Pion elastic and $\pi \to \gamma\gamma^*$ form factors in a broad range of momentum transfers

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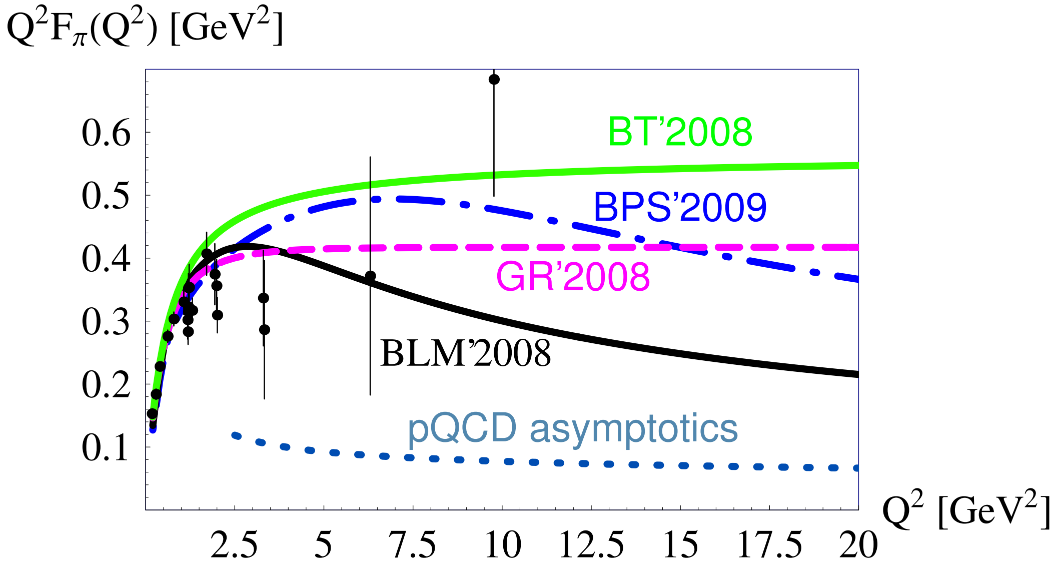
We study $F_\pi(Q^2)$ and $F_{P\gamma}(Q^2)$, $P = \pi, \eta, \eta'$ making use of the local-duality (LD) version of QCD sum rules.

To probe the accuracy of the LD sum rule, we consider in parallel to QCD a potential model: in this case, the exact form factor may be calculated from the solutions of the Schroedinger equation and confronted with the result from the quantum-mechanical LD sum rule, thus probing the accuracy of the method.

We argue that the LD sum rule is expected to give reliable predictions for $F_\pi(Q^2)$ and $F_{\pi\gamma}(Q^2)$ in the region $Q^2 \geq 5-6$ GeV^2.

Based on "Accuracy of local-duality sum rules for the pion elastic form factor" arXiv:1103.3781 and work on $F_{\pi\gamma}$ in progress.

Theoretical description of the pion form factor at $Q^2 \sim 5 - 50$ GeV^2 in QCD is a complicated problem. Some recent results are shown on the plot:



No conclusive results have been obtained and we still have a strong discrepancy between the results from various theoretical approaches.

Our goal is to study $F_\pi(Q^2)$ and $F_{P\gamma}(Q^2)$ making use of the so-called local-duality version of QCD sum rules.

The basic steps leading to a LD sum rule:

The basic object: $\langle 0|T\, j^5_\alpha j_\mu j^5_\beta|0\rangle$.

j^5_α, j^5_β **- are the pion interpolating axial currents.** j_μ **is the electromagnetic current.**

In QCD this correlator may be calculated by applying OPE. Duality assumption says that the contribution of the excited states is dual to the high-energy region of the perturbative diagrams. Using this assumption, the sum rule takes the form

$$f_\pi^2 F_\pi(Q^2) \;=\; \int\limits_0^{s_{\rm eff}(\tau,Q^2)} ds_1 \int\limits_0^{s_{\rm eff}(\tau,Q^2)} ds_2 e^{-\frac{(s_1+s_2)\tau}{2}} \Delta_{\rm pert}(s_1,s_2,Q^2) + \frac{\langle \alpha_s G^2\rangle}{24\pi}\tau + \frac{4\pi\alpha_s\langle\bar{q}q\rangle^2}{81}(13+Q^2\tau)\tau^2 + \cdots$$

$\Delta_{\rm pert}$ **are double spectral densities of 3-point diagrams of perturbation theory.**

We want to study the form factor at large Q. The form factor of a bound state should decrease with Q; however, power corrections on the r.h.s. are polynomials in Q and thus rise with Q. So, this expression cannot be directly used at large Q. To apply sum rule at large Q one of the few possibilities is just set $\tau = 0$.

The Local – duality (LD) limit is $\tau \to 0$. Then ALL power corrections vanish.

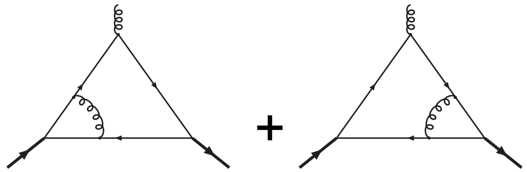
$$F_\pi(Q^2) = \frac{1}{f_\pi^2} \int\limits_0^{s_{\rm eff}(Q^2)} ds_1 \int\limits_0^{s_{\rm eff}(Q^2)} ds_2\, \Delta^{(VAV)}_{\rm pert}(s_1,s_2,Q^2).$$

For any given prediction for the form factor $F_\pi(Q^2)$, one can calculate the equivalent $s_{\rm eff}(Q^2)$. The problem is now how to determine the "true" $s_{\rm eff}(Q^2)$.

Properties of the spectral functions

• **Vector Ward identity at $Q^2 = 0$ relates 3-point and 2-point functions.**

• **Factorization at $Q^2 \to \infty$: the leading $1/Q^2$ behavior of the spectral function is given by**



If we set

$$s_{\text{eff}}(Q^2 = 0) = \frac{4\pi^2 f_\pi^2}{1 + \alpha_s/\pi} \qquad s_{\text{eff}}(Q^2 \to \infty) = 4\pi^2 f_\pi^2,$$

then the form factor obtained from the LD sum rule satisfies the correct normalization at $Q^2 = 0$ and reproduces the asymptotic behavior according to the factorization theorem for the form factor at $Q^2 \to \infty$.

The two values are not far from each other, construct an interpolation function $s_{\text{eff}}(Q^2)$ for all Q^2.

The local – duality model for hadron elastic form factors :

a. Based on a dispersive three-point sum rule at $\tau = 0$ (i.e. infinitely large Borel mass parameter). In this case all power corrections vanish and the details of the non-perturbative dynamics are hidden in one quantity — the effective threshold $s_{\rm eff}(Q^2)$.

b. Makes use of a model for $s_{\rm eff}(Q^2)$ based on a smooth interpolation between its values at $Q^2 = 0$ determined by the Ward identity and at $Q^2 \to \infty$ determined by factorization. Since these values are not far from each other, the details of the interpolation are not essential.

Obviously, the LD model for the effective continuum is a model which does not take into account the details of the confinement dynamics. The only property of theory relevant for this model is factorization of hard form factors.

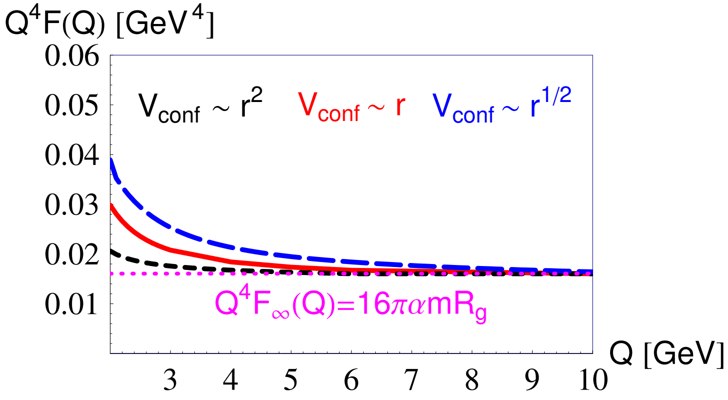
The model may be tested in quantum mechanics for the case of the potential containing the Coulomb and Confining interactions.

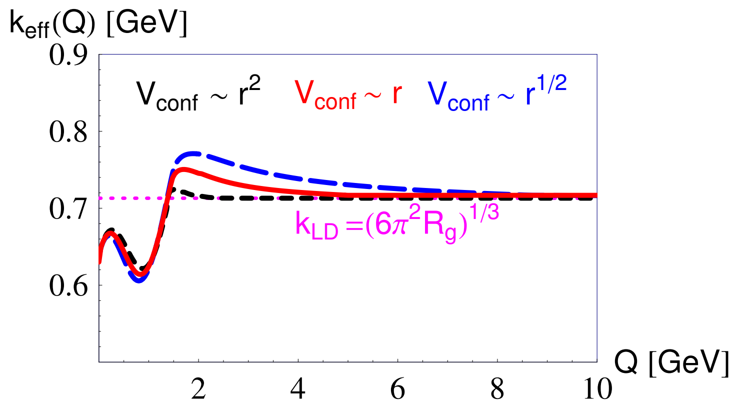
• The form factor satisfies factorization theorem similar to QCD. LD sum rules is very similar to QCD; the spectral densities are calculated from diagrams of NR field theory.

• The exact form factor may be calculated and confronted with LD model, probing its accuracy.

Elastic form factor

Results for elastic form factor in quantum-mechanical potential model





The plots show the results for the elastic form factor in potential model. The potential containing a Coulomb interaction and a confining part for several different confining parts: HO potential, linear potential, and $r^{1/2}$ potential.

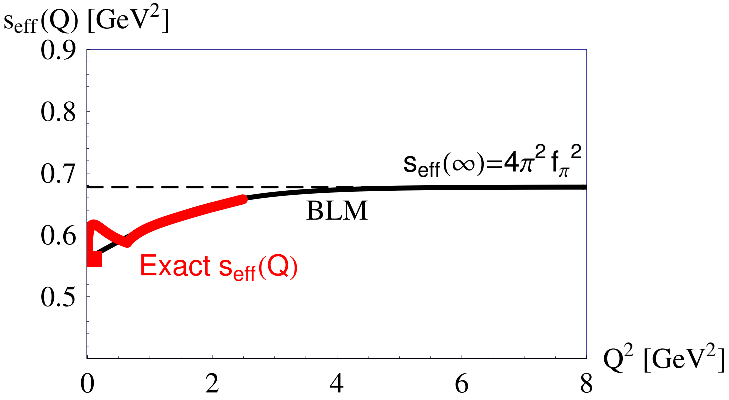
Left: the exact form factors for these potentials

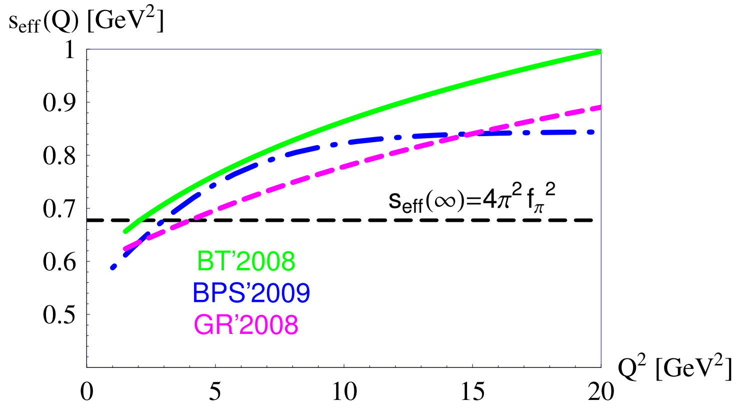
Right: the corresponding equivalent effective thresholds.

An important conclusion from these plots:

Independently of the form of the confining part, the accuracy of the LD model increases with Q already starting with relatively low values $Q \simeq 2-3$ GeV.

Results for elastic pion form factor in QCD:





Left plot: the equivalent threshold extracted from the experimental data and the LD model. So far one can see no disagreement between the two.

Right: the equivalent thresholds for other theoretical predictions are given on the right plot. Obviously, these results imply that the accuracy of the LD model decreases with Q^2 even at Q^2 as large as $Q^2 = 20$ GeV^2. Let is notice that this is in conflict with our experience from quantum mechanics.

The future accurate data expected from JLab in the range up to $Q^2 = 8$ GeV^2 should decide.

$\mathrm{P} \to \gamma\gamma^*$ transition form factor

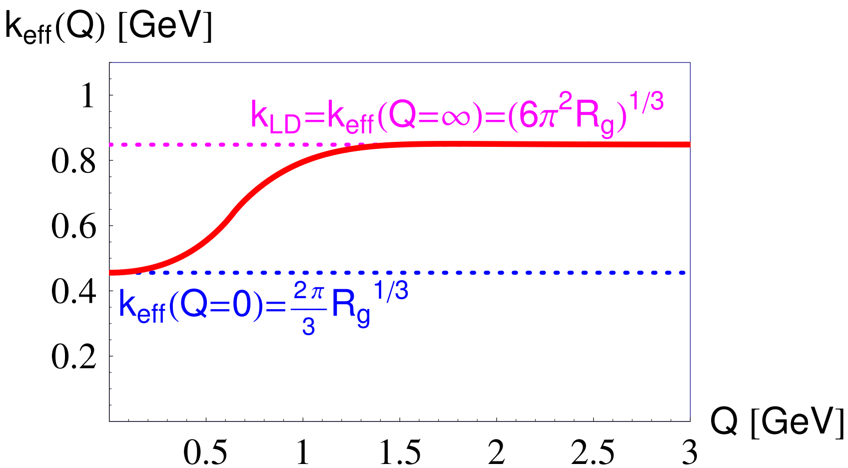
A Borel sum rule for $\langle 0|T j_\alpha j_\beta j^5_\mu|0\rangle$ at $\tau = 0$:

$$F_{\pi\gamma}(Q^2) = \frac{1}{f_\pi} \int\limits_0^{s_{\rm eff}(Q^2)} \Delta^{(AVV)}_{\rm pert}(s, Q^2)\, ds, \qquad s_{\rm eff}(Q^2 \to \infty) \to 4\pi^2 f_\pi^2.$$

$\Delta^{(AVV)}_{\rm pert}(s, Q^2)$ is calculable from diagram of perturbative QCD. Fixing $s_{\rm eff}(Q^2)$ gives $F_{\pi\gamma}(Q^2)$.

Quantum mechanics:

Here is the exact effective threshold obtained for a quantum-mechanical model with HO potential. The parameters are chosen such that the ground state has a typical hadron size 1 Fm.



The LD threshold gives a very good approximation to the exact threshold at $Q > 1.5$ GeV. The accuracy of the LD approximation further increases with Q in this region.

$\eta, \eta' \to \gamma\gamma^*$ transition form factor

$\eta - \eta'$-mixing scheme by Feldmann et al

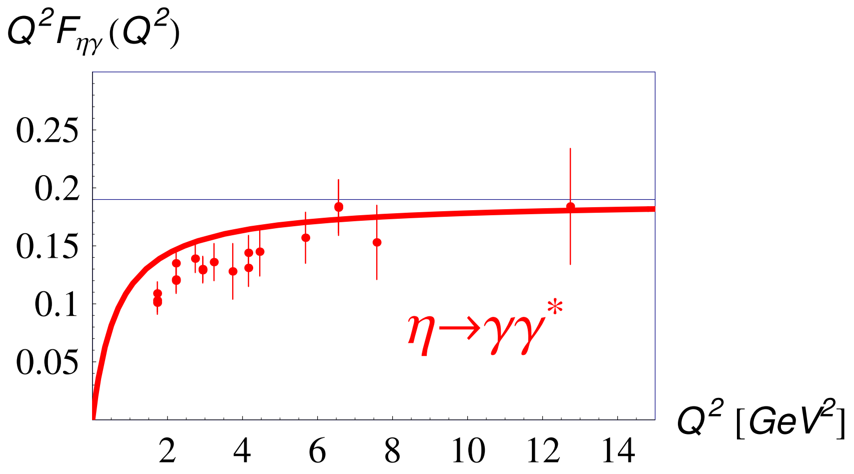
$$F_{\eta\gamma} = \cos(\phi)F_{n\gamma} - \sin(\phi)F_{s\gamma}, \quad F_{\eta'\gamma} = \sin(\phi)F_{n\gamma} + \cos(\phi)F_{s\gamma}, \quad \phi \simeq 38^0$$

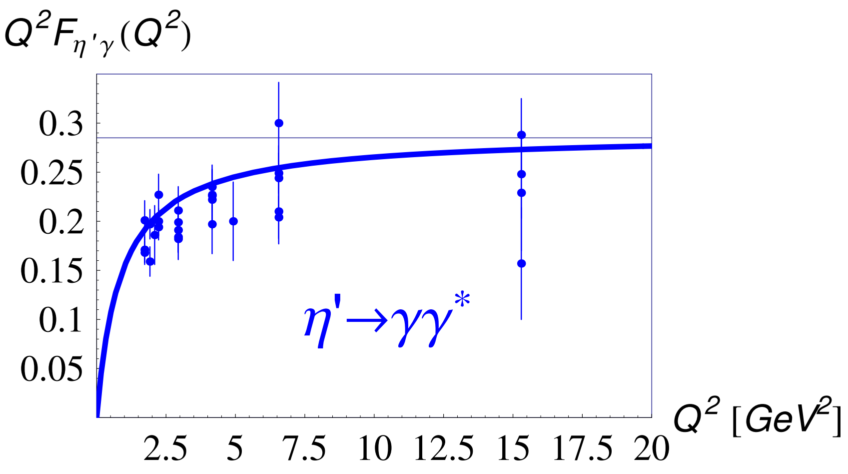
with $n \to \frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$ and $s \to \bar{s}s$.

Two LD expressions for these form factors:

$$F_{n\gamma}(Q^2) = \frac{1}{f_n} \int_0^{s_{\rm eff}^{(n)}(Q^2)} \Delta_n(s, Q^2)\, ds, \qquad F_{s\gamma}(Q^2) = \frac{1}{f_s} \int_0^{s_{\rm eff}^{(s)}(Q^2)} \Delta_s(s, Q^2)\, ds,$$

Two separate effective thresholds: $s_{\rm eff}^{(n)} = 4\pi^2 f_n^2, \quad s_{\rm eff}^{(s)} = 4\pi^2 f_s^2, \quad f_n \simeq 1.07 f_\pi, \quad f_s \simeq 1.36 f_\pi$.

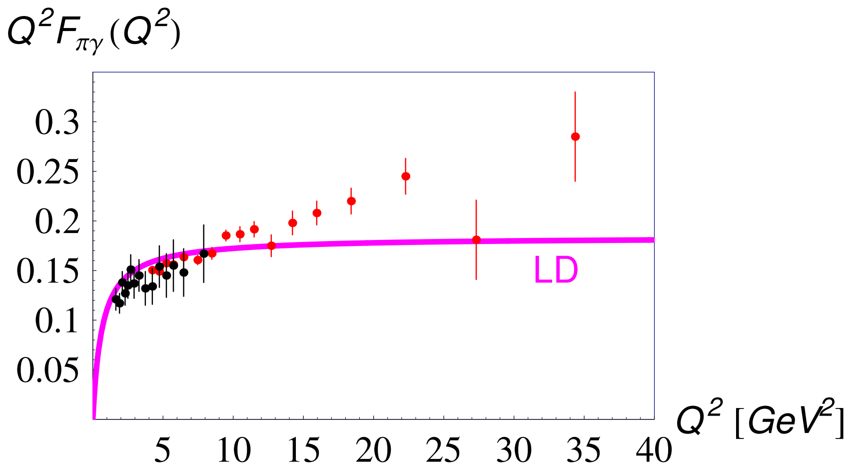


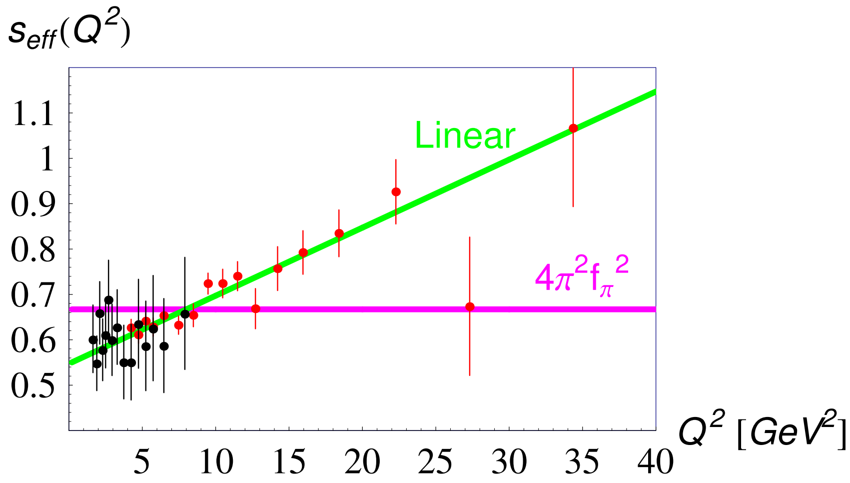


No disagreement between the LD model and the data.

$\pi^0 \to \gamma\gamma^*$ transition form factor

For the pion transition form factor one observes a clear disagreement with the BaBar data.





Left: CLEO+CELLO (black), BaBar (red)data vs LD prediction for F_π.

Right: equivalent threshold for the BaBar data. It may be well approximated by a linear rising function.

This means that - opposite to

(i) the η and η' cases and

(ii) the lessons from quantum mechanics,

the violations of LD rise with Q even in the region $Q^2 \simeq 40$ GeV^2!

Puzzle: it is hard to find a reasonable explanation why nonstrange components in η, η' and π^0 should behave so much differently?

Summary and conclusions

We investigated the pion and η, η' form factors by means of a LD model which may be formulated in any theory where hard exclusive amplitudes satisfy factorization theorems.

Our main conclusions are as follows:

• For the elastic form factor, independently of the details of the confining interaction, the predictions of the LD model are expected to exhibit maximal deviations from the exact form factor in the region $Q^2 \approx 4-8$ GeV^2. As Q increases further, the accuracy of the LD model increases rather fast. For arbitrary confining interaction, the LD model gives very accurate results for $Q^2 \geq 20-30$ GeV^2.

The accurate data on the pion form factor prompt that the LD limit for the effective threshold $s_{\rm eff}(\infty) = 4\pi^2 f_\pi^2$ may be reached already at relatively low values $Q^2 = 5-6$ GeV^2; thus, large deviations from the LD limit at $Q^2 = 20-50$ GeV^2 appear to us unlikely.

• For the $P \to \gamma\gamma^*$ form factors, the LD model should work well in the region $Q^2 \geq$ a few GeV^2.

The LD predictions work well for $\eta \to \gamma\gamma^*$ and $\eta' \to \gamma\gamma^*$ form factors.

For $\pi \to \gamma\gamma^*$ form factor the present BaBar data indicate extreme violation of local duality prompting a linearly rising effective threshold. This puzzle has so far no explanation.