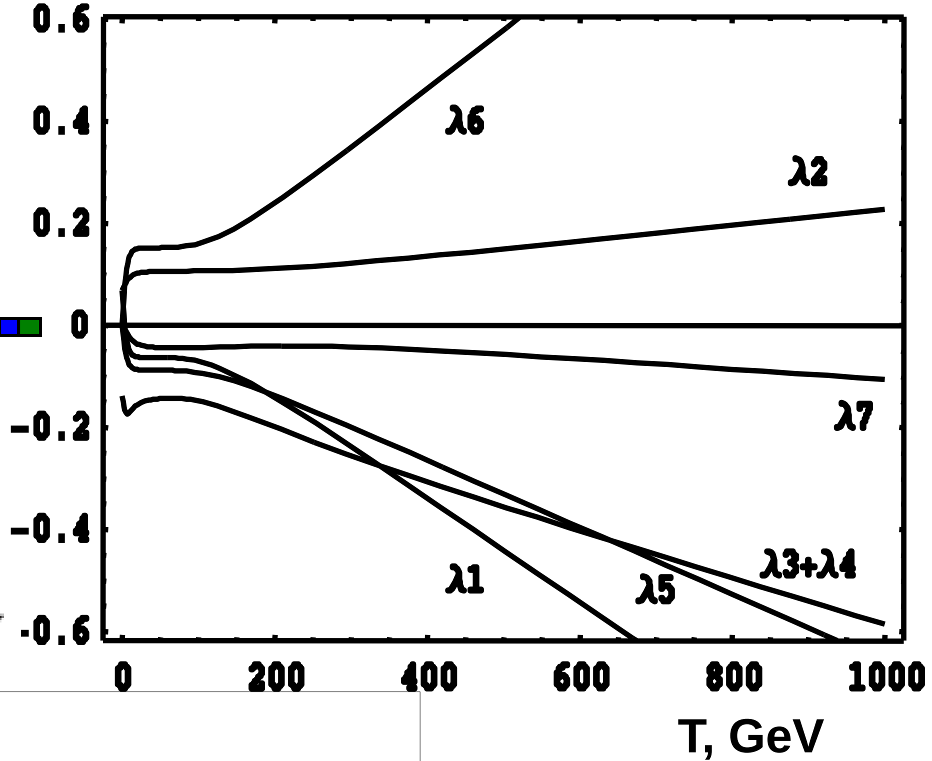
$$A'_{ij} = \{\frac{2\cdot 3h_U^2}{24\,\pi}F(m_Q^2, m_U^2, T)\begin{bmatrix} |\mu|^2 & -\mu^* A_U^* \\ -\mu A_U & |A_U|^2 \end{bmatrix} + \\ + (U \longrightarrow D, A \longleftrightarrow \mu)\}(1-\frac{1}{2}l)$$

$$F(m_1^2, m_2^2, T) = T\sum_{n=-\infty}^{+\infty}\frac{1}{(\sqrt{m_1^2+(2\pi nT)^2}+\sqrt{m_2^2+(2\pi nT)^2})^3} =$$

$$= \frac{T}{(m_1+m_2)^3} + 2T\sum_{n=1}^{+\infty}\frac{1}{(\sqrt{m_1^2+(2\pi nT)^2}+\sqrt{m_2^2+(2\pi nT)^2})^3}.$$

Temperature-dependent parameters with various quantum corrections in CPX-like scenario A_t=A_b=1000 GeV, μ=2000 GeV





CPX: M.Carena, J.Ellis, A.Pilaftsis, C.Wagner, PL B495 (2000) 155

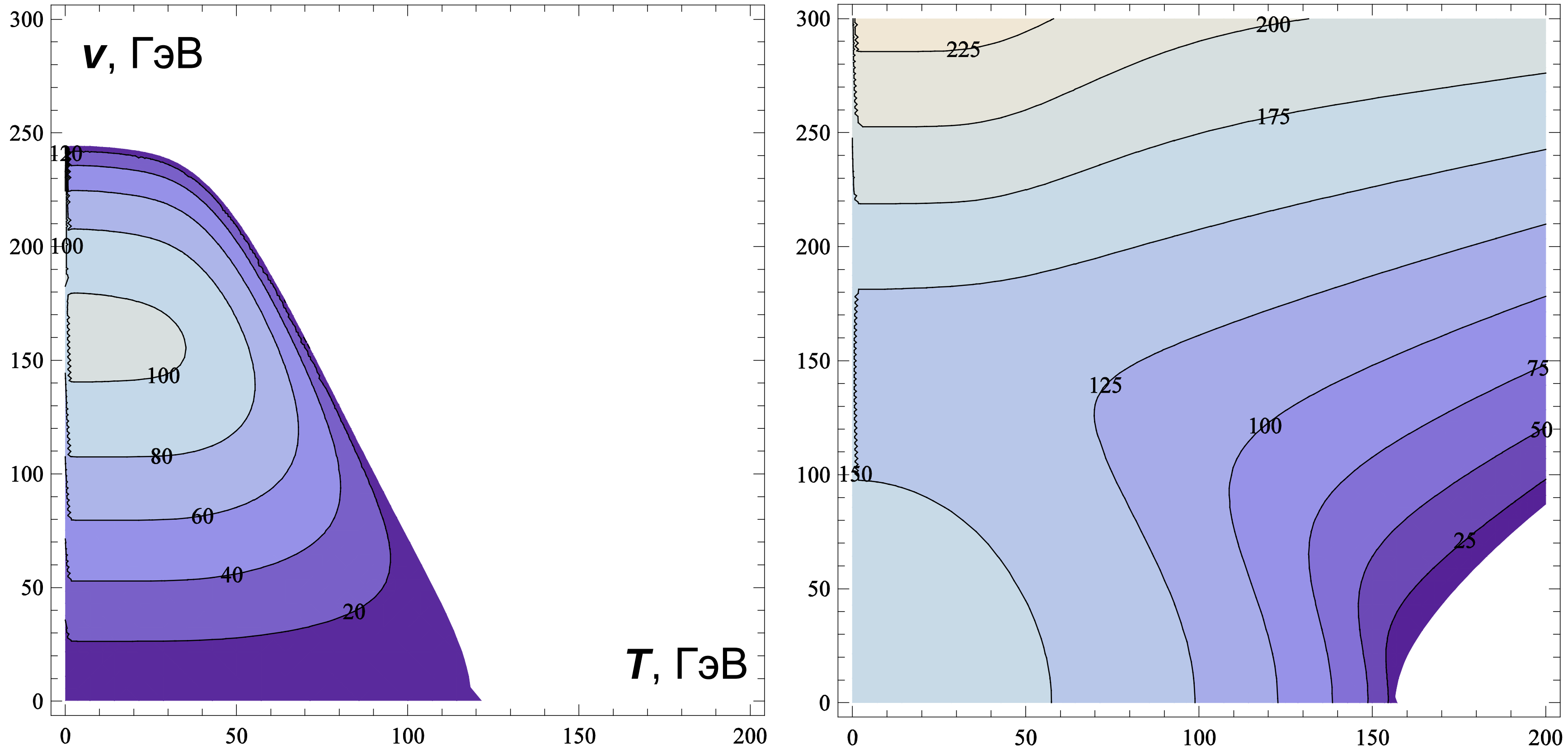
The thermal evolution of the CP-even Higgs bosons h and H is expressed by

$$
\begin{aligned}
m_h^2 = c_{\alpha-\beta}^2 m_A^2 + v^2 (2\lambda_1 s_\alpha^2 c_\beta^2 + 2\lambda_2 c_\alpha^2 s_\beta^2 - 2(\lambda_3 + \lambda_4) c_\alpha c_\beta s_\alpha s_\beta + \\
+ \mathrm{Re}\lambda_5 (s_\alpha^2 s_\beta^2 + c_\alpha^2 c_\beta^2) - 2 c_{\alpha+\beta} (\mathrm{Re}\lambda_6 s_\alpha c_\beta - \mathrm{Re}\lambda_7 c_\alpha s_\beta)), \\
m_H^2 = s_{\alpha-\beta}^2 m_A^2 + v^2 (2\lambda_1 c_\alpha^2 c_\beta^2 + 2\lambda_2 s_\alpha^2 s_\beta^2 + 2(\lambda_3 + \lambda_4) c_\alpha c_\beta s_\alpha s_\beta + \\
+ \mathrm{Re}\lambda_5 (c_\alpha^2 s_\beta^2 + s_\alpha^2 c_\beta^2) + 2 s_{\alpha+\beta} (\mathrm{Re}\lambda_6 c_\alpha c_\beta + \mathrm{Re}\lambda_7 s_\alpha s_\beta)),
\end{aligned}
$$

where *α is* the mixing angle of the CP-even states h and H.

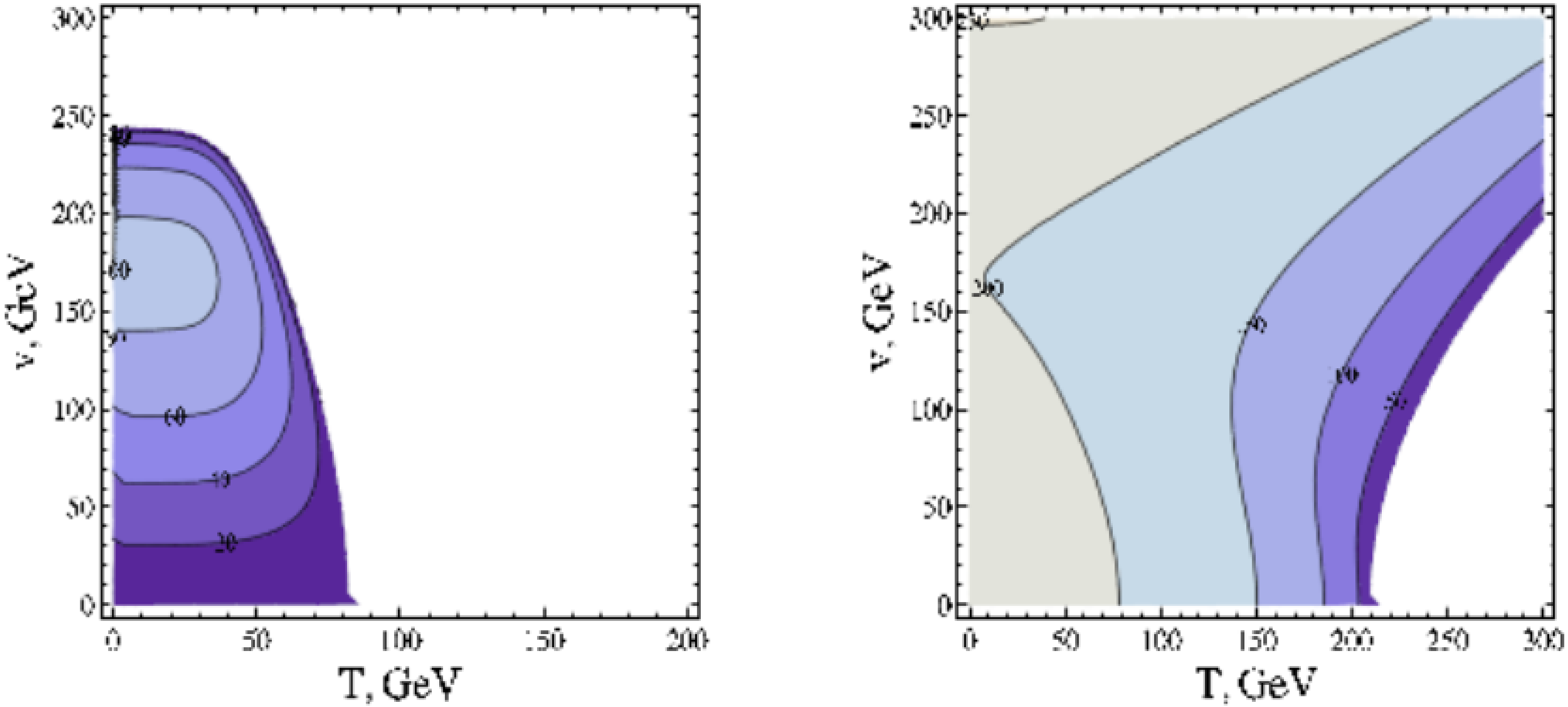
[Akhmetzyanova E.N., Dolgopolov M.V., Dubinin M.N. Higgs Bosons in the Two-Doublet Model with CP Violation // Phys.Rev.D. V.71. N7. 2005. P.075008. (hep-ph/0405264)]

Higgs bosons masses



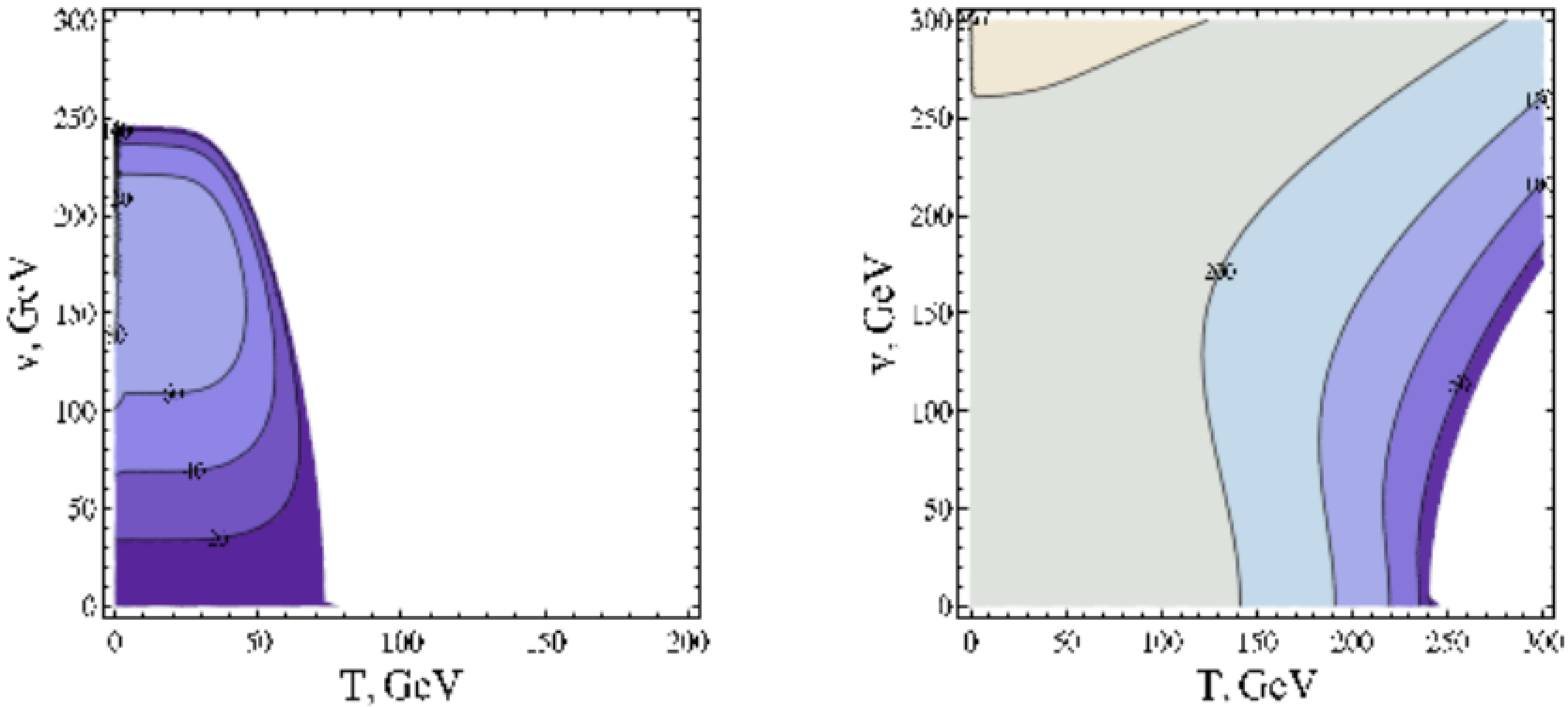
$\mathrm{tg}\beta = 5$, $m_{H^\pm} = 180$ GeV, $A_{t,b} = 1200$ GeV, $\mu = 500$ GeV.

Higgs bosons masses



The same contours at $\mathtt{tg}\beta = 15$, $m_{H^\pm} = 230$ GeV

Higgs bosons masses



The same contours at $\mathtt{tg}\beta = 40$, $m_{H^\pm} = 260$ GeV

Conclusions

1. In the MSSM we calculate the 1-loop finite-temperature corrections from the squarks-Higgs bosons sector, reconstruct the effective two-Higgs-doublet potential in the full MSSM parameter space ($m_{H\pm}$, $tg\beta$, $A_{t,b}$, μ, m_Q, m_U, m_D).

2. At large values of A and μ of around 1 TeV the threshold finite-temperature corrections from triangle and box diagrams with intermediate third generation squarks are very substantial.

3. High sensitivity of the low-temperature evolution to the effective two-doublet and the MSSM squark sector parameters is observed, but rather extensive regions of the full MSSM parameter space allow the first-order electroweak phase transition respecting the phenomenological constraints at zero temperature.