# Models with large extra dimensions

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- Prologue

- Bernhard Riemann, "Über die Hypothesen, welche der Geometrie zu Grunde liegen" (1854 г.)

  (*On the Hypotheses which lie at the Bases of Geometry*, *Translated by William Kingdon Clifford*)

  The questions about the infinitely great are for the interpretation of nature useless questions. But this is not the case with the questions about the infinitely small. It is upon the exactness with which we follow phenomena into the infinitely small that our knowledge of their causal relations essentially depends…

- Now it seems that the empirical notions on which the metrical determinations of space are founded, the notion of a solid body and of a ray of light, cease to be valid for the infinitely small. We are therefore quite at liberty to suppose that the metric relations of space in the infinitely small do not conform to the hypotheses of geometry; and we ought in fact to suppose it, if we can thereby obtain a simpler explanation of phenomena…

- This leads us into the domain of another science, of physics, into which the object of this work does not allow us to go today.

- Ernst Mach, "Erkenntnis und Irrtum" (*Knowledge and Error*) (1905 г.)

- Gunnar Nordström, "Über die Möglichkeit, das elektromagnetische Feld und das Gravitationsfeld zu vereinigen" (*On the possibility of unifying the electromagnetic and the gravitational fields*) (1914 г.)

# 1 Kaluza-Klein theory

The space-time has an extra space dimension, which is macroscopically unobservable.

In the papers by Kaluza and Klein the unobservability of the extra dimension was explained by its compactness and extremely small size, - of the order of the Planck length $\mathbf{l_{Pl} = 1/M_{Pl}}$.

The original Kaluza-Klein model: gravity in five-dimensional space-time E=$M^4$ x $S^1$ with action

$$S = \frac{1}{16\pi\hat{G}} \int_E \hat{R}\sqrt{-g}\, d^5X, \quad X^N = \{x^\nu, y\}, \quad 0 \leq y < L,$$

$\hat{G}$ being the five dimensional gravitational constant, $\hat{R}$ being the five-dimensional scalar curvature and the signature of the metric being

sign $g_{MN}$ = (-,+,+,+, +), M,N = 0,1,2,3,4.

The five-dimensional metric $g_{MN}$ can be decomposed as

$$g_{MN} = \begin{pmatrix} \gamma_{\mu\nu} + \phi A_\mu A_\nu & \phi A_\mu \\ \phi A_\nu & \phi \end{pmatrix}, \quad \phi = g_{44}, \phi A_\mu = g_{\mu 4}.$$

If $g_{MN}$ does not depend on the extra dimension coordinate y, then

$$\begin{aligned} \hat{R} &= R_{(4)} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \phi^{-1} - \frac{1}{2} \phi^{-2} \partial_\mu \phi \partial^\mu \phi \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \end{aligned}$$

$R_{(4)}$ being the scalar curvature of the four-dimensional space-time $M_4$ with metric $\gamma_{\mu\nu}$.

In the papers by Kaluza and Klein the field φ was assumed to be constant, and the field $A_\mu$ was identified with the electromagnetic field.

The theory gives a relation between the five-dimensional (M) and four-dimensional ($M_{Pl}$) Planck masses:

$$M_{Pl}^2 = M^3 L.$$

Any field in space-time $E=M^4 \times S^1$ can be expanded in a Fourier series in the coordinate y.

Thus, for a scalar field

$$\phi(x,y) = L^{-\frac{1}{2}} \sum_n \phi^{(n)}(x) \exp(i\frac{2\pi n y}{L}).$$

the Lagrangian in five-dimensional space-time is

$$\mathcal{L} = -\frac{1}{2}\partial_M \phi \partial^M \phi - \frac{m^2}{2}\phi^2$$

and the equations of motion look like

$$(\partial_M \partial^M - m^2)\phi = 0.$$

The modes $\varphi^{(n)}$ satisfy the equations

$$(\partial_\mu \partial^\mu - m_n^2)\phi^{(n)} = 0, \quad m_n^2 = m^2 + \frac{4\pi^2 n^2}{L^2}.$$

Since *L* is of the order of the Planck length, the observable fields may be only the "zero modes", i.e. they do not depend on the coordinate of the extra dimension.

- For every four-dimensional field there must exist a tower of fields with the same quantum numbers and the masses of the ordre of $M_{Pl}$, which cannot be observed at the energies available nowadays.

# 2 Dimensional reduction and spontaneous compactification

Nonabelian generalization:

R. Kerner

"Generalization of the Kaluza-Klein theory for an arbitrary nonabelian gauge group"

Annales Poincare Phys. Theor. 9 (1968) 143.

Gravity in space-time E = $M^4$xG renders in the four-dimensional space-time $M^4$ a gauge field with the gauge group G.

- The spinor fields of the SM cannot be derived from the metric of the multidimensional space-time and therefore they should be included into the original multidimensional theory.

- The interpretation of gravity theory interacting with matter fields in a multidimensional space-time in terms of four-dimensional fields is called dimensional reduction.

- The dynamical explanation of the factorized structure of the multidimensional space-time arising due to the interaction of gravity with the matter fields was given the name spontaneous compactification.

To construct by these methods the SM one needed large extra dimensions:

Yu.A. Kubyshin, I.P. Volobuev, J. M. Mourao and G. Rudolph
"Dimensional reduction of gauge theories, spontaneous compactification and model building"
Lecture Notes in Physics, 349 (1990)

There remained the problem of the unobservability of such large extra dimensions.

# 3 Large extra dimensions

Localization of fields:

V.A.Rubakov and M.E. Shaposhnikov,
"Do We Live Inside A Domain Wall?"
Phys. Lett. 125 (1983) 136.

"Extra Space-Time Dimensions: Towards A Solution Of The Cosmological Constant Problem",
Phys. Lett. 125 (1983) 139.

The fields of the SM can be localized on a domain wall in a multidimensional space. If the thickness of the domain wall goes to zero, then it turns into a membrane, or just a "brane".

- ADD scenario

N. Arkani-Hamed, S.Dimopoulos and G.R. Dvali, "The hierarchy problem and new dimensions at a millimeter", Phys. Lett. B 429 (1998) 263

A single brane without tension (i.e. energy density) in a space-time with an arbitrary number of compact extra dimensions.

The scenario provides a solution to the hierarchy problem: it gives a strong gravity in the multidimensional space-time and a weak gravity on the brane

$$M_{Pl}^2 = M^{(2+n)} V_n$$

The approximation of the zero brane tension turns out to be rather too rough, and the proper gravitational field of the brane cannot be taken into account perturbatively.

## Football-Shaped Extra Dimensions

Sean M. Carroll, Monica M. Guica,
"Sidestepping the cosmological constant
with football shaped extra dimensions".
e-Print: hep-th/0302067

The solution is based on a factorizable geometry with a metric

$$ds^2 = \eta_{\mu\nu}\, dx^\mu dx^\nu + \gamma_{ij}(y)\, dy^i dy^j$$

and a magnetic flux threading the space of extra dimensions.

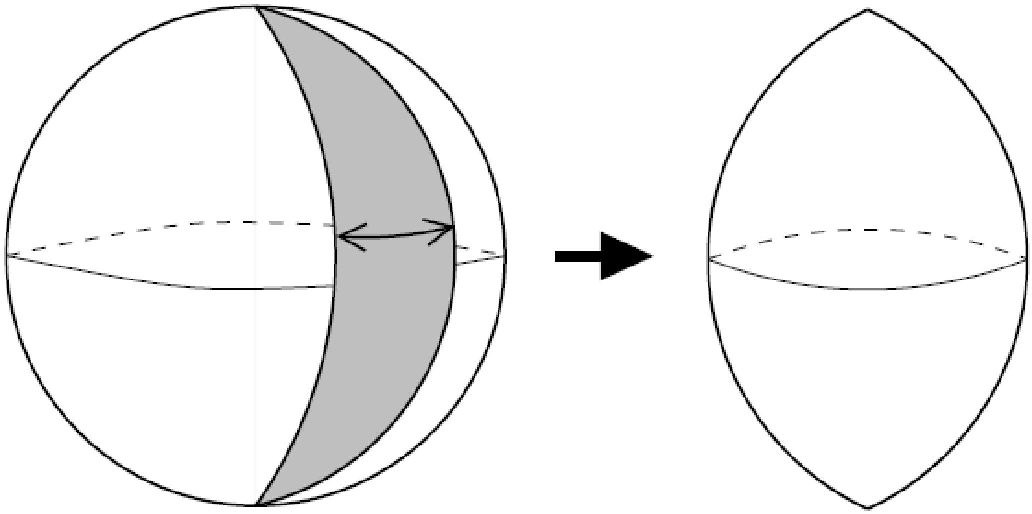


Figure 1: Removing a wedge from a sphere and identifying opposite sides to obtain a football geometry. Two equal-tension branes with conical deficit angles are located at either pole; outside the branes there is constant spherical curvature.

# 4 The Randall-Sundrum model

L. Randall and R. Sundrum,
“A large mass hierarchy from
a small extra dimension”,
Phys. Rev. Lett. 83 (1999) 3370

Two branes with tension at the fixed points of the orbifold $S^1/Z_2$:

$$\begin{aligned} S &= \int d^4x \int_{-L}^{L} dy\, \left(2M^3 R - \Lambda\right) \sqrt{-g} - \\ &- \lambda_1 \int_{y=0} \sqrt{-\tilde{g}} d^4x - \lambda_2 \int_{y=L} \sqrt{-\tilde{g}} d^4x . \end{aligned}$$

The solution for the background metric:

$$ds^2 = \gamma_{MN} dx^M dx^N = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + (dy)^2, \; \sigma(y) = k|y| + c.$$

The parameters k, Λ и $\lambda_{1,2}$ satisfy the fine tuning conditions:

$$\Lambda = -24M^3k^2, \quad \lambda_1 = -\lambda_2 = 24M^3k.$$

The linearized gravity is obtained by the substitution

$$g_{MN} = \gamma_{MN} + \frac{1}{\sqrt{2M^3}} h_{MN}$$

It is a gauge theory with the gauge transformations

$$
\begin{aligned}
h^{(\prime)}_{\mu\nu}(x,y) &= h_{\mu\nu}(x,y) - (\partial_\mu \xi_\nu + \partial_\nu \xi_\mu + 2\gamma_{\mu\nu}\partial_4 \sigma \xi_4) \\
h^{(\prime)}_{\mu 4}(x,y) &= h_{\mu 4}(x,y) - (\partial_\mu \xi_4 + \partial_4 \xi_\mu - 2\partial_4 \sigma \xi_\mu) \\
h^{(\prime)}_{44}(x,y) &= h_{44}(x,y) - 2\partial_4 \xi_4 .
\end{aligned}
$$

The functions $\xi^M(x,y)$ satisfy the orbifold symmetry conditions

$$
\xi^\mu(x,-y) = \xi^\mu(x,y), \quad \xi^4(x,-y) = -\xi^4(x,y).
$$

The field $h_{MN}$ can be transformed to the gauge

$$
h_{\mu 4} = 0, \; h_{44} = \phi(x).
$$

The distance between the branes along the geodesic x = const

$$l = \int_0^L \sqrt{ds^2} \simeq \int_0^L \left(1 + \frac{1}{2\sqrt{2M^3}} h_{44}\right) dy = L\left(1 + \frac{1}{2\sqrt{2M^3}}\phi(x)\right).$$

The equtions for the fields $h_{\mu\nu}(x,y)$ и $\Phi(x)$ can be decoupled by the substitution

$$\begin{aligned} h_{\mu\nu}(x,y) &= b_{\mu\nu}(x,y) + \gamma_{\mu\nu}(y)(\sigma(y) - c)\phi(x) \\ &+ \frac{1}{2k^2}\left[\sigma(y) - c + \frac{1}{2}\right]\partial_\mu\partial_\nu\phi(x) + \frac{c}{4k^2}e^{-2\sigma(y)}\partial_\mu\partial_\nu\phi(x), \end{aligned}$$

The equation for the transverse-traceless field $b_{\mu\nu}(x,y)$ is

$$\frac{1}{2}\left(e^{-2\sigma(y)}\Box b_{\mu\nu}+\frac{\partial^2 b_{\mu\nu}}{\partial y^2}\right)-b_{\mu\nu}[2(\sigma')^2-\sigma'']=0.$$

The massless graviton is described by the solution

$$b_{\mu\nu}(x,y)=e^{-2\sigma(y)}\bar{h}_{\mu\nu}(x).$$

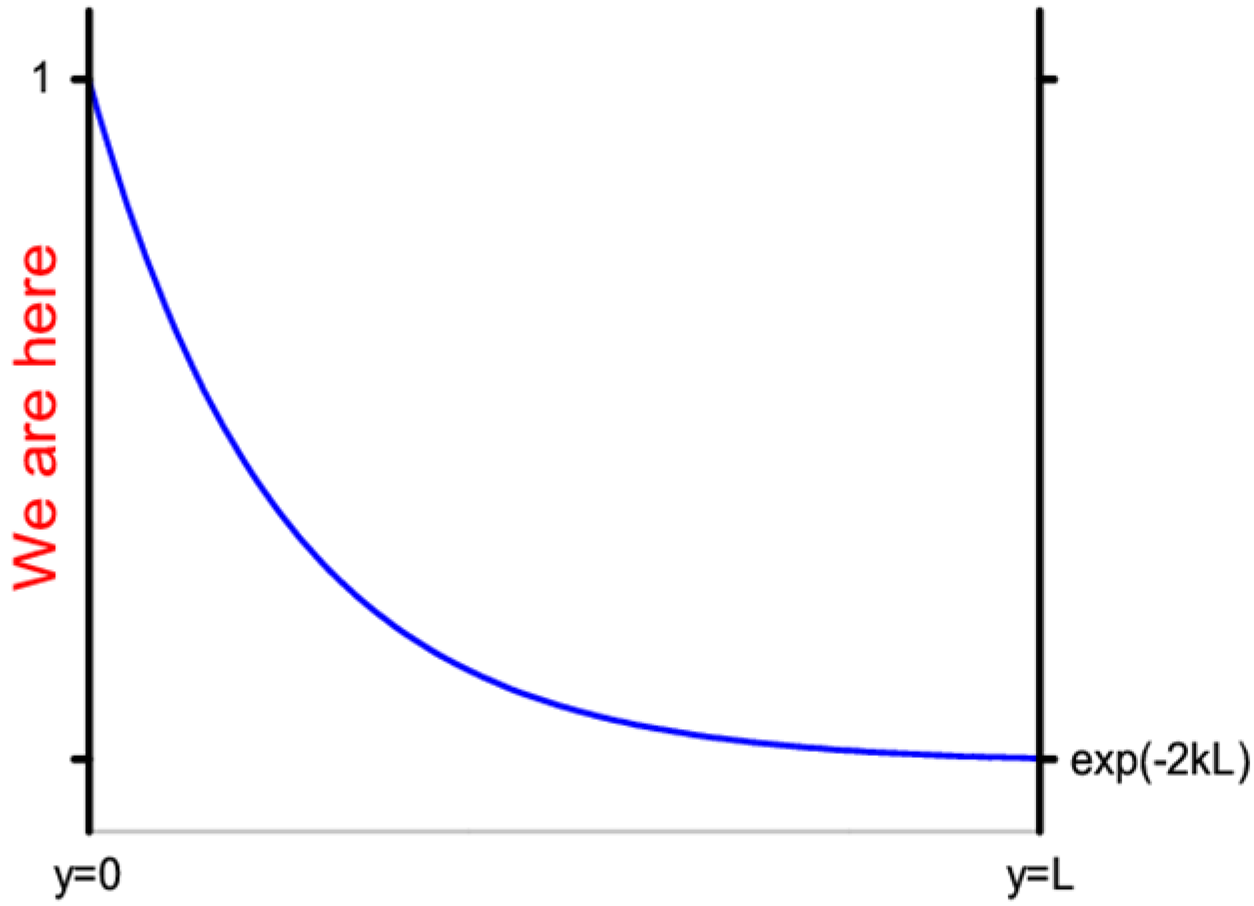
The metric in the zero mode approximation looks like

$$ds^2=e^{-2\sigma(y)}(\eta_{\mu\nu}+\bar{h}_{\mu\nu}(x))dx^\mu dx^\nu+dy^2=e^{-2\sigma(y)}\bar{g}_{\mu\nu}(x)dx^\mu dx^\nu+dy^2.$$

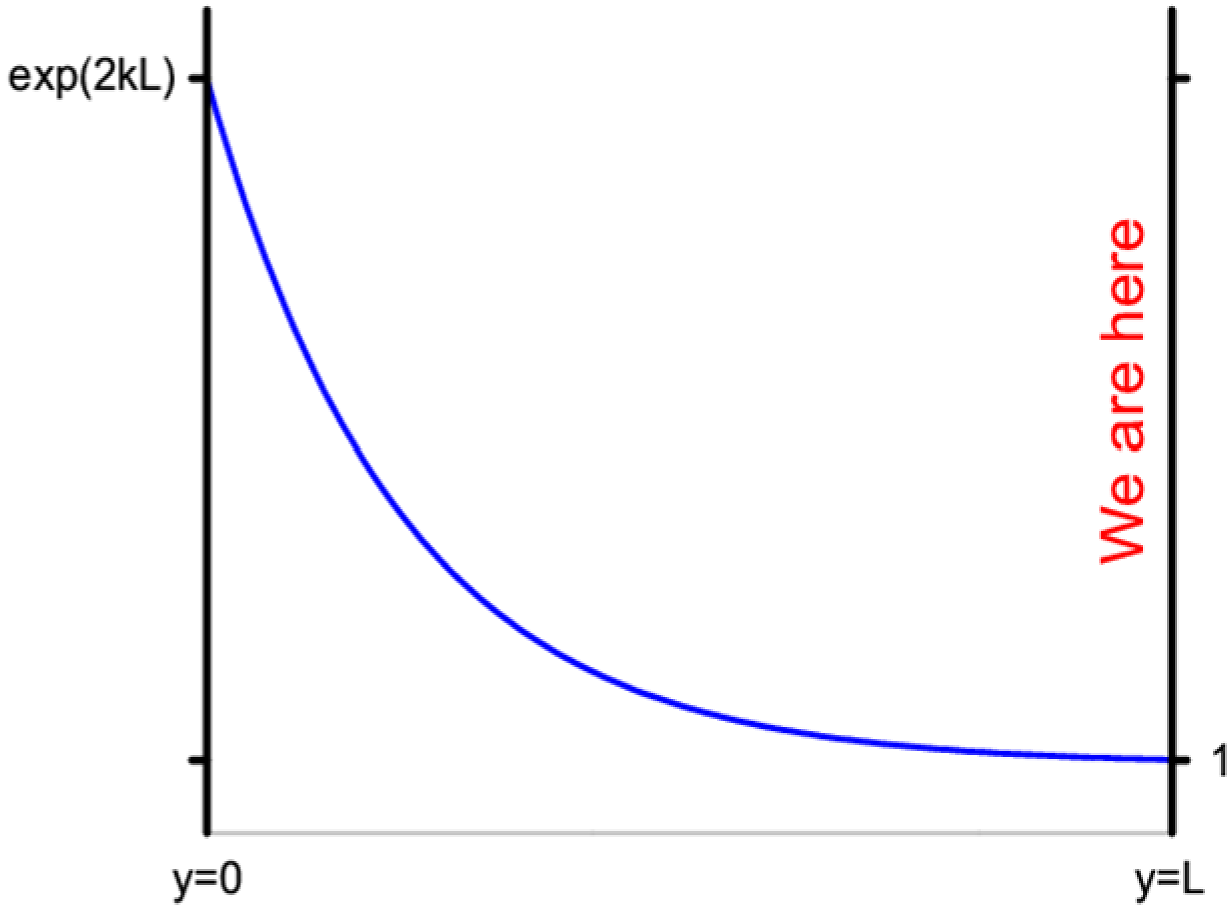
Substituting this metric into the action and integration over the coordinate of the extra dimension one gets an effective action

$$S_{eff} = 2M^3 e^{-2c} \frac{1 - e^{-2kL}}{k} \int d^4x R_4(\bar{g})\sqrt{-\bar{g}}.$$

Galilean coordinates: $g_{\mu\nu}$ = diag(-1,1,1,1).




Coordinates {x} are Galilean for c=0, $M_{Pl}^2 = \frac{M^3}{k}(1 - e^{-2kL}).$




Coordinates {x} are Galilean for c = - kL, $M_{Pl}^2 = \frac{M^3}{k}(e^{2kL} - 1).$

The hiearrchy problem is solved, if $M \sim k \sim 1$ TeV и $kL \sim 35$.

There appears a tower of tensor fields on the brane with the lowest mass of the order of M and the coupling to the SM fields of the order of 1/M.

The branes in the Randall-Sundrum model can oscillate with respect to each other, which manifests itself as a massless four-dimensional scalar field, -- the radion field.

The coupling of the radion to matter on the brane is too strong and contradicts the experimental restrictions even at the level of classical gravity.

- Newton's law and the deflection fo light: the observer and the mass are on brane 2

$$V = -G_{(4)}\left(1+\frac{e^{2kR}}{3}\right)\frac{m}{r},$$

$$\Delta\varphi = \frac{4mG_{(4)}}{r_0}\left(\frac{1}{1+\frac{e^{2kR}}{3}}\right).$$

The Randall-Sundrum model must be stabilized!

# 5 Stabilized Randall-Sundrum model

- Stabilization mechanisms:

W. D. Goldberger and M.B. Wise,
“Modulus stabilization with bulk fields”,
Phys. Rev. Lett. **83** (1999) 4922

O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch,
“Modeling the fifth dimension with scalars and gravity”,
Phys. Rev. D **62** (2000) 046008

The second model is more consistent. We consider such values of the model parameters that the background metric of the stabilized model is close to that of the unstabilized model.

The physical degrees of freedom of the model in the linear approximation were isolated in the paper

E.E. Boos, Y.S. Mikhailov,
M.N. Smolyakov and I.P. Volobuev,
"Physical degrees of freedom
in stabilized brane world models",
Mod. Phys. Lett. A **21** (2006) 1431

## They are:

- tensor fields $b_{\mu\nu}^{n}(x)$, n=0,1, … with masses $m_n$ (m_0 = 0) and wave functions in the space of extra dimension $\psi_n(y)$,
- scalar fields $\varphi_n(x)$, n=1,2, … with masses $\mu_n$ and wave functions in the space of extra dimension $g_n(y)$.

The interaction with the SM fields is described by the Lagrangian

$$L_{int} = -\frac{1}{\sqrt{8M^3}}\left(\psi_0(L) b^0_{\mu\nu}(x) T^{\mu\nu} + \sum_{n=1}^{\infty} \psi_n(L) b^n_{\mu\nu}(x) T^{\mu\nu} + \frac{1}{2}\sum_{n=1}^{\infty} g_n(L)\varphi_n(x) T^{\mu}_{\mu}\right),$$

$T_{\mu\nu}$ being the energy-momentum tensor of the SM.

At low energies this leads to contact interactions of SM fields

$$L_{eff} = \frac{1.82}{\Lambda_\pi^2 m_1^2} T^{\mu\nu} \tilde{\Delta}_{\mu\nu,\rho\sigma} T^{\rho\sigma},$$

$$\tilde{\Delta}_{\mu\nu,\rho\sigma} = \frac{1}{2}\eta_{\mu\rho}\eta_{\nu\sigma} + \frac{1}{2}\eta_{\mu\sigma}\eta_{\nu\rho} - \left(\frac{1}{3} - \frac{\delta}{2}\right)\eta_{\mu\nu}\eta_{\rho\sigma},$$

$m_1$ and $\Lambda_\pi$ being the mass and the coupling constant of the first tensor mode and the constant $\delta$ describing the contribution of the scalar modes.

For M ≈ 2 TeV, k ≈ 1 TeV, k L = 35 and the mass of the first scalar mode of the order of 2 TeV these parameters turn out to be

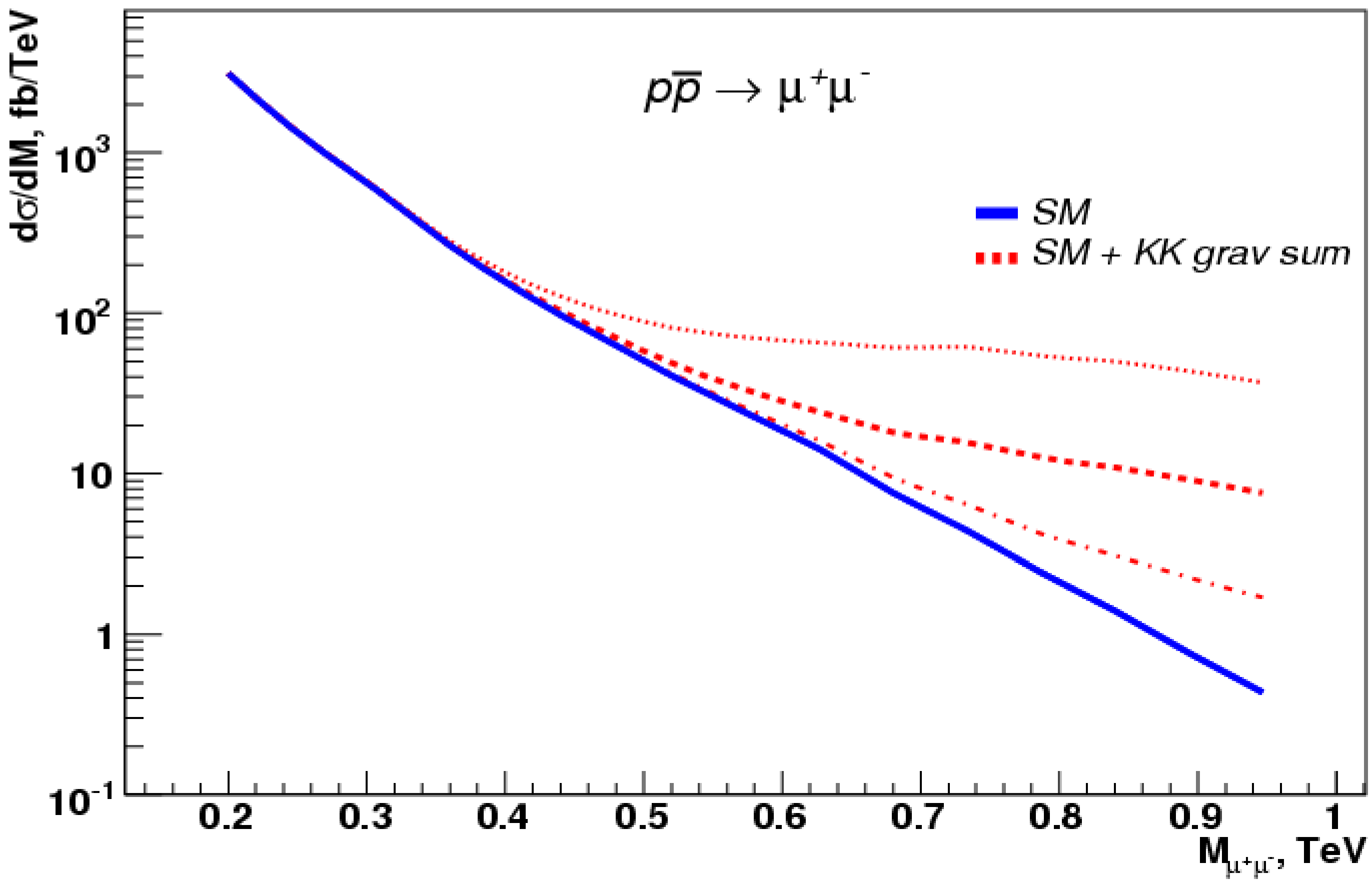
$$\Lambda_\pi \simeq 8\,TeV, \quad m_1 \simeq 3.83\,TeV.$$

# 6 Processes with Kaluza-Klein gravitons

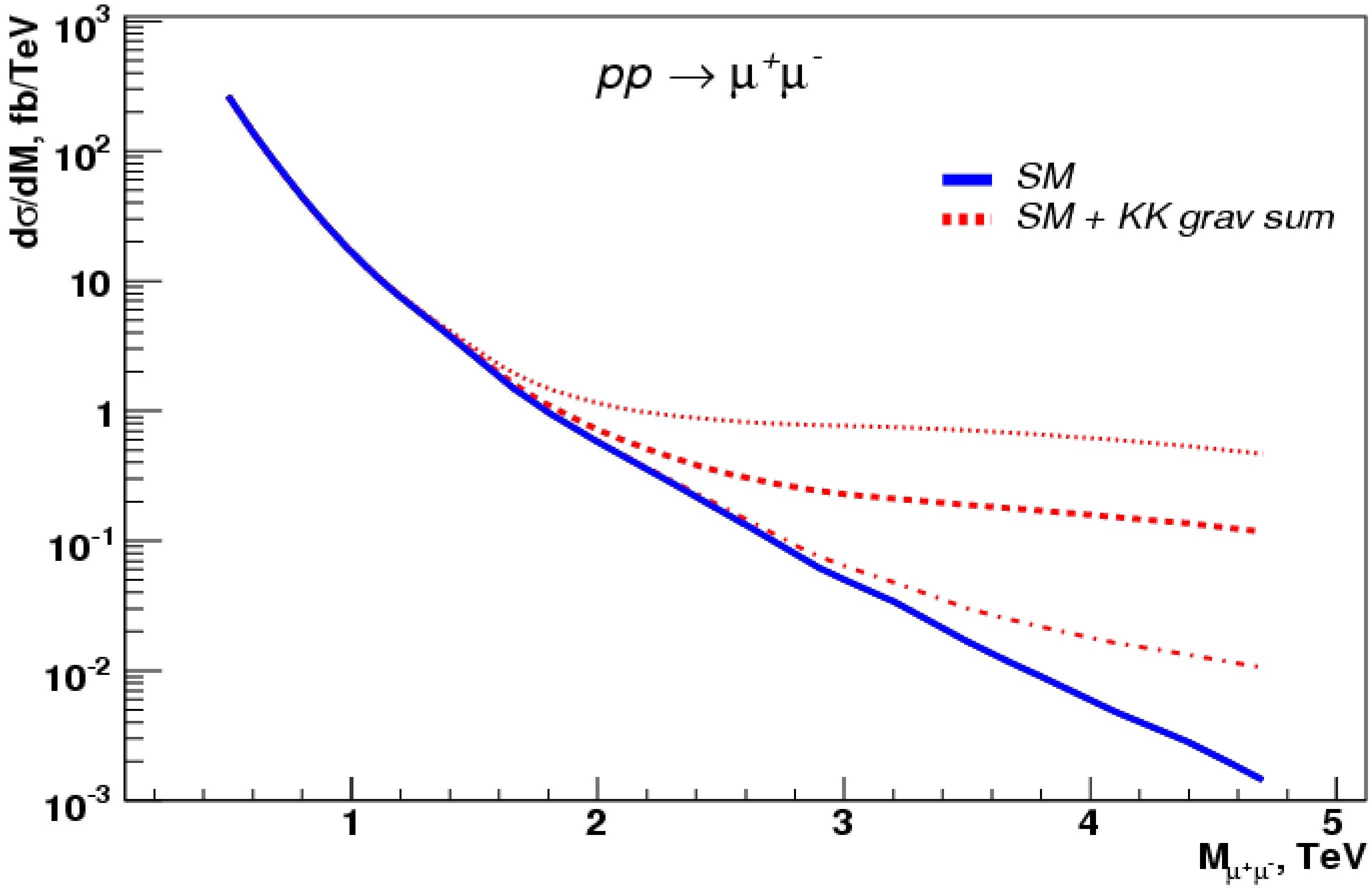
In the first approximation the effective interaction Lagrangian includes a sum of various 4-particle effective operators (not only 4-fermion, but also 2-fermion-2boson and 4-boson), which are invariant with respect to the SM gauge group and lead to a well defined phenomenology.

Various processes due to this Lagrangian were studies with the help of the CompHEP package in the paper

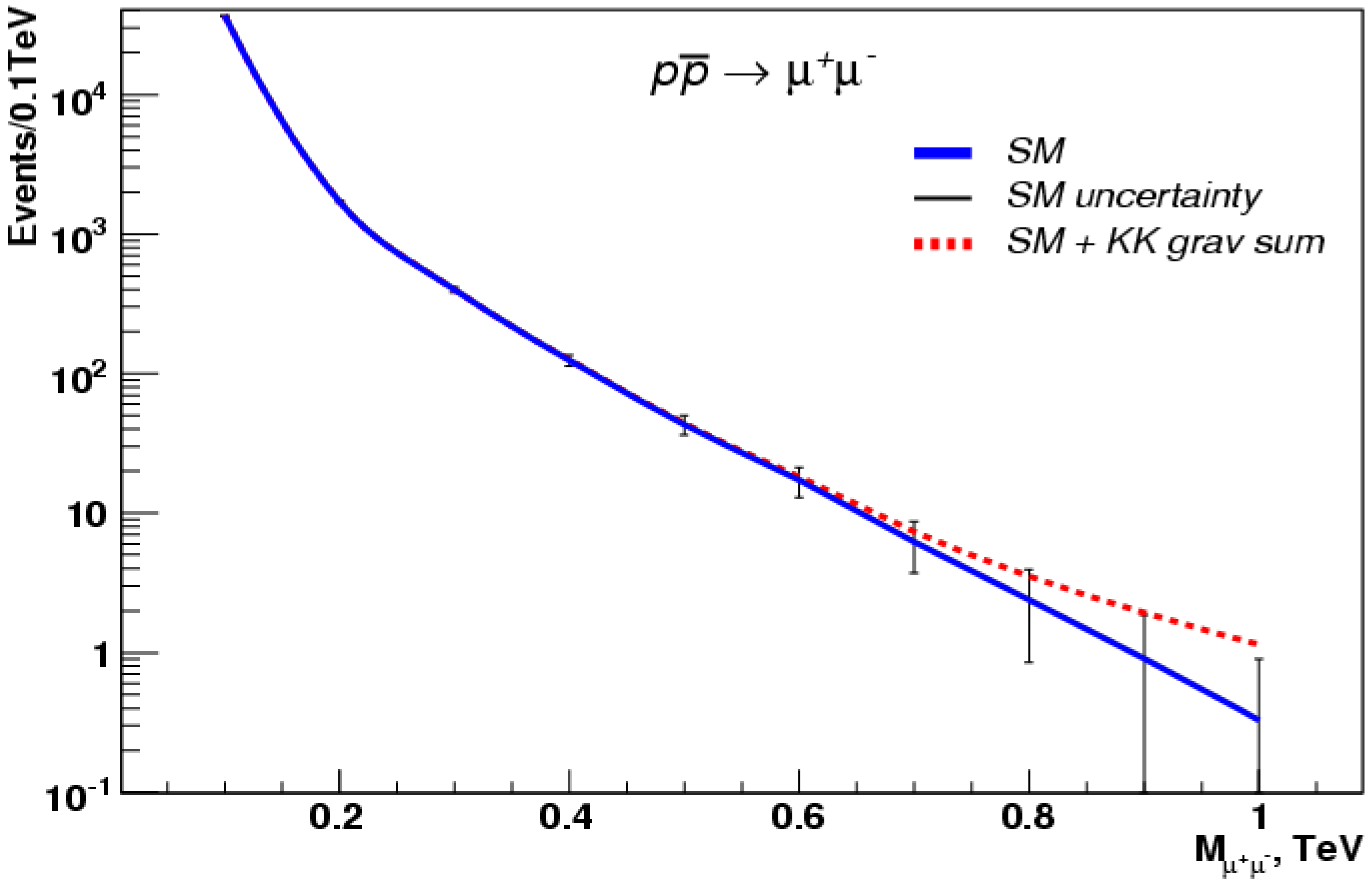
E.E. Boos, V.E. Bunichev, M.N. Smolyakov and I.P. Volobuev, "Testing extra dimensions below the production threshold of Kaluza-Klein excitations" Phys.Rev.D79:104013,2009 arXiv:0710.3100v4 [hep-ph].



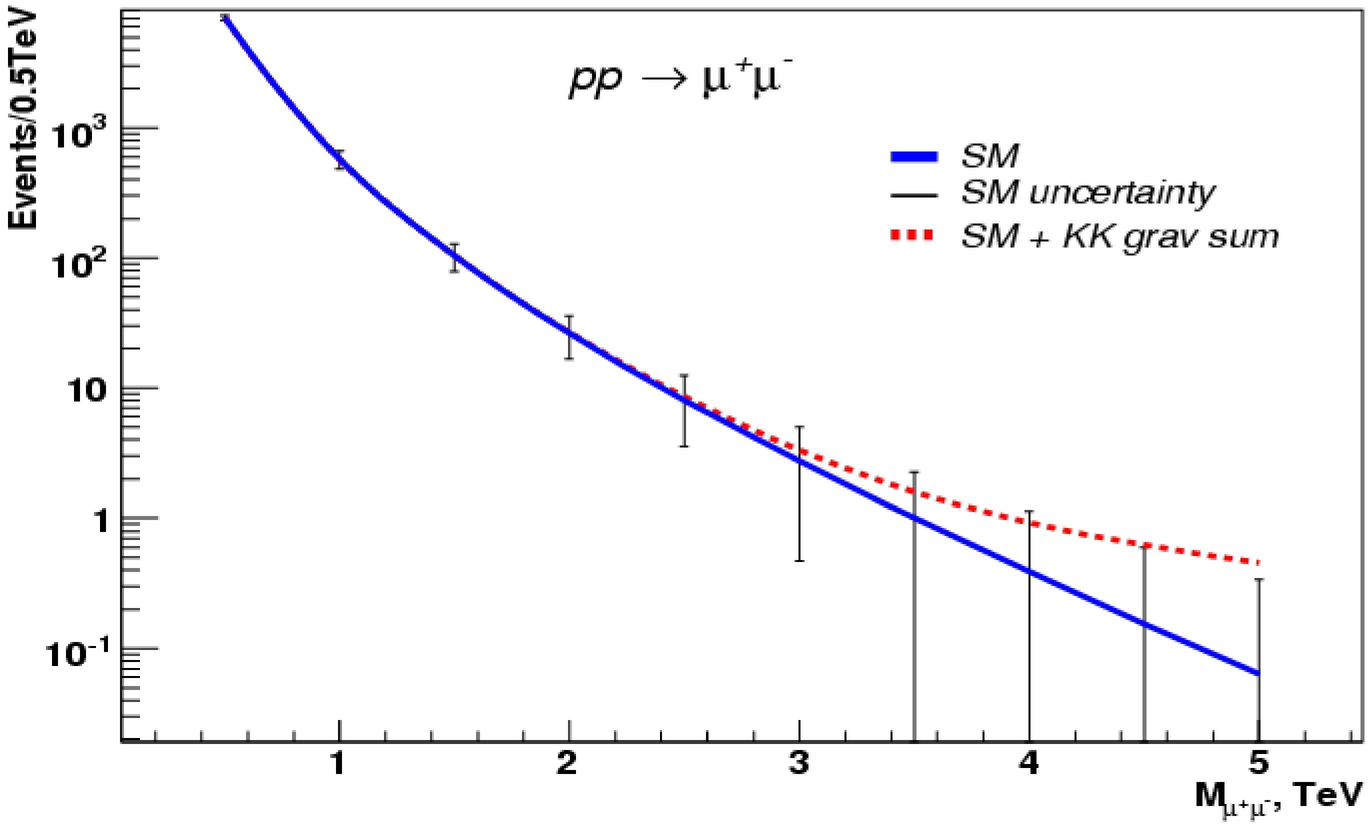

Dilepton invariant mass distribution for parameter $\frac{0.91}{\Lambda_\pi^2\, m_1^2} \times TeV^4$=0.66 (dash-dotted line), 1.82 (dashed line), 4 (dotted line) for the Tevatron




Dilepton invariant mass distribution for parameter $\frac{0.91}{\Lambda_\pi^2 m_1^2} \times TeV^4$=0.0014 (dash-dotted line), 0.0046 (dashed line), 0.01 (dotted line) for the LHC




Dilepton invariant mass distribution for $95\%$ CL parameter $\frac{0.91}{\Lambda_\pi^2 m_1^2} \times TeV^4$=0.66 for the Tevatron ($L = 10 fb^{-1}$)




Dilepton invariant mass distribution for $95\%$ CL parameter $\frac{0.91}{\Lambda_\pi^2 m_1^2} \times TeV^4$=0.0014 for the LHC ($L = 100 fb^{-1}$)

- The restrictions on the coupling constant $0.91/\Lambda_\pi^2 m_1^2$, for which the extra dimension cannot be observed at the Tevatron and the LHC:

$$Tevatron: \ \frac{0.91}{\Lambda_\pi^2 m_1^2} \times TeV^4 < 0.66,$$

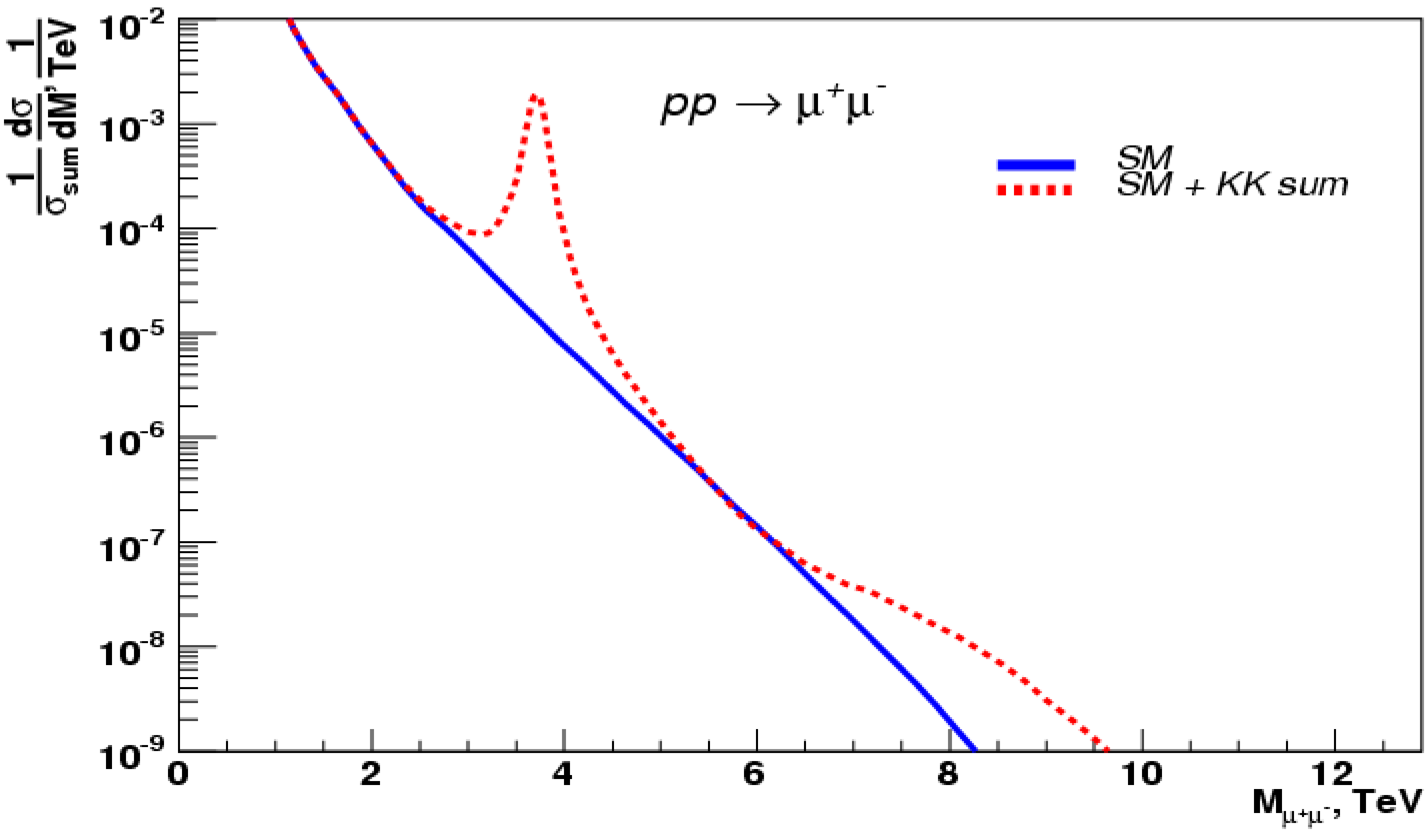
$$LHC: \ \frac{0.91}{\Lambda_\pi^2 m_1^2} \times TeV^4 < 0.0014.$$

- The lowest value of the parameter $\Lambda_\pi$ , for which the extra dimension does not manifest itself, can be found form the demand that the resonance width is lesser than its mass, i.e. $\Gamma_1 = m_1/\xi$ , where $\xi > 1$:

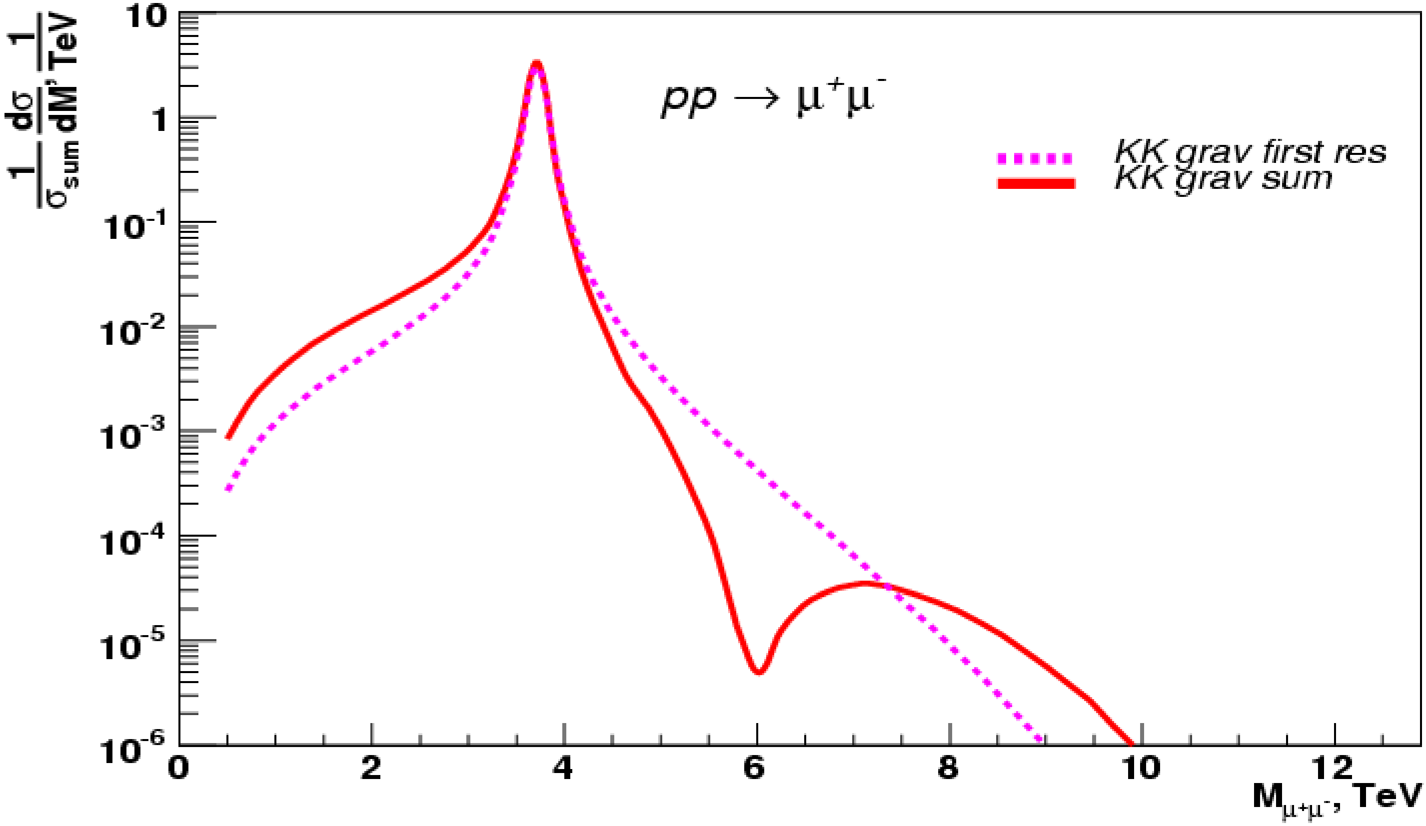
$$Tevatron: \ \Lambda_\pi > 0.61 \cdot \xi^{1/4}\, TeV,$$

$$LHC: \ \Lambda_\pi > 2.82 \cdot \xi^{1/4}\, TeV.$$

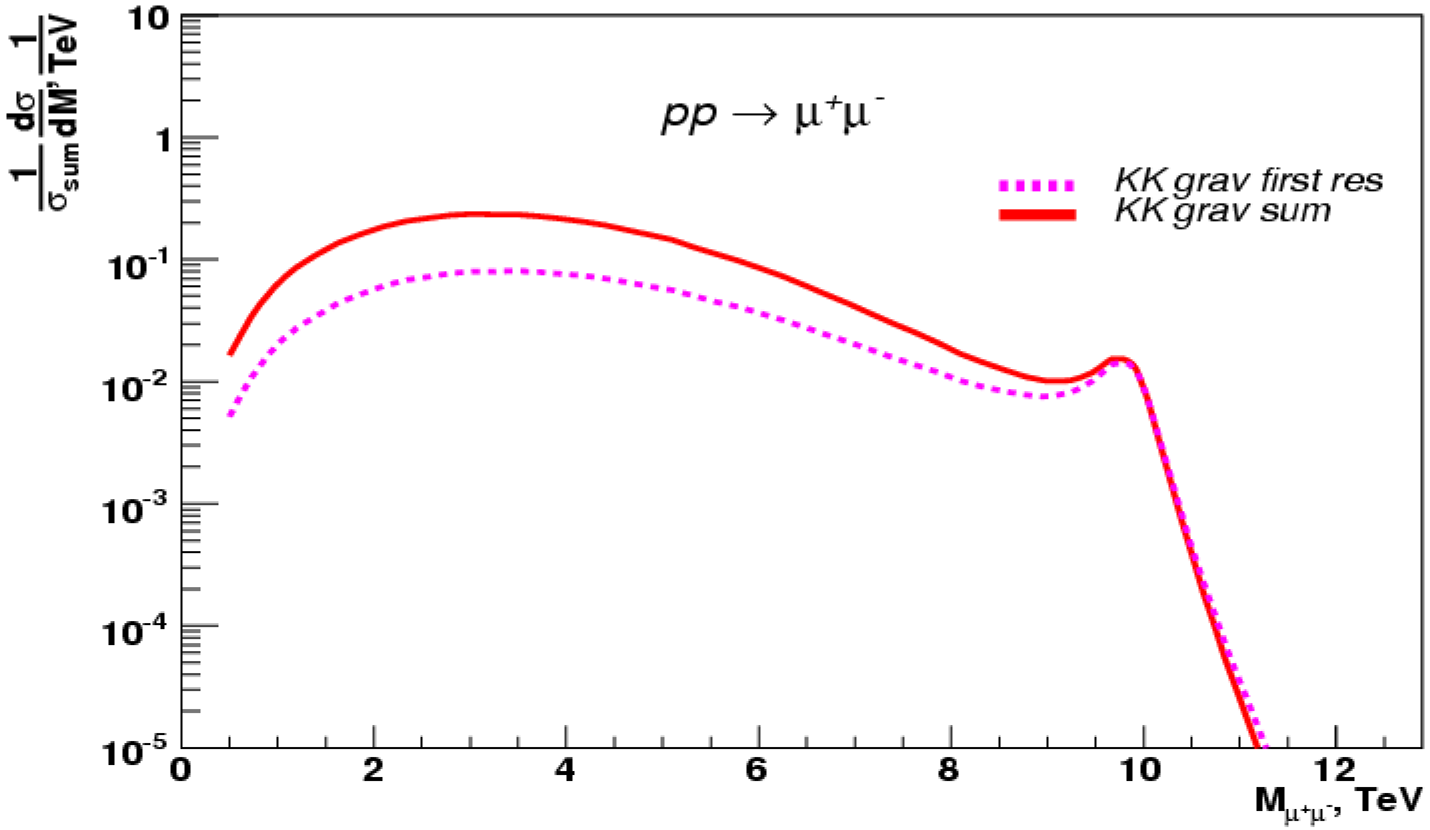
- In a similar way one can consider the situation, where the mass of the first mode lies in the accessible energy range. In this case the contribution of the first mode can be taken into account explicitly and the contribution of all the other modes, starting from the second one, can be again described by the effective contact interaction.




Dilepton invariant mass distribution from the SM (solid line) and from the SM plus sum of KK modes including the first KK resonance with $M_{res} = 3.83\ TeV$, $\Gamma_{res} = 0.08\ TeV$, $\Lambda_\pi = 8\ TeV$ (dashed line) for the LHC

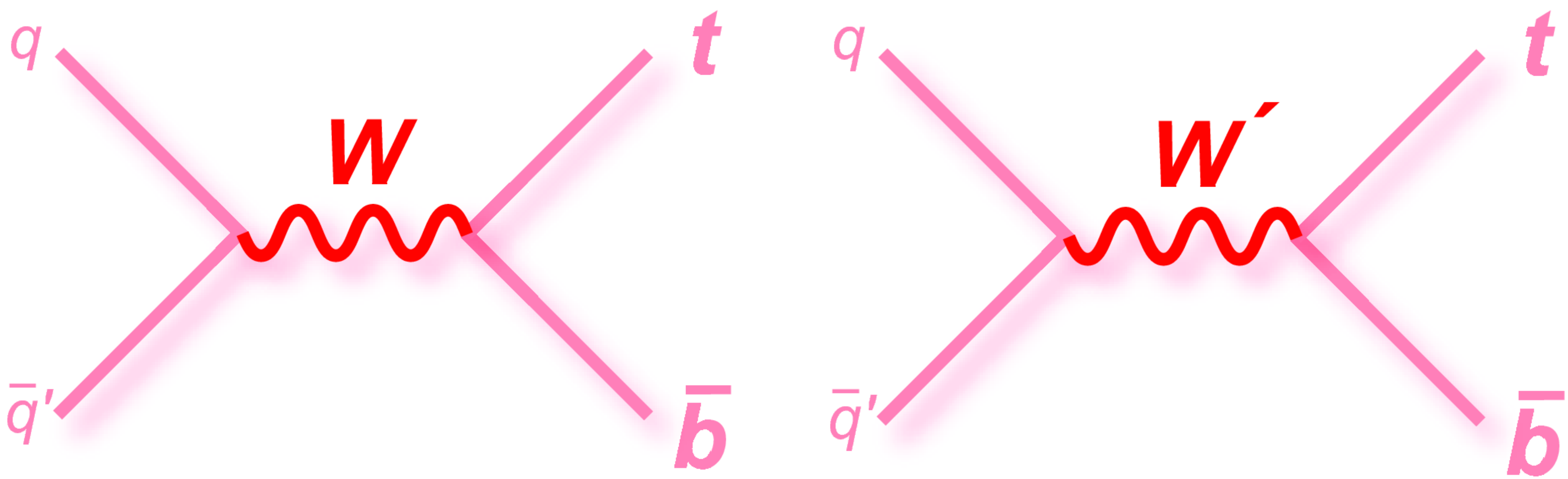



The normalized dilepton invariant mass distribution from the first KK resonance plus the sum of KK tower states starting from the second mode (solid line) and from the first KK resonance only (dashed line) for $M_{res} = 3.83\ TeV$, $\Gamma_{res} = 0.08\ TeV$, $\Lambda_\pi = 8\ TeV$ for the LHC

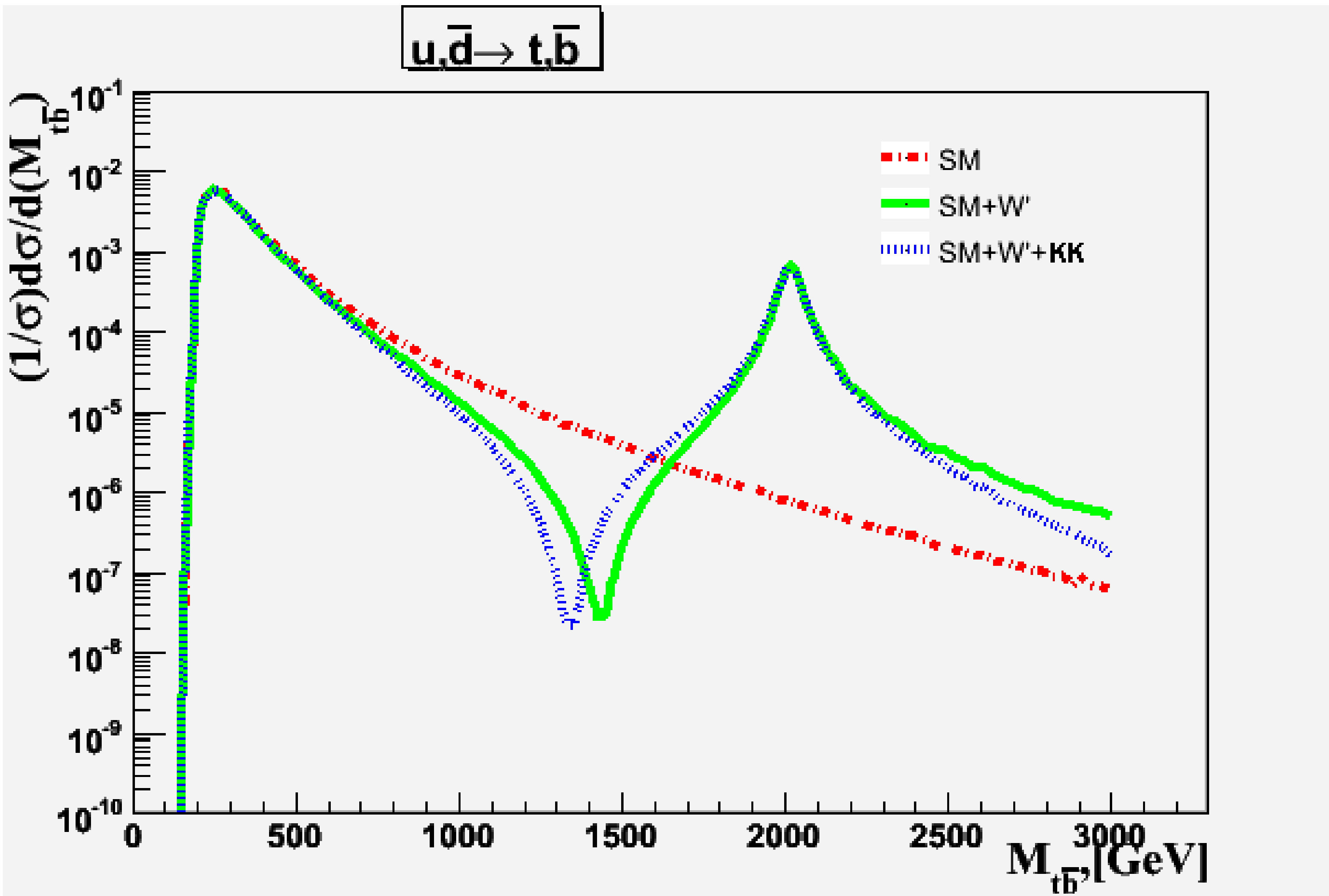



The normalized dilepton invariant mass distribution from the sum of KK tower states starting from the first KK mode (solid line) and from the first KK mode only (dashed line) for $M_{res} = 10\ TeV$, $\Gamma_{res} = 0.5\ TeV$, $\Lambda_\pi = 14\ TeV$ for the LHC

- Similar interference effects take place, when one considers KK towers of the SM fields. For example, the KK excitations of W contribute to the process of single top production:



$$M_{W'} = 2\ \text{TeV}, \qquad M_{W'\_sum} = 2.8\ \text{TeV}, \qquad \Gamma_{W'} = 65.7\ \text{GeV}$$



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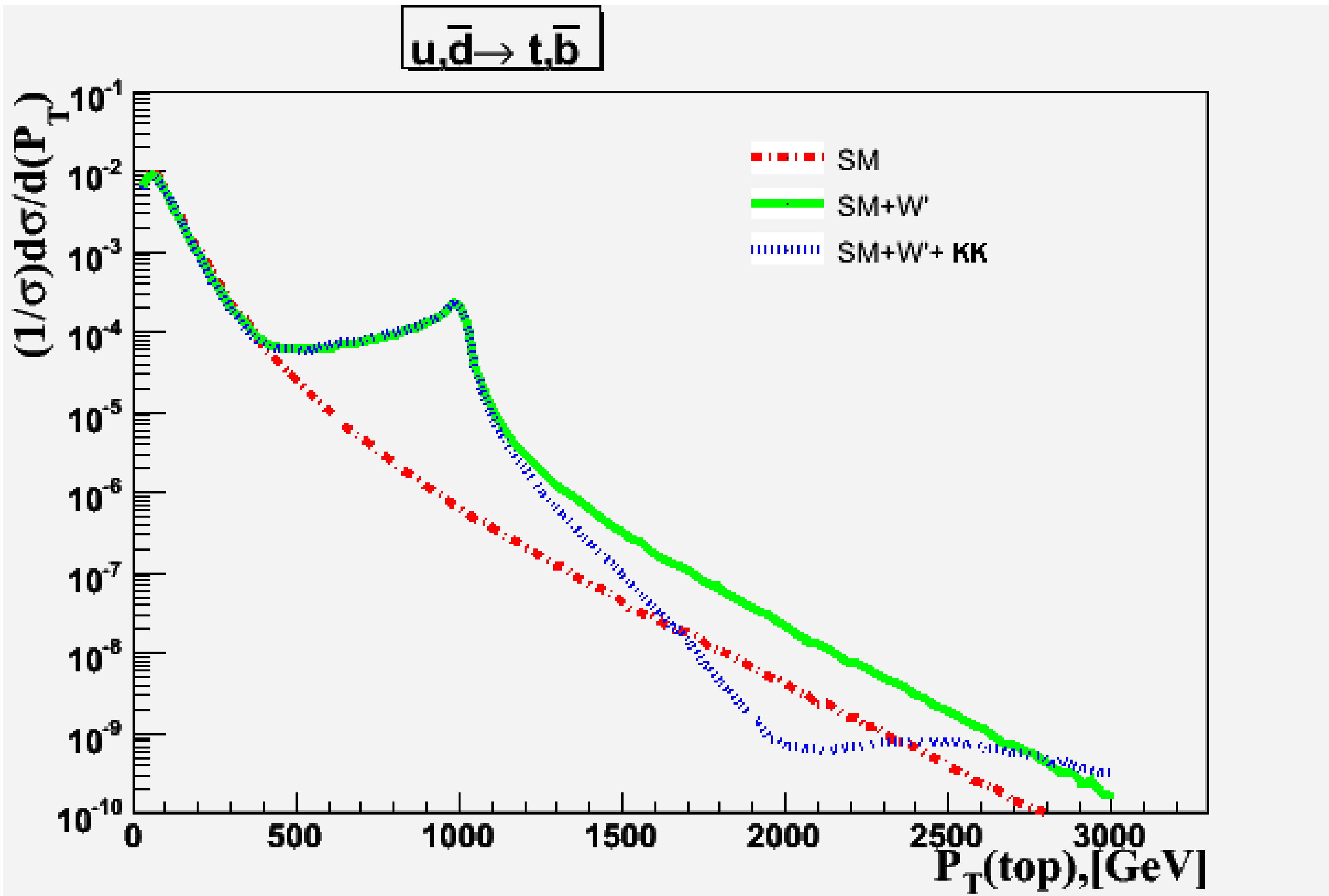

# 7 Conclusion

- The stabilized Randall-Sundrum model is phenomeno-logically acceptable. If the values of the fundamental parameters lies in the TeV energy range, then the effects due to the massive modes can be observed in collider experiments.
- The effective interaction Lagrangian looks like

$$L_{eff} = \frac{1.82}{\Lambda_\pi^2 m_1^2} T^{\mu\nu} \tilde{\Delta}_{\mu\nu,\rho\sigma} T^{\rho\sigma},$$

$$\tilde{\Delta}_{\mu\nu,\rho\sigma} = \frac{1}{2}\eta_{\mu\rho}\eta_{\nu\sigma} + \frac{1}{2}\eta_{\mu\sigma}\eta_{\nu\rho} - \left(\frac{1}{3} - \frac{\delta}{2}\right)\eta_{\mu\nu}\eta_{\rho\sigma},$$

$T_{\mu\nu}$ being the energy-momentum tensor of the SM, $m_1$ and $\Lambda_\pi$ being the mass and the coupling constant of the first tensor mode and δ describing the contribution of the scalar modes.

- The Tevatron data give the estimate

$$\Lambda_\pi > 0.61 \cdot \xi^{1/4}\, TeV.$$

- Similar effective Lagrangians are induced by excitations of the SM fields.

- The effective contact interaction induced by the infinite towers of the massive gravitons or of the KK excitations of the SM particles should be taken into account also in the case, where the centre of mass energy is above the production threshold of the first mode.