Effective actions for high energy scattering in QCD and in gravity

L. N. Lipatov
Petersburg Nuclear Physics Institute,
Gatchina, St.Petersburg, Russia

QFTHEP2011, Sochi

Content

1. Gribov Pomeron calculus
2. BFKL equation
3. Effective action for high energy QCD
4. Production amplitudes in $N = 4$ SUSY
5. Pomeron and graviton in $N = 4$ SUSY
6. High energy amplitudes in gravity and supergravity
7. Effective action for reggeized gravitons
8. Discussion

1 Scattering amplitudes at high energies

High energy kinematics

$$s = 4E^2 \gg -t = \vec{q}^2 \,, \; \theta \approx \frac{|q|}{E} \ll 1$$

t-channel partial wave expansion

$$A^p(s,t) = s \int_{a-i\infty}^{a+i\infty} \frac{d\omega}{2\pi i} \left((-s)^\omega - p s^\omega\right) f^p_\omega(t) \,, \; p = \pm 1$$

Regge pole hypothesis and Pomeron trajectory

$$f^p_\omega(t) = \frac{\gamma^2(t)}{\omega - \omega_p(t)} \,, \; \omega_P(-q^2) \approx \Delta_P - \alpha'_P q^2 \,, \; p = 1$$

Asymptotics of elastic amplitudes and cross-sections

$$A(s,t) \approx is\,\gamma^2(-q^2) s^{\Delta_P - \alpha'_P q^2} \,, \; \sigma = \gamma^2(0)\, s^{\Delta_P}$$

2 Gribov Pomeron calculus

Multi-particle t-channel unitarity

$$\Im_t f_\omega(t) \sim \sum_n \int d\Omega_n |f_\omega^{(n)}|^2$$

Mandelstam cut contribution

$$A_M(s,t) \approx -i \int d^2k\, \Phi^2(k, q-k)\, s^{2\Delta - \alpha'_P k^2 - \alpha'_P (q-k)^2}$$

Separation of particles in their rapidities

$$0 < y_1 < \ldots < y_n < \ln s\,, \; y_k - y_{k-1} \gg 1\,, \; y = \frac{1}{2} \ln \frac{E+p}{E-p}$$

Gribov's Pomeron action

$$S = \int dy\, d^2\rho \left(\phi^* (\partial_y - \Delta) \phi + \frac{(\vec{\nabla}\phi)^2}{2m} + \lambda(\phi^* \phi^2 + \phi \phi^{*2}) + \ldots \right)$$

3 Gluon reggeization in QCD

QCD Born amplitude

$$M_{AB}^{A'B'}(s,t)|_{Born} = 2s\, g\, T^{c}_{A'A}\, \delta_{\lambda_{A'}\lambda_A}\, \frac{1}{t}\, g\, T^{c}_{B'B}\, \delta_{\lambda_{B'}\lambda_B}$$

Leading Logarithmic Approximation

$$M(s,t) = M_{Born}(s,t)\, s^{\omega(t)}\,,$$

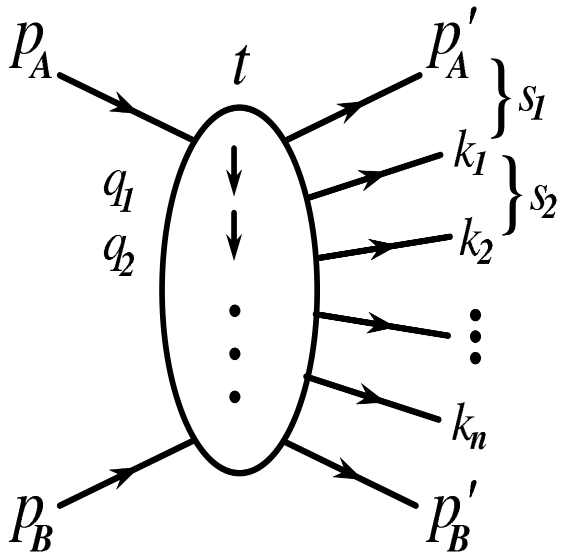
Its region of applicability

$$\alpha_s \ln s \sim 1\,,\ \alpha_s = \frac{g^2}{4\pi} \ll 1$$

Gluon trajectory in LLA

$$\omega(-|q|^2) = -\frac{\alpha_s N_c}{4\pi^2} \int d^2k\, \frac{|q|^2}{|k|^2|q-k|^2} \approx -\frac{\alpha_s N_c}{2\pi} \ln \frac{|q^2|}{\lambda^2}$$

4 Amplitudes in multi-Regge kinematics



$$M^{BFKL}_{2\to 2+n} \sim \frac{s_1^{\omega_1}}{|q_1|^2}\, gT^{d_1}_{c_2c_1} C(q_2,q_1)\, \frac{s_2^{\omega_2}}{|q_2|^2} ... gT^{d_n}_{c_{n+1}c_n}\, C(q_{n+1},q_n)\, \frac{s_{n+1}^{\omega_{n+1}}}{|q_{n+1}|^2}\,,$$

$$\omega_r = -\frac{\alpha_s N_c}{2\pi}\left(\ln\frac{|q_r^2|}{\mu^2} - \frac{1}{\epsilon}\right)\,,\ C(q_2,q_1) = \frac{q_2\, q_1^*}{q_2^* - q_1^*}\,,\ \sigma_t = \sum_n \int d\Gamma_n\, |M_{2\to 2+n}|^2$$

5 BFKL equation (1975)

Balitsky-Fadin-Kuraev-Lipatov equation

$$E\,\Psi(\vec{\rho}_1,\vec{\rho}_2) = H_{12}\,\Psi(\vec{\rho}_1,\vec{\rho}_2)\,,\ \sigma_t \sim s^{\Delta}\,,\ \Delta = -\frac{\alpha_s N_c}{2\pi}\,E_0$$

Hamiltonian for the Pomeron wave function

$$H_{12} = \frac{1}{p_1 p_2^*}(\ln|\rho_{12}|^2)p_1 p_2^* + \frac{1}{p_1^* p_2}(\ln|\rho_{12}|^2)p_1^* p_2 + \ln\,|p_1 p_2|^2 - 4\psi(1)\,,$$

$$\rho_{12} = \rho_1 - \rho_2\,,\ \rho_r = x_r + iy_r\,,\ \Delta = 4\alpha N_c\,\ln 2\,/\pi$$

Möbius invariance and Pomeron intercept

$$\rho_k \to \frac{a\rho_k + b}{c\rho_k + d}\,,\ m = \gamma + n/2\,,\ \widetilde{m} = \gamma - n/2\,,\ \gamma = 1/2 + i\nu\,,$$

$$E = \epsilon_m + \epsilon_{\widetilde{m}}\,,\ \epsilon_m = \psi(m) + \psi(1-m) - 2\psi(1)\,,\ \Delta = \frac{g^2 N_c}{\pi^2}\ln 2$$

6 Effective action in QCD

Locality of the theory in the rapidity space

$$y = \frac{1}{2} \ln \frac{\epsilon_k + |k|}{\epsilon_k - |k|}\,, \;\; |y - y_0| < \eta\,, \;\; \eta << \ln\, s$$

Gluon and Reggeized gluon fields

$$v_\mu(x) = -iT^a v_\mu^a(x)\,, \;\; A_\pm(x) = -iT^a A_\pm^a(x)\,, \, \delta A_\pm(x) = 0$$

Effective action for their interactions (L., 1995)

$$S = \int d^4x \left(L_{QCD} + Tr(V_+ \partial_\mu^2 A_- + V_- \partial_\mu^2 A_+) \right) \,,$$

$$V_+ = -\frac{1}{g} \partial_+ \, P \exp \left(-g \int_{-\infty}^{x^+} v_+(x') d(x')^+ \right) = v_+ - g v_+ \frac{1}{\partial_+} v_+ + ...$$

7 Production amplitudes in $N=4$ SUSY

Relative correction R to the BDS amplitude $A_{2\to4}$ (F.,L., 2011)

$$R\,e^{i\pi\delta} = cos\,\pi\omega_{ab}+i\,\frac{a}{2}\sum_{n=-\infty}^{\infty}(-1)^n e^{i\phi n}\int_{-\infty}^{\infty}\frac{|w|^{2i\nu}d\nu}{\nu^2+\frac{n^2}{4}}\,\Phi(\nu,n)\left(\frac{1}{u_1-1}\right)^{\omega(\nu,n)},$$

$$u_1=\frac{ss_2}{s_{012}s_{123}}\,,\;\; u_2=\frac{s_1t_3}{s_{012}t_2}\,,\;\; u_3=\frac{s_3t_1}{s_{123}t_2}\,,\;\; |w|^2=\frac{u_2}{u_3}\,,$$

$$\cos\phi=\frac{1-u_1-u_2-u_3}{2\sqrt{u_2u_3}}\,,\;\; \delta=\frac{\gamma_K}{8}\,\ln\frac{|w|^2}{|1+w|^4}\,,\;\; \omega_{ab}=\frac{\gamma_K}{8}\,\ln|w|^2\,,$$

$$\omega(\nu,n)=-aE_{\nu,n}-a^2(\epsilon_{\nu n}+3\zeta(3))\,,\;\; E_{\nu n}=-\frac{1}{2}\,\frac{|n|}{\nu^2+\frac{n^2}{4}}+2\Re\psi(1+i\nu+\frac{|n|}{2})-2\psi(1)\,,$$

$$\epsilon_{\nu n}=-\frac{\Re}{2}\left(\psi''(1+i\nu+\frac{|n|}{2})-\frac{2i\nu\psi'(1+i\nu+\frac{|n|}{2})}{\nu^2+\frac{n^2}{4}}\right)-\zeta(2)\,E_{\nu n}-\frac{1}{4}\,\frac{|n|\,\left(\nu^2-\frac{n^2}{4}\right)}{\left(\nu^2+\frac{n^2}{4}\right)^3}$$

8 Pomeron and graviton in $N = 4$ SUSY

BFKL Pomeron in a diffusion approximation

$$j = 2 - \Delta - D\,\nu^2\,,\;\; \gamma = 1 + \frac{j-2}{2} + i\nu$$

Constraint from the energy-momentum conservation

$$D = \Delta$$

AdS/CFT relation for the graviton Regge trajectry

$$j = 2 + \frac{\alpha'}{2}\,t\,,\;\; t = E^2/R^2\,,\;\; \alpha' = \frac{R^2}{2}\,\Delta$$

Large coupling asymptotics for γ and Δ (KLOV, BPST)

$$\gamma = -\Delta\sqrt{j-2+\Delta}\,,\;\; \Delta = \frac{1}{\sqrt{g^2 N_c}}$$

9 Perturbation theory in gravity

Einstein-Hilbert action

$$S_{EH} = -\frac{1}{2\kappa^2} \int d^4x \, \sqrt{-g} \, R \, , \;\; R = R_{\mu\nu} \, g^{\mu\nu}$$

Riemann tensor

$$R_{\mu\nu} = R^{\sigma}_{\mu,\sigma\nu} \, , \;\; R^{\sigma}_{\mu,\alpha\beta} = \partial_\beta \Gamma^{\sigma}_{\mu\alpha} - \partial_\alpha \Gamma^{\sigma}_{\mu\beta} + \Gamma^{\rho}_{\mu\alpha} \, \Gamma^{\sigma}_{\rho\beta} - \Gamma^{\rho}_{\mu\beta} \, \Gamma^{\sigma}_{\rho\alpha}$$

Christophel symbol and gravity field

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} \, g^{\rho\sigma} \, \left(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu} \right) \, , \;\; g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

General coordinate transformation

$$\delta h_{\mu\nu} = D_\mu \chi_\nu + D_\nu \chi_\mu \, , \;\; D_\mu \chi_\nu = \partial_\mu \chi_\nu - \Gamma^{\rho}_{\mu\nu} \chi_\rho$$

10 High energy amplitudes in gravity

Production amplitudes in LLA (L.L. (1982))

$$A_{2\to n} = -s^2 \Gamma^{\mu'\nu'}_{\mu\nu} \frac{s_1^{\omega(q_1^2)}}{q_1^2} \Gamma_{\rho_1\sigma_1} \frac{s_2^{\omega(q_2^2)}}{q_2^2} \Gamma_{\rho_2\sigma_2} ... \Gamma^{\rho'\sigma'}_{\rho\sigma}$$

Graviton-graviton-reggeon vertex

$$\Gamma^{\mu'\nu'}_{\mu\nu} = \frac{\kappa}{4} \left(\Gamma_{\mu\mu'} \Gamma_{\nu\nu'} + \Gamma_{\mu\nu'} \Gamma_{\nu\mu'} \right)$$

Gluon-gluon-reggeized gluon vertex

$$\Gamma_{\mu\mu'} = -\delta_{\mu\mu'} + \frac{p^A_{\mu'} p^B_{\mu} + p^{A'}_{\mu} p^B_{\mu'}}{p^A p^B} + \frac{q^2}{2} \frac{p^B_{\mu} p^B_{\mu'}}{(p^A p^B)^2}$$

Reggeon–reggeon-graviton vertex

$$\Gamma_{\rho\sigma} = \frac{\kappa}{4} \left(C_\rho C_\sigma - N_\rho N_\sigma \right) \, , \; N = \sqrt{q_1^2 q_2^2} \left(\frac{p^A}{kp^A} - \frac{p^B}{kp^B} \right)$$

11 Graviton trajectory and amplitudes

Graviton Regge trajectory (L. (1982))

$$\omega(q^2) = \frac{a}{\pi} \int \frac{q^2\, d^2k}{k^2(q-k)^2} f(k,q)\,,\;\; a = \frac{\kappa^2}{8\pi^2}\,,$$

$$f(k,q) = (k, q-k)^2 \left(\frac{1}{k^2} + \frac{1}{(q-k)^2} \right) - q^2 + \frac{N}{2}(k, q-k)$$

Gravitino action

$$S_{3/2} = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma} \int d^4x \sum_{r=1}^{N} \bar{\psi}^r_\mu \gamma_5 \gamma_\nu \partial_\rho \psi^r_\sigma$$

Amplitudes in the DL approximation

$$A = A_{Born} s^{-a\ln\frac{q^2}{\lambda^2}} \int \frac{d\omega}{2\pi i}\, s^\omega\, \frac{f_\omega}{\omega}\,,\;\; f_\omega = 1 - \frac{N-4}{2}\, a\, \frac{f_\omega^2}{\omega^2} - \frac{a}{2\omega}\, \frac{d}{d\omega}\, f_\omega$$

12 Effective action for gravity

Locality in the rapidity space

$$y=\frac{1}{2}\ln\frac{\epsilon_k+|k|}{\epsilon_k-|k|}\,,\;\;|y-y_0|<\eta\,,\;\;\eta<<\ln\,s$$

Reggeized graviton fields

$$\delta A^{++}(x)=\delta A^{--}(x)=0\,,\;\partial_+A^{++}(x)=\partial_-A^{--}(x)=0$$

Effective action for the high energy gravity (L. 2011)

$$S=-\frac{1}{2\kappa}\int d^4x\left(\sqrt{-g}\,R+\frac{1}{2}\left(\partial_+j^-\,\partial_\mu^2A^{++}+\partial_-j_+\,\partial_\mu^2A^{--}\right)\right)$$

Hamilton-Jacobi equation for effective currents

$$g^{\mu\nu}\partial_\mu j^\pm\partial_\nu j^\pm=0\,,\;\partial_\pm j^\mp=h_{\pm\pm}-\left(h_{\rho\pm}-\frac{1}{2}\frac{\partial_\rho}{\partial_\pm}h_{\pm\pm}\right)^2+...$$

13 Effective currents for shock waves

Aichelburg - Sexl metric

$$(ds)^2 = \eta_{\mu\nu}\, dx^\mu dx^\nu + a\, \ln|\overrightarrow{x}|\delta(x^-)\,(dx^-)^2\,,\ a = \frac{8}{\sqrt{2}}\, G\, \mu\,,\ z = a\,\frac{x^-}{|\overrightarrow{x}|^2}$$

Effective current for the shock wave

$$j^+ = -a\,\mu \left(\ln|\overrightarrow{x}| + \ln f(z) - \frac{1}{4}\,\frac{z}{f^2(z)}\right)\,,\ f(z) = \frac{1}{2} + \frac{\sqrt{1+2z}}{2}$$

Perturbative expansion

$$j^+ = -a\ln|\overrightarrow{x}| + \frac{a^2}{\partial_-}\left(\frac{x_\sigma}{2|\overrightarrow{x}|}\right)^2) - \frac{a^3}{\partial_-}\,\frac{x_\mu}{2|\overrightarrow{x}|}\,\frac{\partial_\mu}{\partial_-}\left(\frac{x_\sigma}{2|\overrightarrow{x}|}\right)^2 + \ldots$$

Variational principle for j^+

$$j^+ = \int_{-\infty}^{x^-}\left(g^{++}(y^-, \overrightarrow{\rho}(y^-)) + (\partial_-\overrightarrow{\rho})^2\right)\,,\ \frac{\delta j^+}{\delta\overrightarrow{\rho}(y^+)} = 0$$

14 Discussion

1. Locality of reggeon interactions in the rapidity space.
2. BFKL equation for Pomeron wave function
3. High energy effective action for gluons in QCD.
4. Pomeron-graviton duality in $N = 4$ SUSY.
5. Multi-regge processes in gravity.
6. The graviton trajectory and double logarithms.
7. Effective action for the high energy gravity.
8. Hamilton-Jacobi equation for effective currents.