Charged wall-wall collision in AdS_5

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based on work with I. Ya. Aref'eva and A. Bagrov

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• In a series of papers Amati, Ciafaloni, Veneziano and t'Hooft[1] conjectured that black hole occur in the collision of two light particles at planking energies:

$$E > M_{pl}$$

• I.Ya. Arefeva, K.S. Viswanathan, I.V. Volovich mad a conjecture that ultra-relativistic particle generates a gravitational wave[2] Then these plane gravitational waves collide and produce a black hole.

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[1]D.Amati, M. Ciafaloni and G. Veneziano, *Phys.Lett*B 197(1987)81, G.t Hooft, *Phys.Lett.*B 198(1987);

[2]I.Ya. Arefeva, K.S. Viswanathan, I.V. Volovich, *Nucl.Phys.*B 452:346-368,1995.

• The higher-dimensional, fundamental theory has a new scale for gravity, M_*, that is related to the effective 4-dimensional one through the equation [3]:

$$M_p^2 \simeq R^n M_*^{2+n}, \quad D = 4 + n.$$

R is size of extra spacelike dimensions.

• New development has been started, when people realized that M_{pl} can be about 1 TeV.

[3] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B 429, 263 (1998); Phys. Rev. D 59, 086004 (1999).
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 436, 257 (1998).

• After this time the problem of black hole production has been considered as accessible at about Tev and estimation of BH production for $b < R_{Sch}(D)$, b is impact parameter.[4]

• The technic of entropy finding developed by S. B. Giddings and S. Thomas [5] were applied in AdS/dS space for without charge case [6] and for charged case [7]

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[4] T. Banks and W. Fischler, A Model for High Energy Scattering in Quantum Gravity **hep-th/9906038**.
I.Ya.Aref'eva, High Energy Scattering in the Brane-World and Black Hole Production **hep-th/9910269** Part.Nucl. **31**, 169-180 (2000)

[5] S. B. Giddings and S. Thomas *Phys. Rev.* D 65, 056010 (2002)

[6] S.S. Gubser, S.S. Pufu and A. Yarom *Phys.Rev.* D **78:066014**(2008)
S.S. Gubser, S.S. Pufu and A. Yarom *JHEP* **0911:050**(2009)
L. Alvarez-Gaume, C. Gomez, A. Sabio Vera, A. Tavanfar, M. A. Vazquez-Mozo *JHEP* **0902:009** (2009)

[7] I.Ya. Aref'eva, A.A. Bagrov, E.A. Guseva *JHEP* **0912:009**(2009)
I.Ya. Aref'eva, A.A. Bagrov, L.V. Joukovskaya *JHEP* **1003:002** (2010)

• The metric of arbitrary gravitational shock wave in AdS_5 space-time in Poincare coordinates [8]:

$$ds^2 = L^2 \frac{-dx^+ dx^- + dx_\perp^2 + \phi(z)\delta(x^+)dx^{+2} + dz^2}{z^2}, \qquad (1)$$

where $x^+ = t + x^1$, $x^- = t - x^1$ are light-cone coordinates, $x_\perp$ are coordinates transversal to the direction of moving.

[8] S.S. Gubser, S.S. Pufu and A. Yarom *Phys.Rev.* D **78:066014**(2008)

•The Einstein equation for the membrane wall without charge [9] in Poincare coordinates

$$(\partial_z^2 - \frac{3}{z}\partial_z)\phi(x^+, z) = -16\pi G_5 \mu \frac{z_0^3}{L^3}\delta(x^+)\delta(z - z_0),$$

where $\phi(x^+, z) = \phi(z)\delta(x^+)$. That is equivalent to

$$(\partial_z^2 - \frac{3}{z}\partial_z)\phi(z) = -16\pi G_5 \mu \frac{z_0^3}{L^3}\delta(x^+).$$

• We obtain the charged membrane wall Einstein equation of the form:

$$(\partial_z^2 - \frac{3}{z}\partial_z)\phi(z) = -16\pi G_5 \mu \frac{z_0^3}{L^3}\delta(z - z_0) - 16\pi G_5 T_{x^+x^+}(\bar{Q}, z) \qquad (2)$$

[9] S. Lin, E. Shuryak, *Phys.Rew. D*, **83**, 045025 (2011)

• In AdS_5 charged wave shape in Poincare coordinates has the form [10]

$$F = \frac{3\pi\bar{M}}{a}F_1 + \frac{5\pi\bar{Q}^2}{64a^2}F_2,$$

$$F_1 = \frac{(8q^2+8q+1) - 4(2q+1)\sqrt{q(1+q)}}{2\sqrt{q(1+q)}},$$

$$F_2 = \frac{144q^2+16q-1+128q^4+256q^3-64(2q+1)(q(q+1))^{3/2}}{q(1+q)\sqrt{q(1+q)}}$$

where $q = \dfrac{(x_\perp)^2+(z-z_0)^2}{4zz_0}$.

[10] I. Ya. Aref'eva, A. Bagrov and L. Joukovskaya, *JHEP* **1003:002** (2010).

• Now we apply the charged wave shape [11] to the expression [12] related the shape wave with corresponding plane stress-energy tensor x^+x^+ component :

$$q(q+1)F_{qq} + (3/2)(1+2q)F_q - 3F = -16\pi G_5 L^2 \rho, \quad (3)$$

$$J_{x^+x^+} = \frac{L}{z}\rho, \quad (4)$$

$J_{x^+x^+}$ is the bulk stress-energy tensor x^+x^+ for charged case Q. After that we make renormalization and spread the stress-energy tensor $J_{x^+x^+}$ all over surface.

[11] I. Ya. Aref'eva, A. Bagrov and L. Joukovskaya, *JHEP* **1003:002** (2010).
[12] S.S. Gubser, S.S. Pufu and A. Yarom *JHEP* **0911:050** (2009).

• Let us return to the part of Einstein equation for the charged membrane wall

$$(\partial_z^2 - \frac{3}{z}\partial_z)\phi(z) = -16\pi G_5 \mu \frac{z_0^3}{L^3}\delta(z - z_0) - 16\pi G_5 T_{x^+x^+}. \quad (5)$$

The charged part of x^+x^+ stress-energy tensor component corresponding to plane homogenous source has form

$$-16\pi G_5 T_{x^+x^+} = -\frac{40\pi G_5 Q^2 E}{3L^4 M} \times \quad (6)$$

$$\times \left(\frac{z^4 z_0(-z^2 + 3z_0^2)}{(-z^2 + z_0^2)^3}\Theta(z_0 - z) + \frac{z_0^5(-3z^2 + z_0^2)}{(-z^2 + z_0^2)^3}\Theta(z - z_0) \right).$$

Later we will consider the cases $z < z_0$, $z > z_0$ separately.

- Let us consider the following two equations separately:

$$(\partial_z^2 - \frac{3}{z}\partial_z)\phi_{qz_0>z} = -\frac{40\pi G_5 Q^2 E}{3L^4 M}\frac{z^4 z_0(-z^2+3z_0^2)}{(-z^2+z_0^2)^3},\ z_0 > z;$$

$$(\partial_z^2 - \frac{3}{z}\partial_z)\phi_{qz>z_0} = -\frac{40\pi G_5 Q^2 E}{3L^4 M}\frac{z_0^5(-3z^2+Z_0^2)}{(-z^2+z_0^2)^3},z > z_0.$$

• The function $\phi(z) = zL\psi(z)$ is solution to the equation. Here the function $\psi(z)$ can be represented in the form:

$$\psi_{qz_0>z} = \frac{C_1}{zL^4} + \frac{z^3 C_2}{L^4} - \frac{10}{3}\frac{z_0 Q^2 \pi E G_5 z^3}{L^5 M(|-z^2+z_0^2|)}, \quad z_0 > z,$$

$$\psi_{qz>z_0} = \frac{z^3 C_2}{L^4} + \frac{C_1}{zL^4} - \frac{10}{3}\frac{z_0^5 \pi G_5 Q^2 E}{L^5 M(|-z^2+z_0^2|) z}, \quad z > z_0$$

or

$$\psi_{qz_0>z} = \frac{C_1}{zL^4} + \frac{z^3 C_2}{L^4} - \frac{10}{3}\frac{z_0 z^3 Q^2 \pi\, E G_5}{L^5 M\left(-z^2+Z_0^2\right)}, \quad z_0 > z,$$

$$\psi_{qz>z_0} = \frac{z^3 C_2}{L^4} + \frac{C_1}{zL^4} + \frac{10}{3}\frac{z_0^5 Q^2 \pi\, E G_5}{L^5 M z\left(-z^2+z_0^2\right)}, \quad z > z_0$$

• A sufficient condition for a black hole formation is the existence of trapped surface.

• At the trapped surface the solution to the Einstein equation for the membrane wall must satisfy to conditions [13]:

1. $\psi(z_a) = \psi(z_b) = 0,$

2. $\left(\psi'(z_a)\frac{z_a}{L}\right)^2 = \left(\psi'(z_b)\frac{z_b}{L}\right)^2 = 4,$

where z_a and z_b are the boundaries of trapped surface.

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[13] S. Lin, E. Shuryak, *Phys.Rew. D*, **83**, 045025 (2011).

• In [14] solution to non-charged membrane wall Einstein equation constructed by such way that conditions 1 satisfied automatically

$$\Psi = \begin{cases} \psi_a = C\left(\dfrac{z^3}{z_a^3} - \dfrac{z_a}{z}\right), \; C = -\dfrac{4\pi G_5 \mu}{L^4} \dfrac{\left(\dfrac{z_0^4}{z_b^4} - 1\right) z_b}{\dfrac{z_b^4 - z_a^4}{z_a^3 z_b^3}}, \; z < z_0 \\ \\ \psi_b = D\left(\dfrac{z^3}{z_b^3} - \dfrac{z_b}{z}\right), \; D = -\dfrac{4\pi G_5 \mu}{L^4} \dfrac{\left(\dfrac{z_0^4}{z_a^4} - 1\right) z_a}{\dfrac{z_b^4 - z_a^4}{z_a^3 z_b^3}}, \; z_0 < z \end{cases}$$

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[14] S. Lin, E. Shuryak, *Phys.Rew. D*, **83**, 045025 (2011).

• We want that the solution to charged membrane wall Einstein equation satisfied automatically to the property 1 too.
Thus, the solution part corresponding to charge must satisfy to the properties 1 and can be represented in the form:

$$\begin{cases} \psi_{aq} = -\frac{10}{3}\frac{\pi\mu G_5 Q^2}{L^5 M} z_0 z^3 \frac{-z_a^2 + z^2}{(-z^2 + z_0^2)(-z_a^2 + z_0^2)}, \ z < z_0 \\ \\ \psi_{bq} = \frac{10}{3}\frac{\pi\mu G_5 Q^2}{L^5 M}\frac{z_0^5}{z}\frac{-z_b^2 + z^2}{(-z^2 + z_0^2)(-z_b^2 + z_0^2)}, \quad z_0 < z \end{cases} \tag{7}$$

• Let us consider the property 2

$$\left(\psi'(Z_a)\frac{Z_a}{L}\right)^2 = \left(\psi'(Z_b)\frac{Z_b}{L}\right)^2 = 4$$

for the complete solution:

$$\Psi = \begin{cases} \psi_a + \psi_{qa} \\ \psi_b + \psi_{qb} \end{cases} \tag{8}$$

• The first order derivatives of the solution to the Einstein equation at points z_a, z_b are the following:

$$\psi'(z_a) = -\frac{16\pi G_5 E}{L^4}\frac{\left(\frac{z_0^4}{z_b^4}-1\right)\frac{z_b}{z_a}}{\frac{z_b^4-z_a^4}{z_a^3 z_b^3}} - \frac{20}{3}\frac{\pi E G_5 Q^2}{L^5 M}\frac{z_0 z_a^4}{(-z_a^2+z_0^2)^2},$$

$$\psi'(z_b) = -\frac{16\pi G_5 E}{L^4}\frac{\left(\frac{z_0^4}{z_a^4}-1\right)\frac{z_a}{z_b}}{\frac{z_b^4-z_a^4}{z_a^3 z_b^3}} + \frac{20}{3}\frac{\pi E G_5 Q^2}{L^5 M}\frac{Z_0^5}{(-z_b^2+z_0^2)^2}.$$

• The condition 2 gives us the following equations:

$$F_a = -\frac{8\pi G_5 E\left(z_0^4 - z_b^4\right) z_a^3}{L^5(z_b^4 - z_a^4)} - \frac{10}{3}\frac{\pi E G_5 Q^2}{L^6 M}\frac{z_0 z_a^5}{(-z_a^2 + z_0^2)^2} = 1,$$

$$F_b = -\frac{8\pi G_5 E\left(z_0^4 - z_a^4\right) z_b^3}{L^5(z_b^4 - z_a^4)} + \frac{10}{3}\frac{\pi E G_5 Q^2}{L^6 M}\frac{z_0^5 z_b}{(-z_b^2 + z_0^2)^2} = -1.$$

These equations have not analytical solutions. We look for numerical solutions and graphical analyse.

Numerical calculations and graphical analyse

Example of parameters choosing. Now let us use the following initial paraments:

$$E = 19.7\,TeV \tag{9}$$

$$L = Z_0 = 4.3fm,$$

$$\frac{L^3}{G_5} = 1.9,$$

$$fm = 10^{-15}m, \quad TeV = 10^{12}eV. \tag{10}$$

Using $1TeV = 10^3GeV = (1.2)^{-1} \cdot 10^3 fm^{-1} \approx 0.8(3) \cdot 10^3 fm^{-1}$ and direct calculations we obtain:

$$E \approx 16416.7fm^{-1} \tag{11}$$

Analogical numerical calculations we apply for another E.

The function F_a for $E = 19.7$ TeV is in 1.

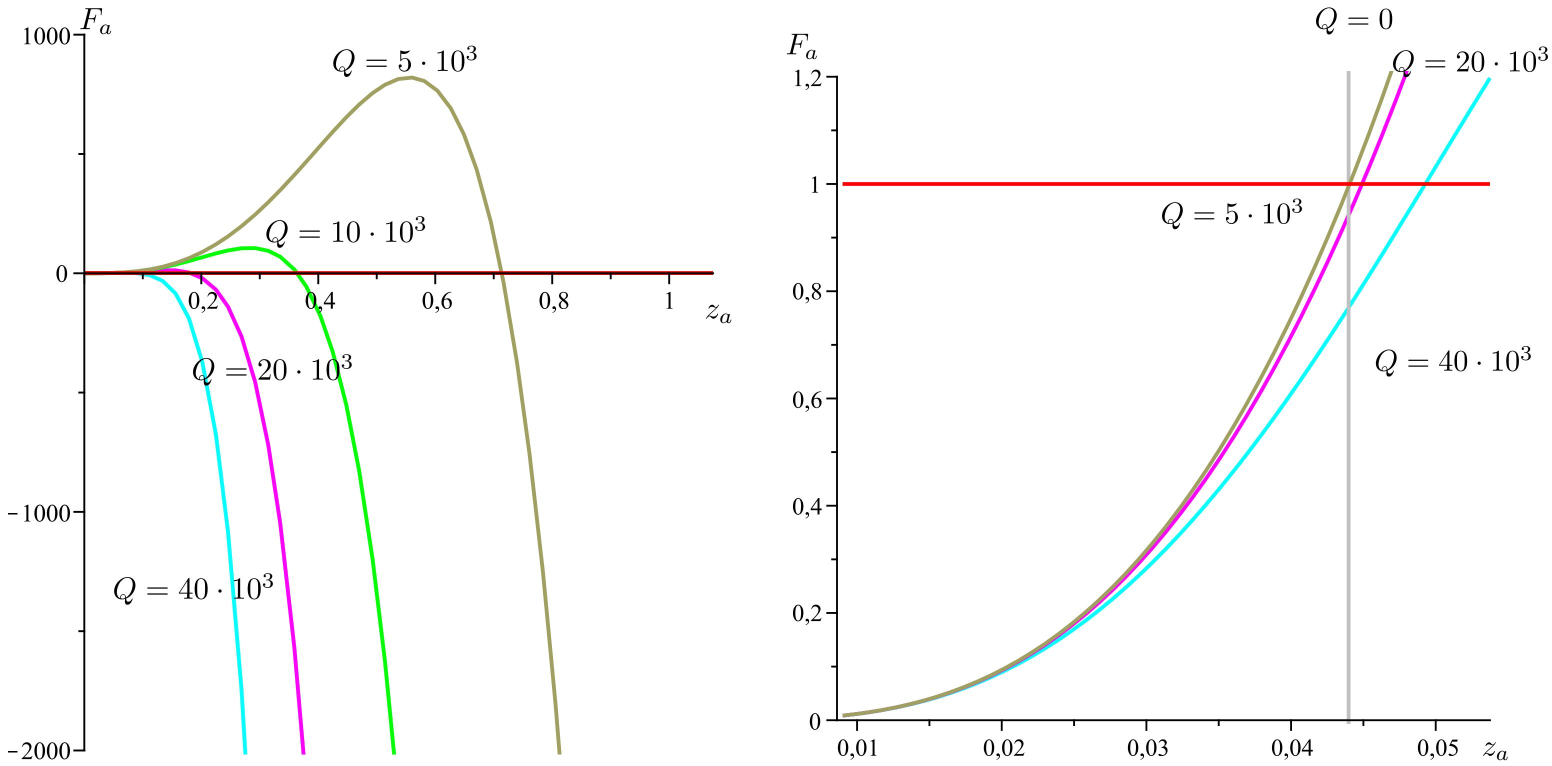

Figure 1: The function $F_a(z_a)$ and the function $F_a(z_a)$ near the root

The function F_b for $E = 19.7$ TeV is in 2.

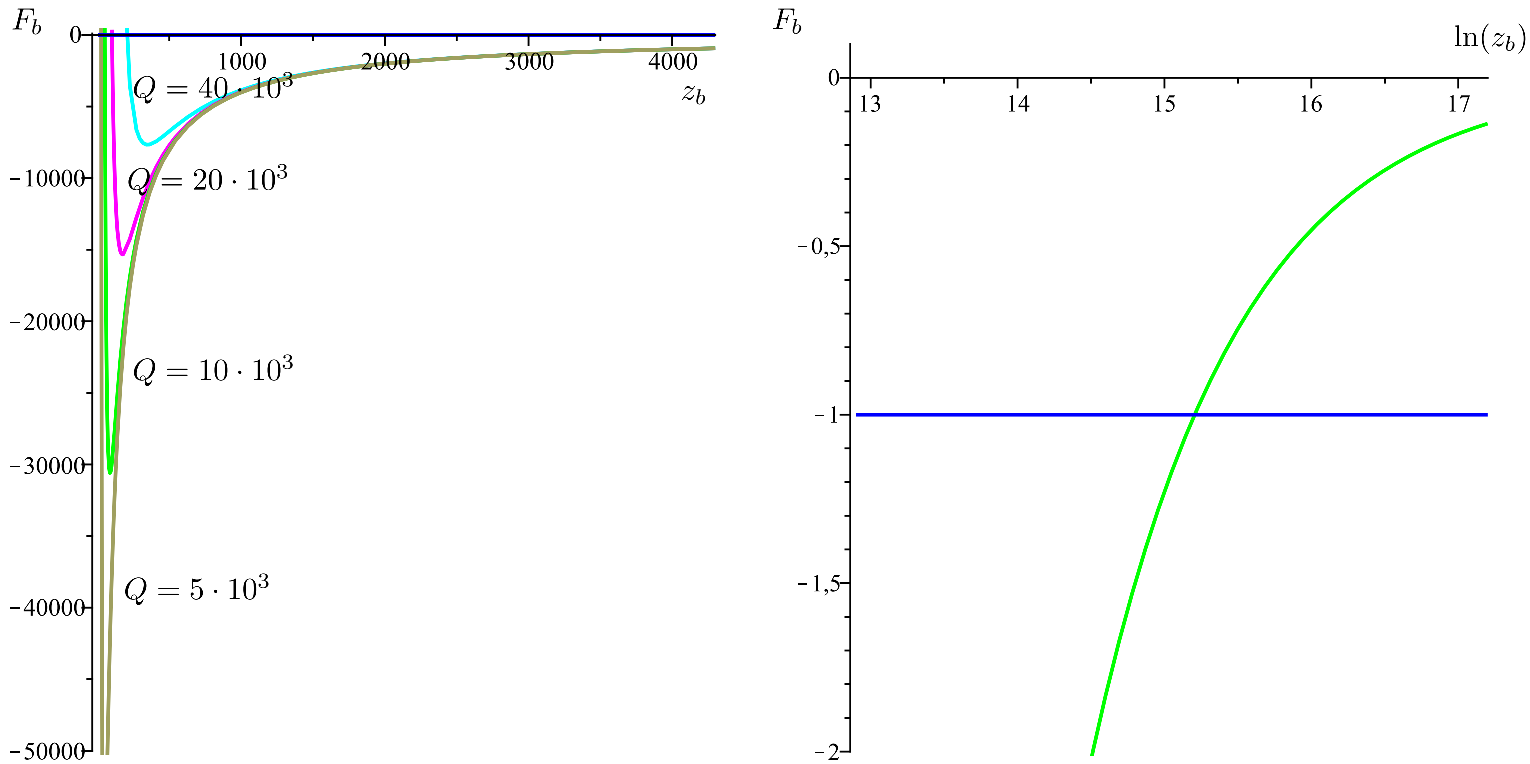

Figure 2: The function $F_b(z_b)$ and the function $F_b(z_b)$ near the root

The square trapped surface calculation

$$S = \frac{2A}{4G_5} = \frac{\int \sqrt{g} dz d^2 x_\perp}{2G_5}, \tag{12}$$

$$s \equiv \frac{S}{d^2 x_\perp} = \frac{L^3}{4G_5} \left(\frac{1}{z_a^2} - \frac{1}{z_b^2} \right). \tag{13}$$

The trapped surface decreases with growth of a charge. The corresponding graphical representations are in the picture (3):

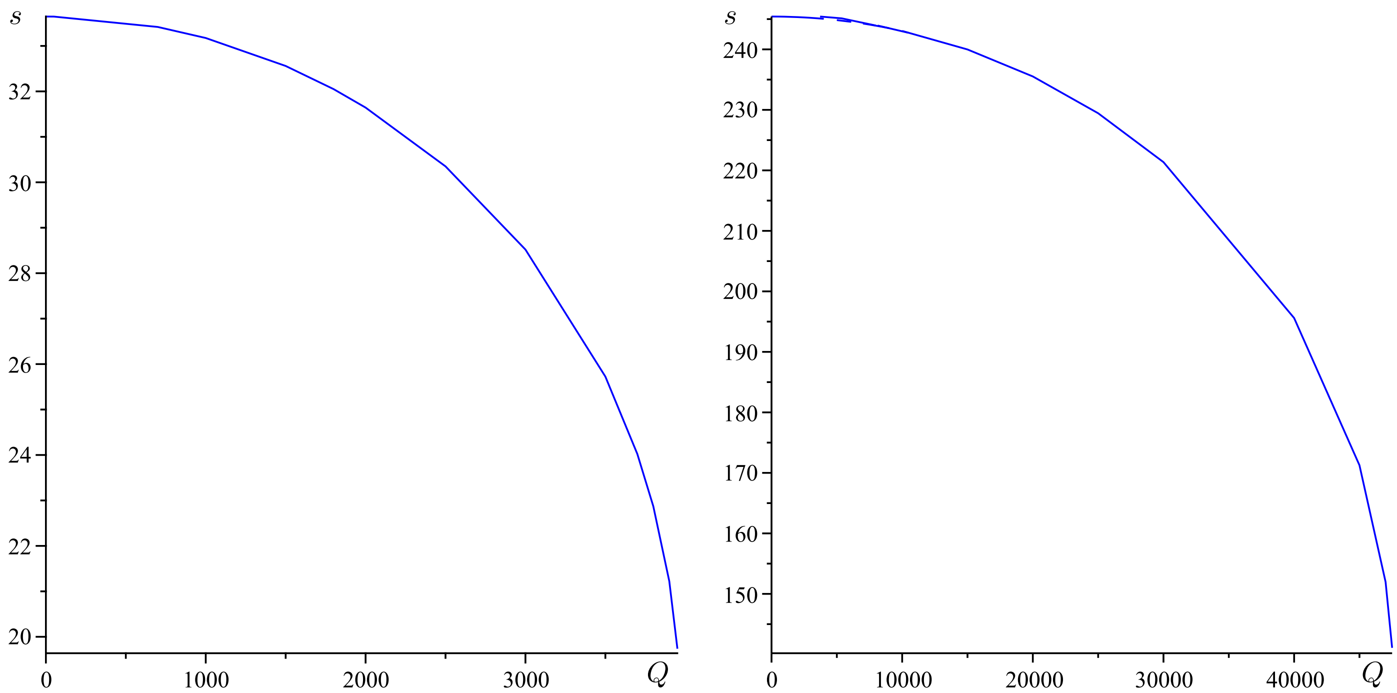

Figure 3: The trapped surface area s(Q) for $E = 1\,TeV$ $E = 19.7\,TeV$

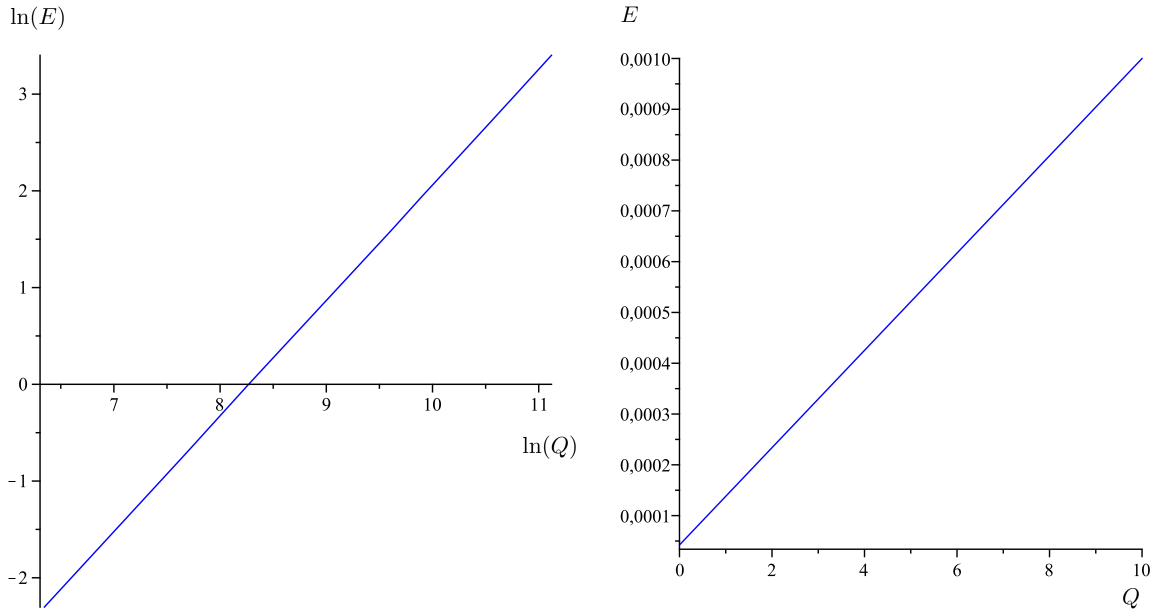

Figure 4: Phase diagram in the logarithmic scale for large Q and large E. The phase diagram for small E and small Q.

Summarizing

At the present report we have considered the charged membrane walls collision in AdS_5 using Einstein equation and shock waves approach. We have studied the influence of the charges to the trapped surface formation in the charged walls collision and obtain the diagram with allowed and forbidden zones for black hole formation.