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Non-linear transformation in 2HDM

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Introduction

It is assumed that the early universe was rather hot, with temperatures of the order of a few hundred GeV. The $SU(2)_W \times U(1)_Y$ symmetry of the ground state (characterized by zero Higgs field vacuum expectation value - VEV) was broken spontaneously down to $U(1)_{em}$. It was coupled with the decrease of temperature to the state with nonzero Higgs field VEV, so called electroweak phase transition.

Investigation of the temperature evolution beyond the Standard Model is important as a basis for models which describe

- generation of the baryon asymmetry,
- nature of the dark matter.

In the following we are going to analyze the equilibrium states of the effective finite-temperature potential (free energy) for the two-Higgs doublet model (2HDM)

Higgs potential

General form

$$U_{eff} = -\mu_1^2\,(\Phi_1^+\Phi_1) - \mu_2^2\,(\Phi_2^+\Phi_2) - \mu_{12}^2\,(\Phi_1^+\Phi_2) - \mu_{12}^{*2}\,(\Phi_2^+\Phi_1) +$$
$$\lambda_1(T)\,(\Phi_1^+\Phi_1)^2 + \lambda_2(T)\,(\Phi_2^+\Phi_2)^2 +$$
$$\lambda_3(T)\,(\Phi_1^+\Phi_1)(\Phi_2^+\Phi_2) + \lambda_4(T)\,(\Phi_1^+\Phi_2)(\Phi_2^+\Phi_1) +$$
$$\frac{1}{2}\lambda_5(T)\,(\Phi_1^+\Phi_2)^2 + \frac{1}{2}\lambda_5^*(T)\,(\Phi_2^+\Phi_1)^2 +$$
$$\lambda_6(T)\,(\Phi_1^+\Phi_1)(\Phi_1^+\Phi_2) + \lambda_6^*(T)\,(\Phi_1^+\Phi_1)(\Phi_2^+\Phi_1) +$$
$$\lambda_7(T)\,(\Phi_2^+\Phi_2)(\Phi_1^+\Phi_2) + \lambda_7^*(T)\,(\Phi_2^+\Phi_2)(\Phi_2^+\Phi_1)$$

Vacuum expectation values

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_1\end{pmatrix} \qquad \langle\Phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_2\end{pmatrix}$$

Higgs potential in terms of VEVs:

$$U_{eff} = \frac{1}{2}\mu_1^2 v_1^2 - \frac{1}{2}\mu_2^2 v_2^2 - \mathrm{Re}\mu_{12}^2\, v_1 v_2 +$$
$$\frac{\lambda_1}{4} v_1^4 + \frac{\lambda_2}{4} v_2^4 + \frac{\lambda_{345}}{4} v_1^2 v_2^2 + \frac{\mathrm{Re}\lambda_6}{2} v_1^3 v_2 + \frac{\mathrm{Re}\lambda_7}{2} v_2^3 v_1,$$

where $\lambda_{345} = \lambda_3 + \lambda_4 + \mathrm{Re}\lambda_5$

Morse lemma

Thermodynamic evolution of the 2HDM potential is a function of the two variables of state v_1 and v_2 and seven temperature-dependent control parameters $\lambda_i(T)$.

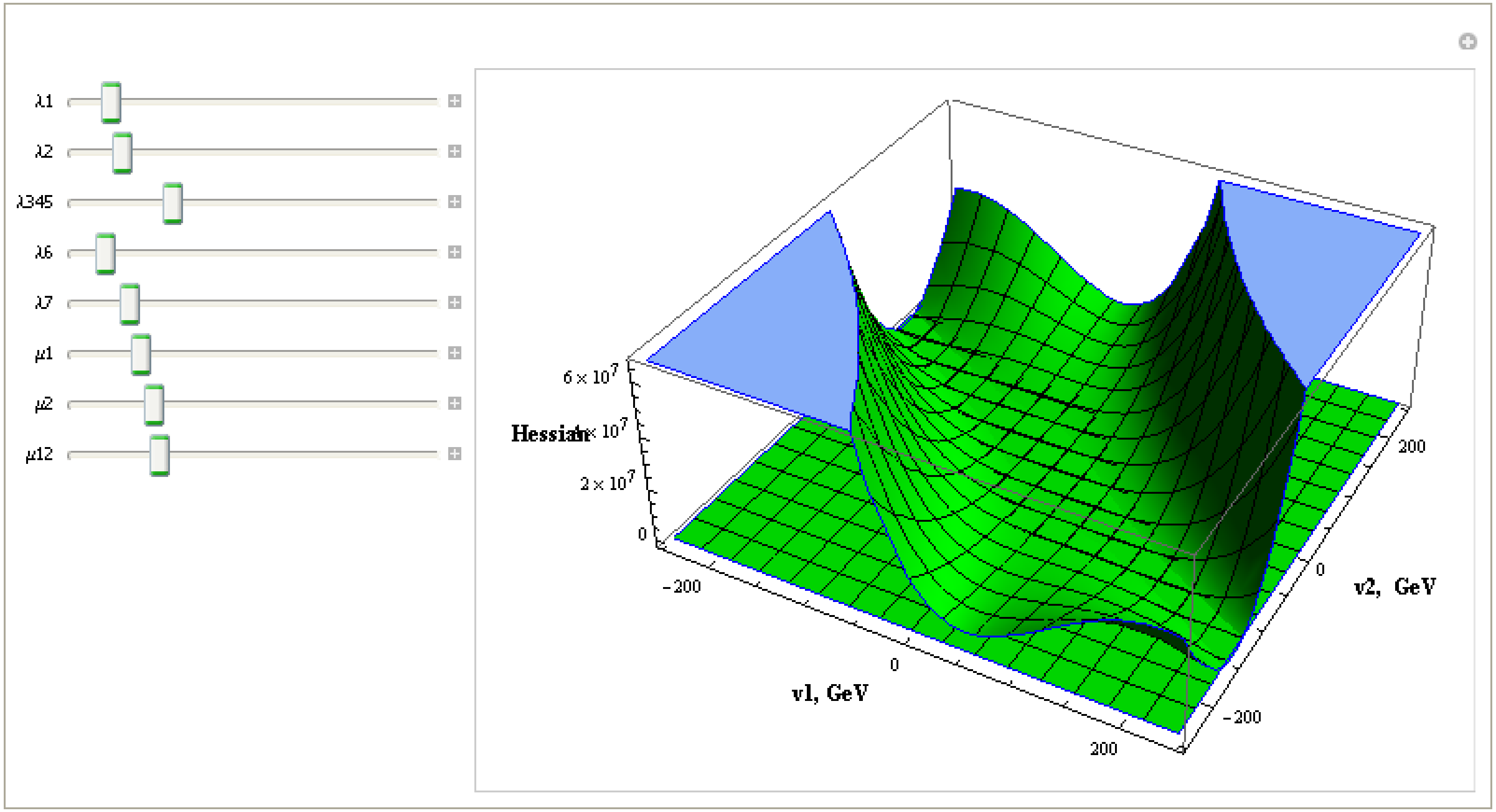
Local properties of the potential are defined in the framework of the catastrophe theory. We use the Morse lemma for the reduction of a potential function to the canonical form by a non-linear transformation.

$$(\nabla U_{eff} = 0) \wedge \left(\det \frac{\partial^2 U_{eff}}{\partial v_i\, \partial v_j} \neq 0\right) \Longrightarrow U_{eff} \longrightarrow U_{canon} = \tilde{\mu}_1\, \tilde{v}_1{}^2 + \tilde{\mu}_2\, \tilde{v}_2{}^2$$

The first condition is obviously the case of potential minimum. The second one sets certain limits for model parameters, which we obtain from the Hessian plot.

Hessian plot

As the Hessian is the function of all coefficients μ and λ, we can simply find areas in the parametric space that contain values which meet the condition.



We consider the experimental value $\sqrt{\nu_1{}^2 + \nu_2{}^2} = 246\ \mathrm{GeV}$ by omitting values of ν_1 and ν_2 that exceed 246 GeV.

Transformation

According to the Morse lemma we state the following transformation stages:

- Linear rotation, diagonalization

$$v_1 = \text{Cos}[\theta]\,\bar{v}_1 - \text{Sin}[\theta]\,\bar{v}_2$$

$$v_2 = \text{Sin}[\theta]\,\bar{v}_1 + \text{Cos}[\theta]\,\bar{v}_2$$

$$\frac{1}{2}\,\mu_1{}^2\,\bar{v}_1{}^2 - \frac{1}{2}\,\mu_2{}^2\,\bar{v}_1{}^2 - \text{Re}\mu_{12}^2\,\bar{v}_1\,\bar{v}_2 + \ldots \longrightarrow \bar{\mu}_1{}^2\,\bar{v}_1{}^2 - \frac{1}{2}\,\bar{\mu}_2{}^2\,\bar{v}_1{}^2 + \ldots$$

where $\bar{\mu}_{1,2} = \frac{1}{2}\left(\mu_1^2 + \mu_2^2 \pm \sqrt{\left(\mu_1^2 - \mu_2^2\right)^2 + 4\,\mu_{12}^4}\right)$

- Non-linear axes-keeping transformation

$$V1 = \bar{v}_1 + \left(A_{20}\,\bar{v}_1^2 + A_{11}\,\bar{v}_1\,\bar{v}_2 + A_{02}\,\bar{v}_2^2\right) + \left(A_{30}\,\bar{v}_1^3 + A_{21}\,\bar{v}_1^2\,\bar{v}_2 + A_{12}\,\bar{v}_1\,\bar{v}_2^2 + A_{03}\,\bar{v}_1^3\right)$$

$$V\,2 = \bar{v}_2 + \left(B_{20}\,\bar{v}_1^2 + B_{11}\,\bar{v}_1\,\bar{v}_2 + B_{02}\,\bar{v}_2^2\right) + \left(B_{30}\,\bar{v}_1^3 + B_{21}\,\bar{v}_1^2\,\bar{v}_2 + B_{12}\,\bar{v}_1\,\bar{v}_2^2 + B_{03}\,\bar{v}_1^3\right)$$

New form calculation

Obviously, the coefficients in terms of similar power should be equal to each other in both expressions. This leads to a single representation of the model potential onto a non-lineary converted coordinates system.

$$U_{eff}(\overline{v}_1, \overline{v}_2) = U_{canon}(V1, V2)$$

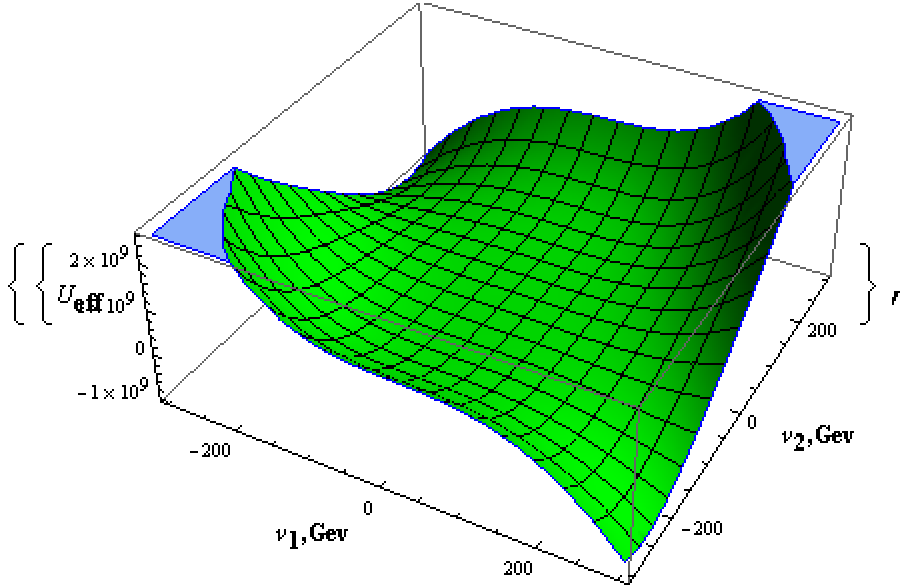
$$\overline{\mu}_1 \overline{v}_1^2 + \overline{\mu}_2 \overline{v}_2^2 + \ldots = \overline{\mu}_1 V1^2 + \overline{\mu}_2 V2^2$$

Therefore, in system of non-physical fields we can easily see the distribution of local minima of the initial effective potential.

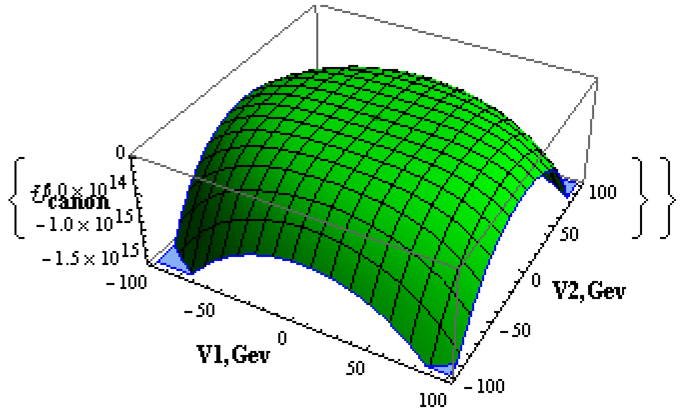
Topological comparison

Here we compare two plots of the potential minima surfaces in physical and non-physical fields. Parameters control both plots simultaneously.

{{μ_1= 600, }, {μ_2= 798, }}
{{μ_{12}= 190, }, {λ_1= 0.44, }}
{{λ_2= 0.64, }, {λ_{345}= 0.23, }}
{{λ_6= 0.8, }, {λ_7= 0.04, }}



Physical fields



Non-physical fields

Conclusion. Perspectives

The most important point in our paper is, we can explore the behaviour of the stable minimum existance conditions. The next step for us is to consider temperature evolution, what makes the effective potential form much more complicated. That's why our approach using the Morse lemma is very helpful as we now can simplify it significantly.

Here the modified potential is not written with true coefficients, it just represents the right form. We are going to add some precise calculation for model parameters in the nearest future.

Thanks for your attention