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Brane Black Holes

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References

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- T. Tanaka, *Prog.Theor.Phys.Suppl.* 148, 307 (2003);
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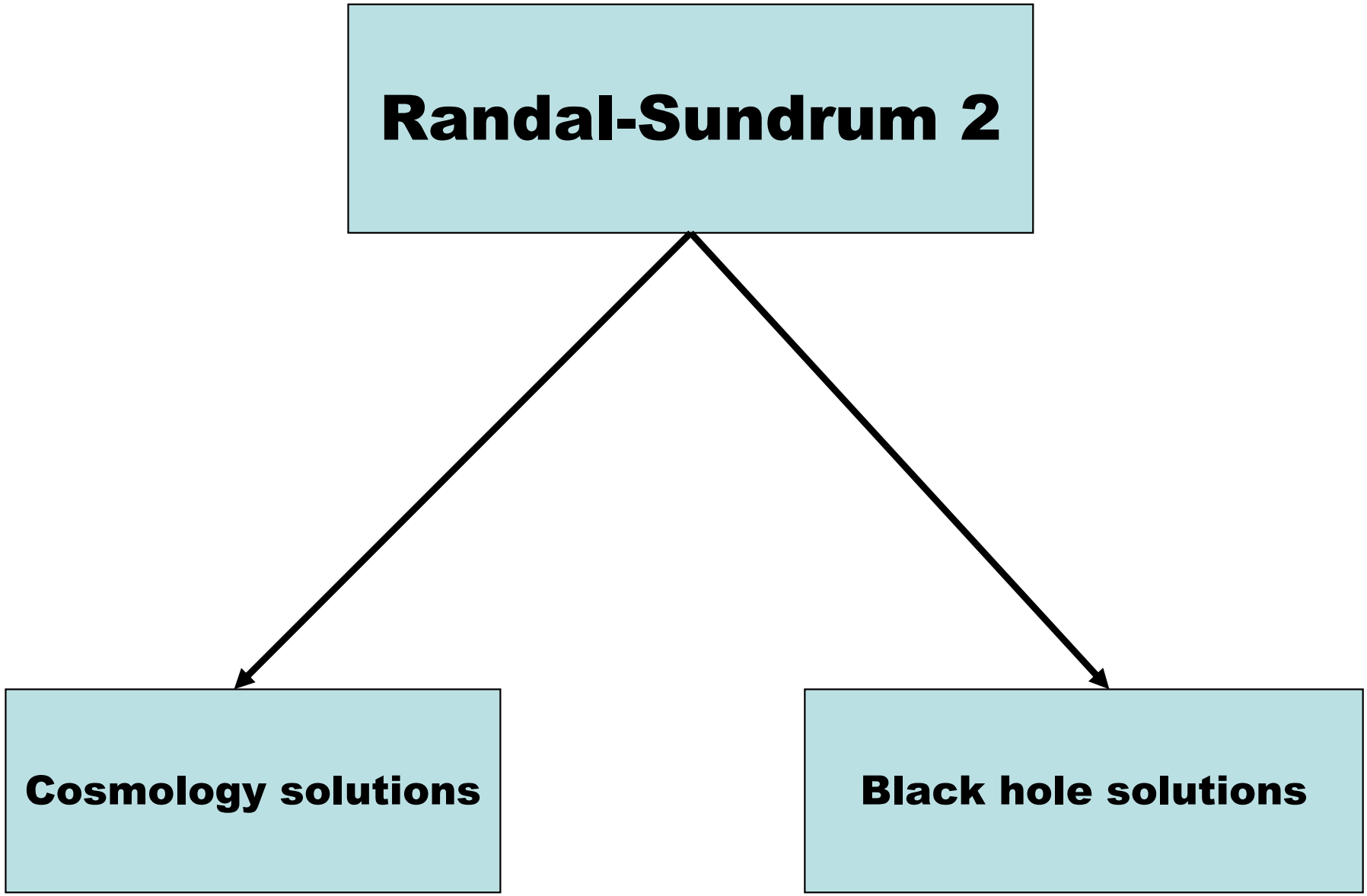

Randal-Sundrum 2
Cosmology solutions
Black hole solutions

5D bulk equations

$$\overline{G}_{AB} = \overline{\kappa}^{2}\left[-\overline{\overline{\Lambda}}\,\overline{g}_{AB} + \delta(\chi)\left(-\lambda\delta^{\mu}_{A}\delta^{\eta}_{B}g_{\mu\eta} + \delta^{\mu}_{A}\delta^{\eta}_{B}T_{\mu\eta}\right)\right]$$

4D brane equations
with extra dimensions contribution

$$G_{\mu\eta} = -\Lambda g_{\mu\eta} + \kappa^{2}T_{\mu\eta} + \overline{\kappa}^{4}S_{\mu\eta} - \Sigma_{\mu\eta}$$

Black hole solution on the brane

$$ds^2 = \Delta(r)dt^2 - \frac{dr^2}{\Delta(r)} - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

where $\Delta(r) = 1 + \frac{\alpha}{r} + \frac{\beta}{r^2}$

α, β – constants,

$$\alpha = -2M / M_{pl}^2$$

$$\beta = q / \overline{M}_{pl}^2$$

q – «tidal charge»

BH solution with $\beta < 0$

contains only one horizon

$$r_h = \frac{M}{M_{pl}^2}\left[1+\sqrt{1-q\frac{M_{pl}^4}{M^2\overline{M}_{pl}^2}}\right]$$

Limitation on «tidal charge» value

from *N.Dadhich, R.Maartens, P.Papadopoulos,V.Rezania (2000)*

- **Gravitational potential:** $\Phi = -\frac{M}{M_{pl}^{2} r} + \frac{q}{2\overline{M}_{pl}^{2} r^{2}}$

- **Requirement: correction term must be much less than Schwarzschild one at Sun mass range, therefore**

$$|q| \ll 2\left(\frac{\widetilde{M}_{pl}}{M_{pl}}\right)^{2} M_{\odot} R_{\odot}$$

where $M_{pl} = \sqrt{\frac{3}{4\pi}}\left(\frac{\overline{M}_{pl}^{2}}{\sqrt{\lambda}}\right)\overline{M}_{pl}$

Time-like geodesics

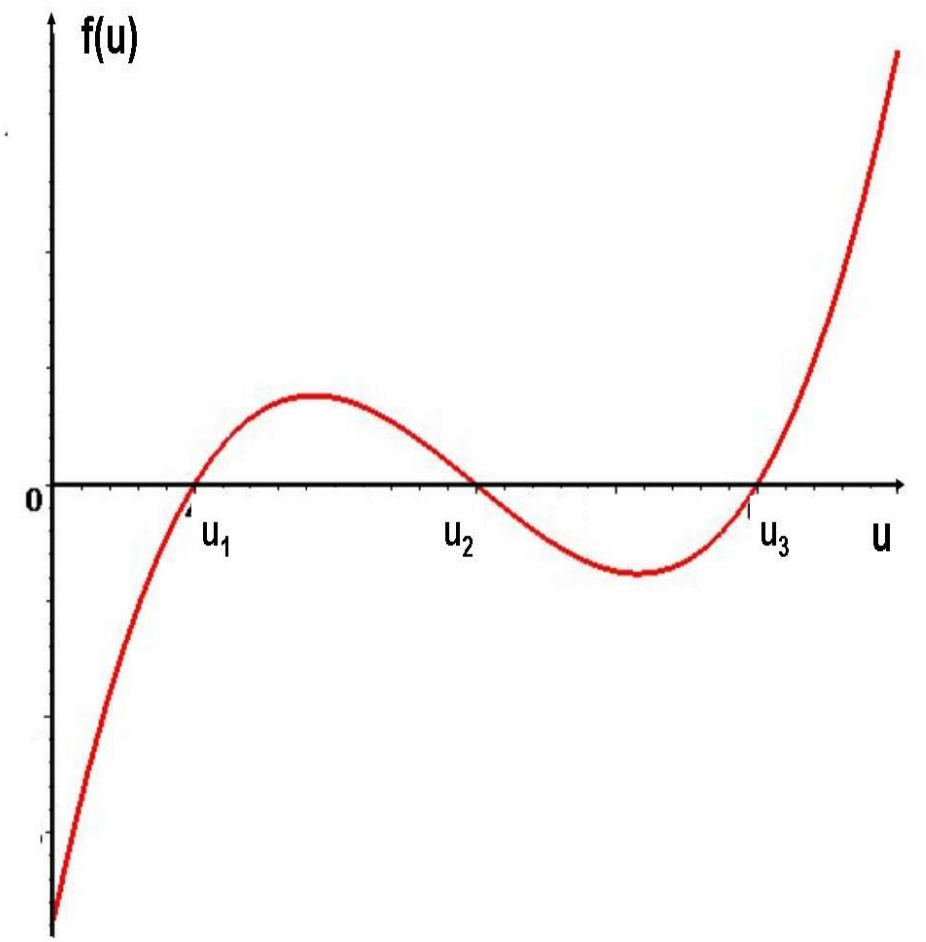
Could be established from

$$\left(\frac{du}{d\phi}\right)^2 = \frac{E^2-1}{L^2} - \frac{\alpha u + \beta u^2}{L^2} + \left(u^2 + \alpha u^3 + \beta u^4\right) = f(u)$$

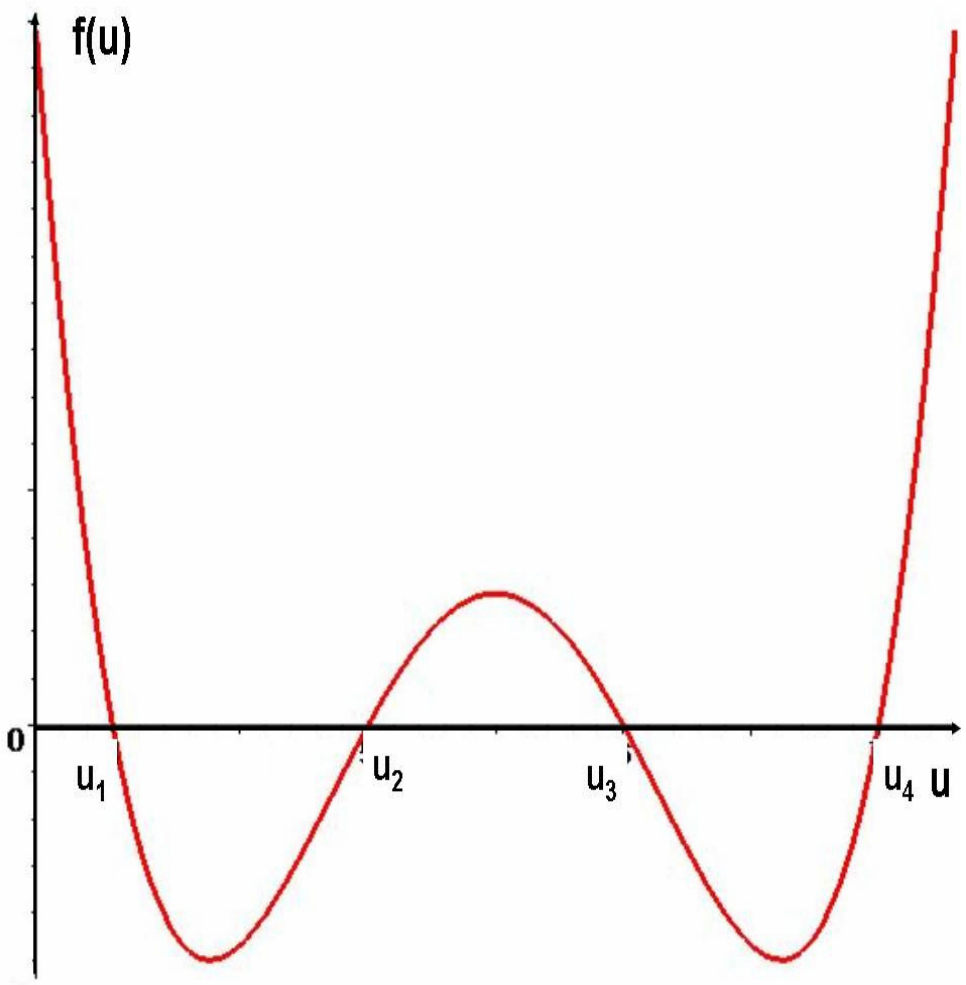
by solving the equation: $f(u) = 0$

Solutions $f(u)=0$

- Schwarzschild BH



- BH with «tidal charge»



Solutions $f(u)=0$

- Schwarzschild BH

3 roots

- BH with «tidal charge»

4 roots,

but minimum one of them is negative

➔

no new types of circular orbits

Last stable circular orbit

is defined by the equation

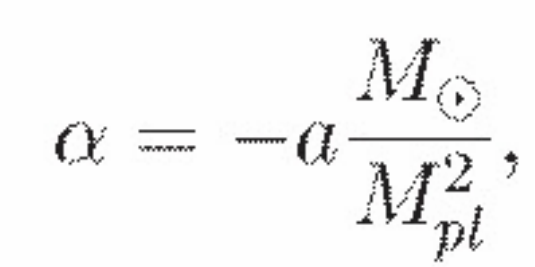
$$8\beta^2 u_c^3 + 9\alpha\beta u_c^2 + 3\alpha^2 u_c + \alpha = 0$$

where $u_c = 1/r_c$,

r_c – radius of a stable circular orbit

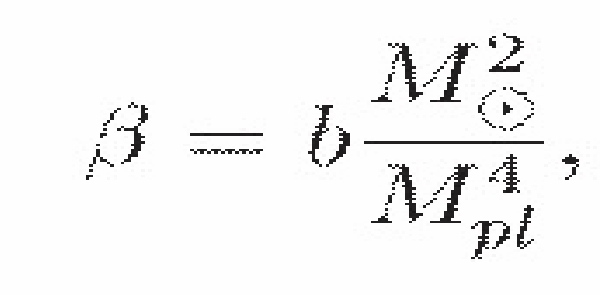
«Tidal charge» contribution

Dimensionless notations:



$$\alpha = -a\frac{M_{\odot}}{M_{pl}^2},$$

a – normalized Schwarzschild mass at Sun mass range, a ~ 1



$$\beta = b\frac{M_{\odot}^2}{M_{pl}^4},$$

b – normalized «tidal charge»

Limitations on normalized «tidal charge» value

- **From** *N.Dadhich, R.Maartens, P.Papadopoulos, V.Rezania (2000)*

$$|b| \ll 2R_{\odot}\frac{M_{pl}^2}{M_{\odot}}.$$

- **For Sun mass range:**

|b| << 10^6

- **From the equation for last stable circular orbit:**

$$8b^2\tilde{u}_c^{\,3} + 9ab\tilde{u}_c^{\,2} + 3a^2\tilde{u}_c + a = 0.$$

where $\tilde{u}_c = u_c M_{\odot}/M_{pl}^2$

- **according to astronomical data real BH are Kerr-like ones, therefore, the conditions of a-terms leading contribution in last stable circular orbit equation in Sun mass range leads to**

|b| << 1

Conclusions

- The “tidal charge” changes geodesic equations, but at the range of Sun and larger masses the presence of this one must not introduce new types of geodesics to avoid contradictions with the current astrophysical data. The suggested limit on “tidal charge” value makes impossible the possibility of its direct observation. Perhaps, the indirect consequences of “tidal charge” could be established in high energy physics.

- Finally, the Black hole solution from RS2 model, discussed here has no any contradictions with astrophysical observational data at the chosen parameter values range. More strong limitation on “tidal charge” value than suggested by *N.Dadhich, R.Maartens, P.Papadopoulos,V.Rezania* could be obtained from last stable circular orbits analysis.


Thank you
for your
attention!!!


And for
the questions!!!