Particle pair production in strong EM field, Imaginary temperature and field emission

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- Tunneling processes (Nature, adv. technologies)
- Electron field emission from metals:
  quantum theory by Fowler and Nordheim
- Results by Sauter and Schwinger for electric field in vacuum
- Condensed matter .vs. Vacuum & EM field

Critical electric field in QED

$$E_{\rm crit} = \frac{m^2c^3}{e\hbar} = \frac{mc^2}{e\lambda\!\!\!^{-}_C} = 1.3 \times 10^{16}\ {\rm V/cm}$$

(F. Sauter, 1931)

$$\Delta t \Delta \mathcal{E} \geqslant \hbar, \quad \Delta \mathcal{E} = m_e c^2, \quad \Delta t \geqslant \frac{\hbar}{m_e c^2}$$

$$\mathcal{A} = eE_{cr} \cdot \Delta r = eE_{cr} c \Delta t = eE_{cr} \frac{\hbar}{m_e c} = m_e c^2$$

A similar phenomenon already was studied in quantum mechanics in 1928 !

Theory of cold field emission from metals (Fowler-Nordheim ,1928)

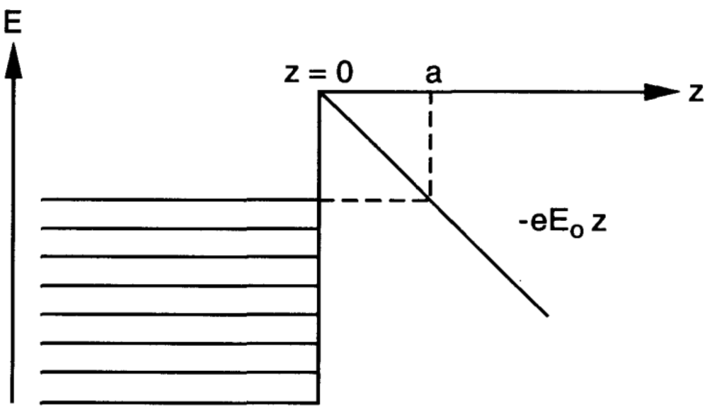


Image potential + external field F

$$V(x) = -\frac{e^2}{4x} - eFx, \quad x > 0$$

Penetration coefficient (WKB)

$$D \approx \exp\{-\frac{2}{\hbar}\int_{x_1}^{x_2} |p(x)|dx\}$$

Electron emission current

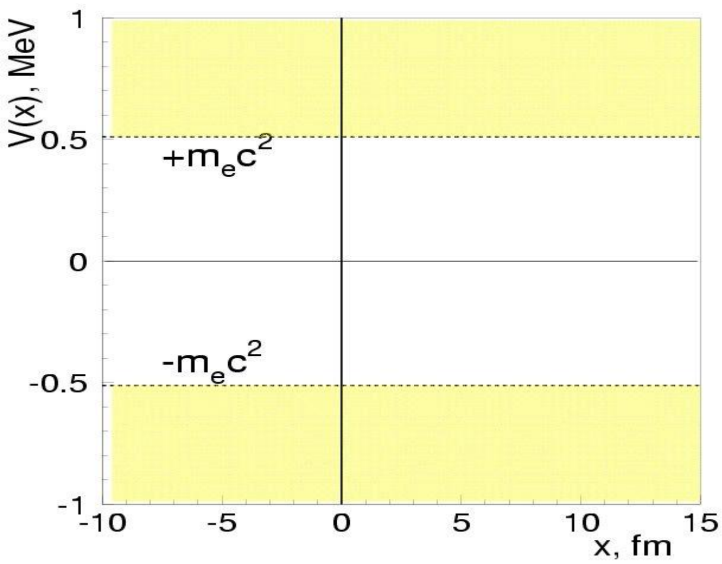
$$J_{FN} = e\int n(\mathcal{E}_x)D(\mathcal{E}_x, E)d\mathcal{E}_x,$$

$$J_{FN}(E) = \frac{A}{t^2(y)}\frac{\alpha}{\pi^2}E^2 \exp\left(-\frac{B}{E}\vartheta(y)\right)$$

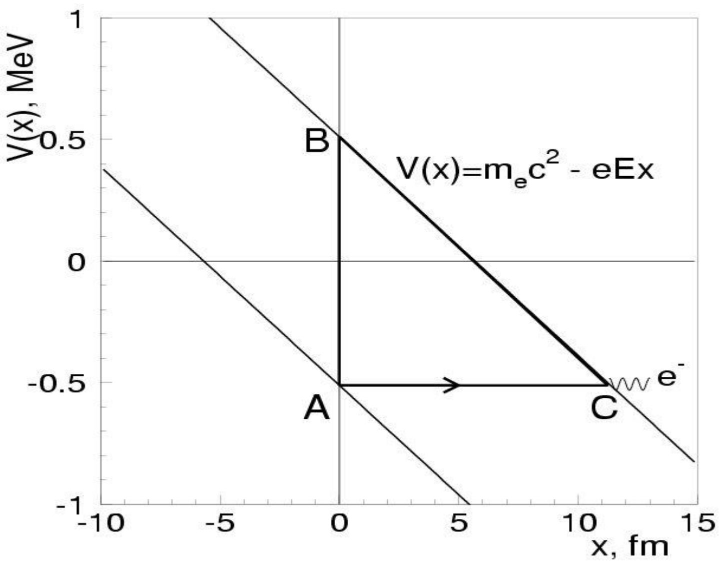
$$A = \frac{ec}{16\varphi}, \quad B = \frac{4\sqrt{2m}}{3e\hbar}\varphi^{3/2}, \quad y = \frac{\sqrt{e^3 E}}{\varphi}$$

$$B = 6.8\text{x}10^7\phi^{3/2}V/\text{см} = 5.4\text{x}10^8 V/cm(\phi = 4eV)$$

Results by Sauter and Schwinger for strong field in vacuum



Figures from the paper by
W. Heisenberg,
H. Euler (1936)



WKB method

$$D \approx \exp\{-\frac{2}{\hbar}\int_{x_1}^{x_2} |p(x)|dx\} \longrightarrow D = \exp\left\{ -\pi\frac{E_{cr}}{E}\right\}$$

QED: The probability of vacuum   decay in a uniform and static electric field with   e+e- pairs creation (Schwinger, 1951)

$$w = \frac{\alpha E^2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left\{ -\pi\frac{nE_{cr}}{E}\right\} \approx \frac{\alpha}{\pi^2}E^2\exp\left\{ -\pi\frac{E_{cr}}{E}\right\}$$

Wick rotation: some examples

Imagine that we are calculating a certain correlation function in the euclidean formulation of the gauge theory. This means averaging over all possible fields Aμ with the weight equal to:

- **Minkowski space-time $\longrightarrow$ Euclid space by t = iτ**

$$\exp(iS(A)) \quad \rightarrow \quad \exp\{-S(A)\} = \exp\left\{-\frac{1}{4g^2}\,\mathrm{Sp}\int F_{\mu\nu}^2\, d^4x\right\}$$

- **Statistical Mechanics $\leftrightarrow$ Quantum Mechanics**

$$\sum Q(j)e^{-E_j/k_BT} \qquad \frac{1}{k_BT} \Rightarrow \frac{it}{\hbar} \qquad \langle Q|e^{-iHt/\hbar}|\psi\rangle = \sum Q_j e^{-E_j it/\hbar}$$

Instantons:

Minkowski space-time -> Euclid space by t -> iτ

BPST-instanton, self-dual equation (Belavin, Polyakov, Schwartz, Tyupkin)

U(1) (‘t Hooft)

$$F_{\alpha\beta} = \pm\frac{1}{2}\epsilon_{\alpha\beta\gamma\delta}F_{\gamma\delta}$$

Field emission at T≠0 vs. Particle production at H≠0

FE from a metal at T=0

$$J_{FN}(F) = \mathrm{A} \cdot F^2 \exp(-\frac{F_{cr}}{F})$$

Scalar or e+e- pair production in electric field, H=0
(at WKB level, the Dirac and Fock-Klein-Gordon equations are identical)

$$w(E) = \frac{\alpha}{\pi^2} \cdot E^2 \exp(-\pi \frac{E_{cr}}{E})$$

Field emission at T≠0 vs. Particle production at H≠0

FE from a metal at T ≠ 0 (Murphy, Good, 1956)

$$J^{(0)}(F,T) = A\frac{\pi c_0\Theta/F}{\sin(\pi c_0\Theta/F)}F^2\exp\Big(-\frac{F_{cr}}{F}\Big), \quad \Theta = k_B T$$

Pair of scalar particle production from vacuum, H ≠ 0 (Ritus,1969),

$$w^{(0)}(E,H) = \bar{A}\frac{\pi H/E}{\mathrm{sh}(\pi H/E)}E^2\exp\Big(-\frac{E_{cr}}{E}\Big)$$

We are observing a very intriguing features !

Field emission atT≠0 vs. Particle production at H≠0

If recall that

$$\sin(i\beta) = i\,\mathrm{sh}(\beta), \quad \cos(i\beta) = \mathrm{ch}(\beta)$$

With the replacement (a sort of Wick rotation)

$$\Theta = i\frac{H}{c_0}, \quad F \equiv E$$

One get from FE Eq, the equation for a pair production from vacuum

$$J(F,T) \Leftrightarrow w(E,H)$$

Magnetic field in vacuum plays a role of the imaginary temperature

How is unusual our conclusion ?

Ising Model in D=2

$$\mathcal{E} = -\sum_{j=1}^{L_v}\sum_{k=1}^{L_h}\{E^h\sigma_{j,k}\sigma_{j,k+1} + E^v\sigma_{j,k}\sigma_{j+1,k} + H\sigma_{j,k}\}$$

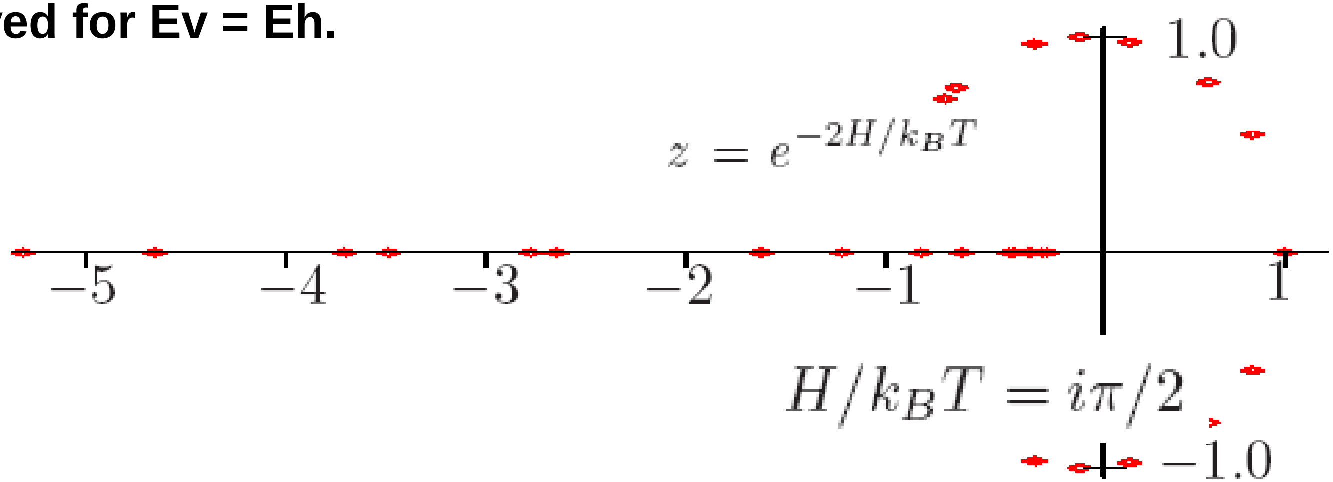
The first result obtained for H ≠ 0 is the famous **circle theorem** **of Lee and Yang (1952): the zeroes of the partition function of an Ising model on a finite size lattice in D dimensions lie on the unit circle |z| = 1:**

$$z = e^{-2H/k_BT}$$

The imaginary magnetic field

Lee and Yang also found that for the free energy of the Ising model can also be exactly solved for Ev = Eh.

$$H/k_BT = i\pi/2$$



Weierstrass theorem

When V is finite the grand partition function $Q^{gr}(z,T;V)$ is an entire function of z which is positive on the positive z axis.
We may use the Weierstrass factorization theorem to express Q^{gr} in terms of its zeros and thus

$$Q^{gr}(z,T;V) = e^{az}\prod_{j=1}^{\infty}[1-(z/z_j)]e^{z/z_j}$$

QFT pays a reward ?

Production a pair of spinor particles from vacuum (Nikishov, 1969)

$$w^{(1/2)}(E,H) = 2\bar{A}\left(\frac{\pi H}{E}\right)\mathrm{cth}\left(\frac{\pi H}{E}\right)E^2\exp\left(-\frac{E_{cr}}{E}\right)$$

Inverse rotation (anzac)

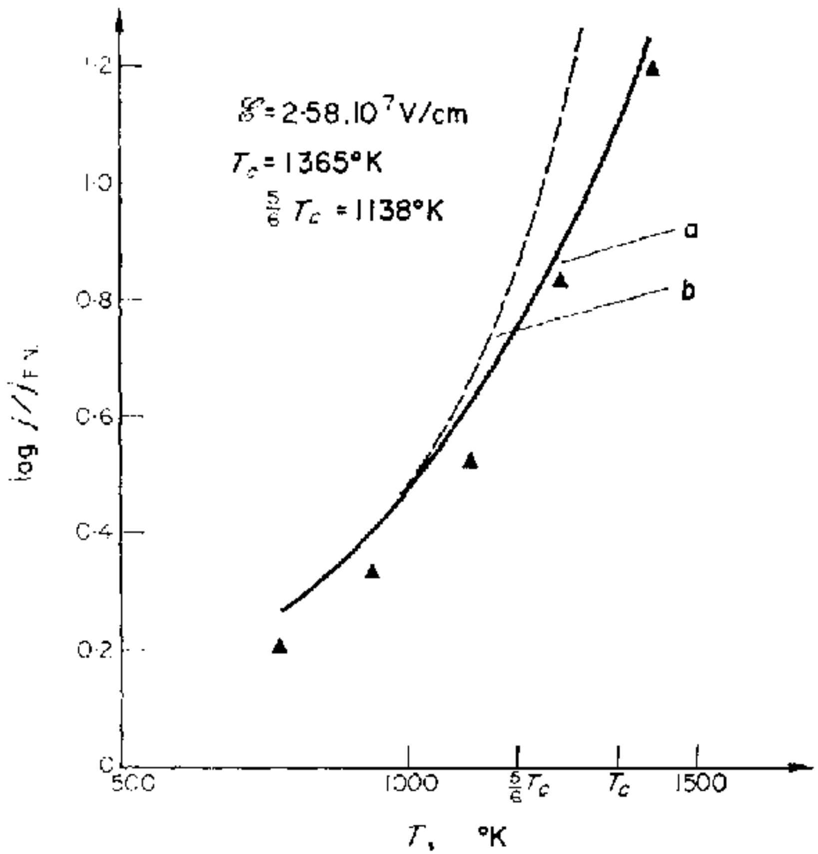
$$H = -ic_0\Theta \qquad \frac{\pi H}{E}\mathrm{cth}\left(\frac{\pi H}{E}\right) \to \frac{\pi c_0\Theta}{F}\mathrm{ctg}\left(\frac{\pi c_0\Theta}{F}\right)$$

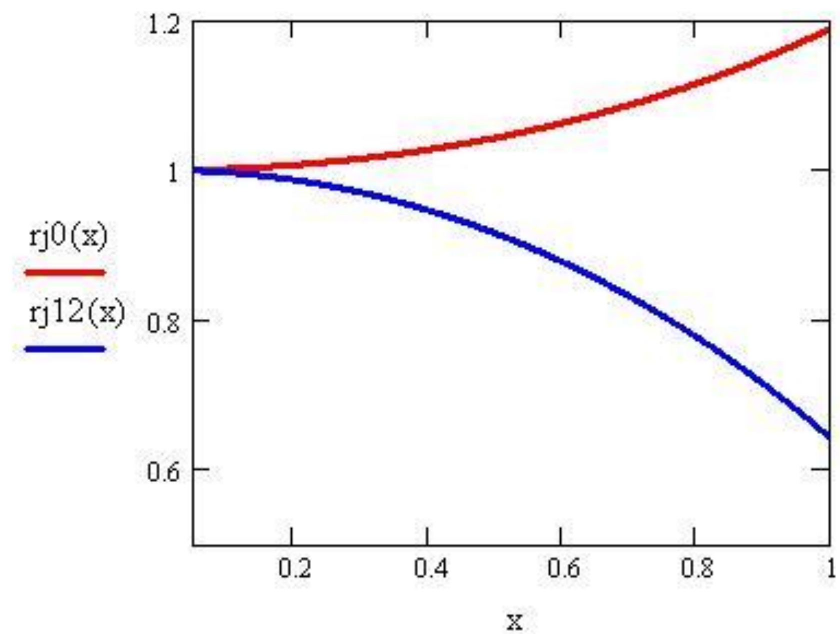
Do we get the emission current from a metal with account of the electron spin ?

$$J^{(1/2)}(T) = J_{FN}(0)\cdot\frac{\pi c_0\Theta}{F}\mathrm{ctg}\left(\frac{\pi c_0\Theta}{F}\right)$$

?

No that simple.
Experimental data on FE compared to a theory (FE+thermionic em. + image ch. eff.)



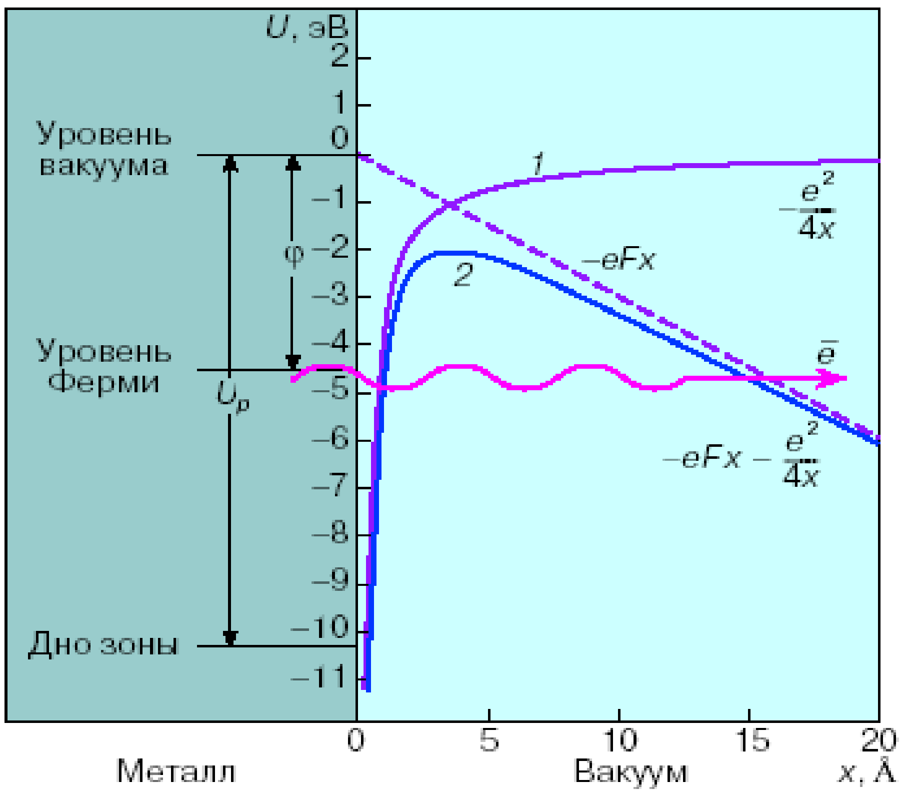


$$\frac{x}{\sin x}$$

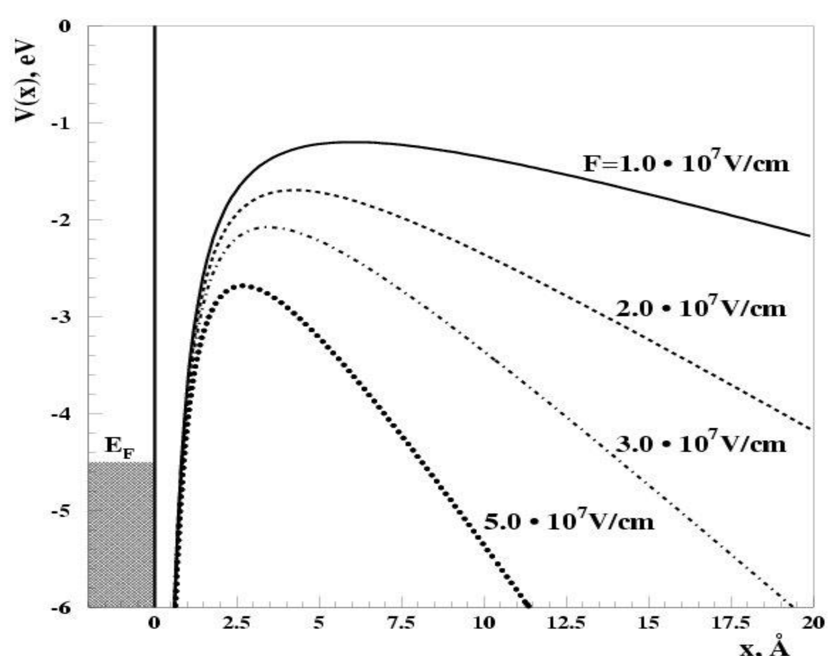
$$\frac{x}{\tan x}$$

Conclusions

- **FE from a metal and particle production from vacuum in strong EM field are compared**
- **Both phenomena are related via a type of Wick rotation** $H = -ic_0 k_B T$
- **Magnetic field in vacuum plays a role of the (imaginary) temperature for scalar particles**



U, эВ
2
1
0
−1
−2
−3
−4
−5
−6
−7
−8
−9
−10
−11
Уровень вакуума
Уровень Ферми
Дно зоны
φ
U_p
1
2
$-\frac{e^2}{4x}$
$-eFx$
$\bar{e}$
$-eFx-\frac{e^2}{4x}$
0
5
10
15
20
Металл
Вакуум
x, Å



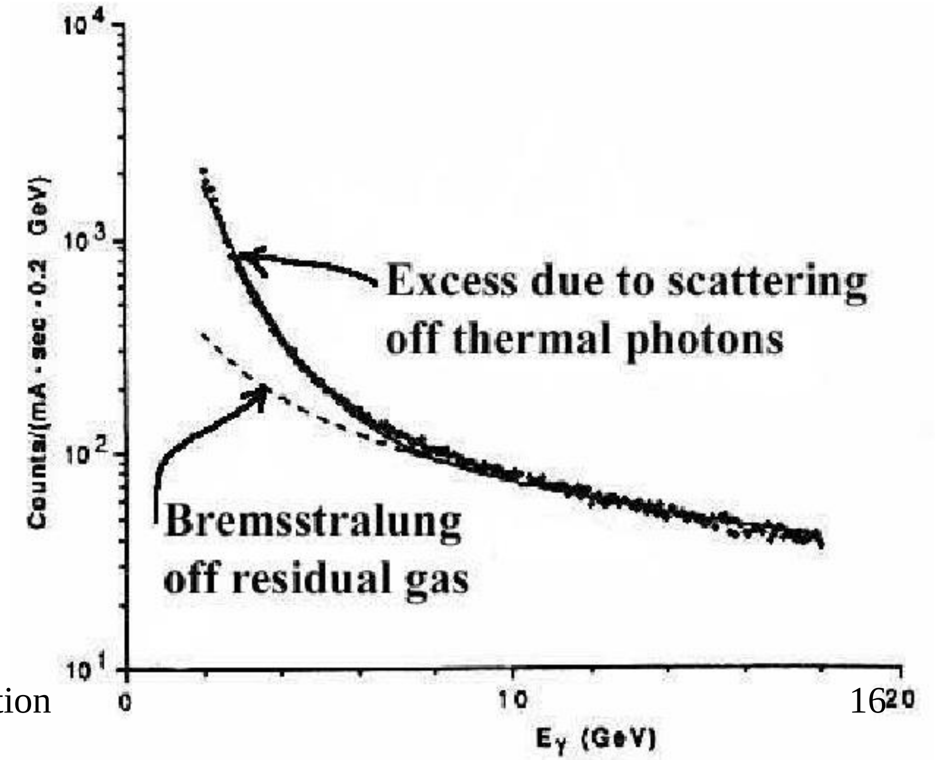
V(x), eV
0
-1
-2
-3
-4
-5
-6
E_F
F=1.0 • 10^7V/cm
2.0 • 10^7V/cm
3.0 • 10^7V/cm
5.0 • 10^7V/cm
0
2.5
5
7.5
10
12.5
15
17.5
20
x, Å

Backups

Scattering Off the Vacuum

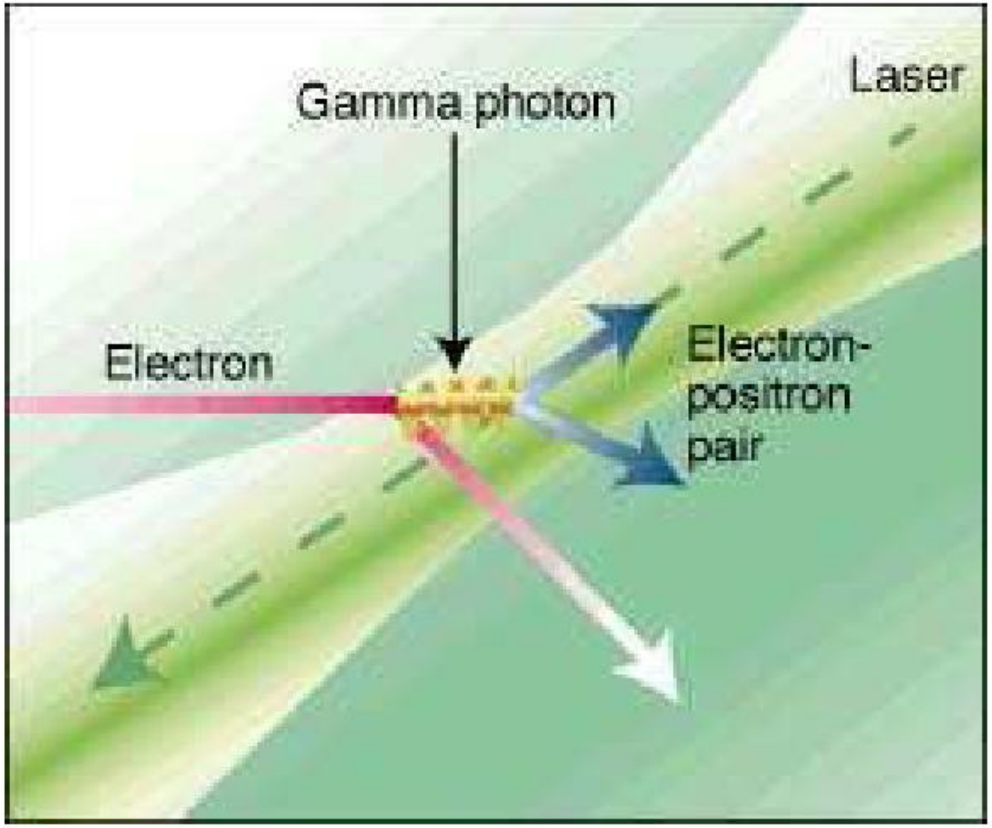
a study of the scattering of an electron beam off the vacuum. The issue is that the “vacuum” pipe of a particle accelerator is not a true vacuum, but contains room-temperature blackbody radiation as well as residual gas molecules. In the process called “inverse” Compton scattering by astrophysicists, a beam electron can amplify the energy of a blackbody photon by a factor $4\gamma^2$.
A room-temperature photon an energy 1/40 eV can attain energies of order 1 GeV when scattered by a 50 GeV electron beam.

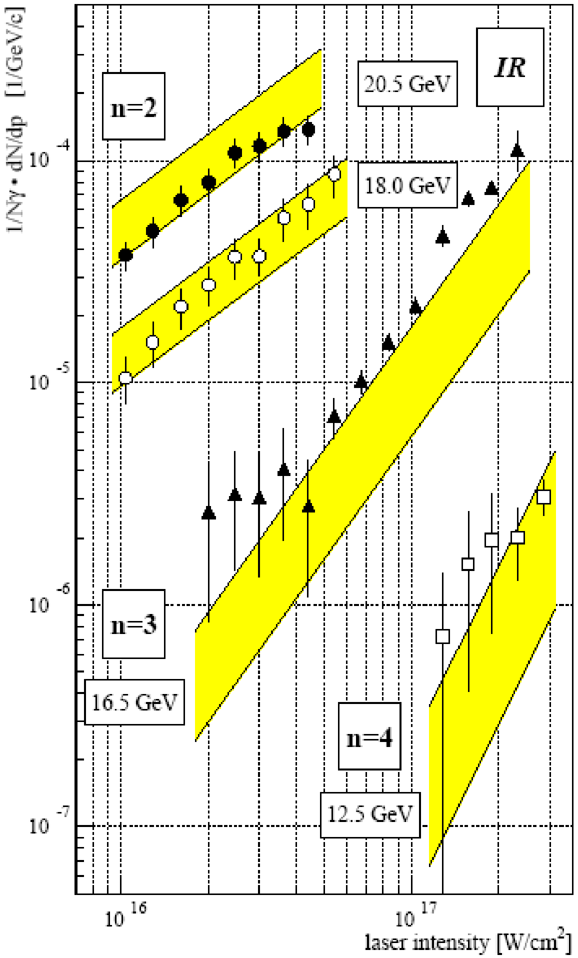
The experiment consisted of the 50-GeV LEP beam at CERN, a single lead glass block, and “no” target.



Scheme of probing strong-field QED in the collisions of a 50-GeV electron beam with a focused laser beam.

$$e + n\omega_0 \rightarrow e' + \omega$$

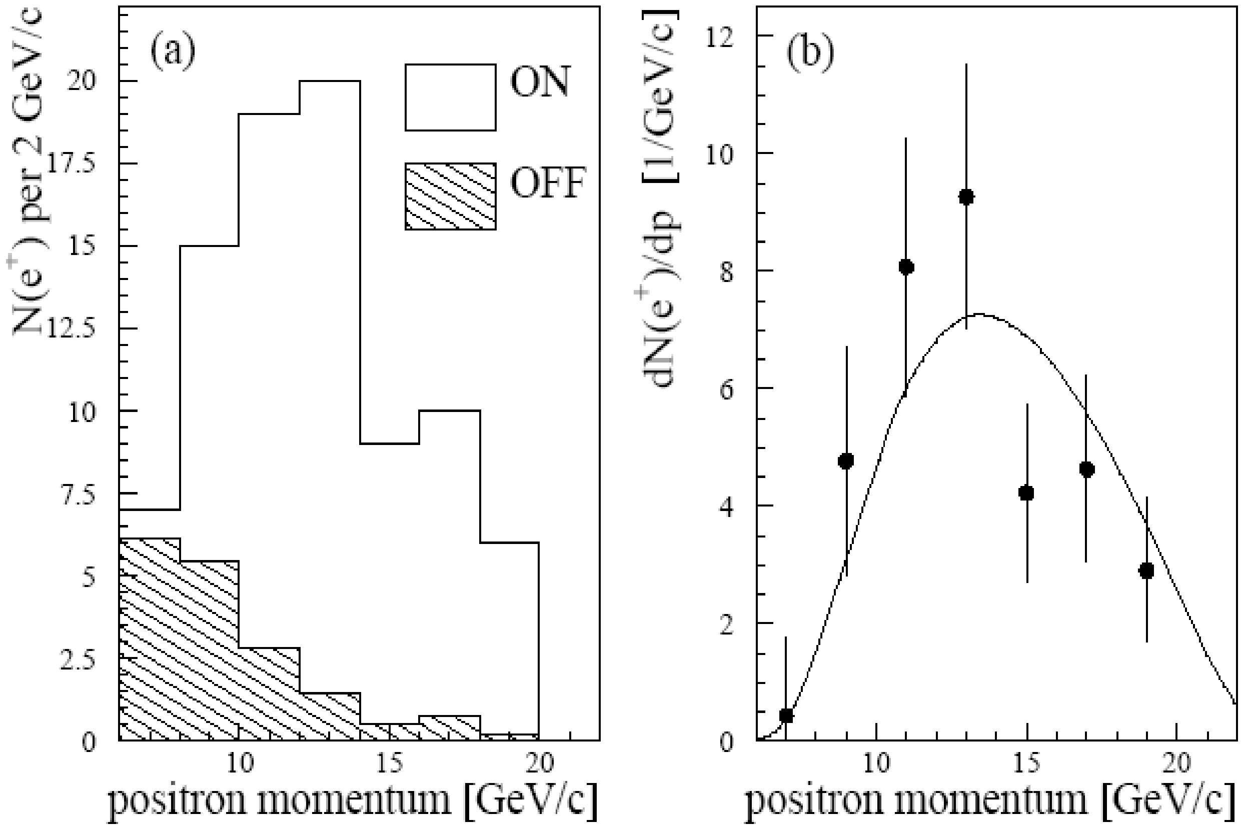




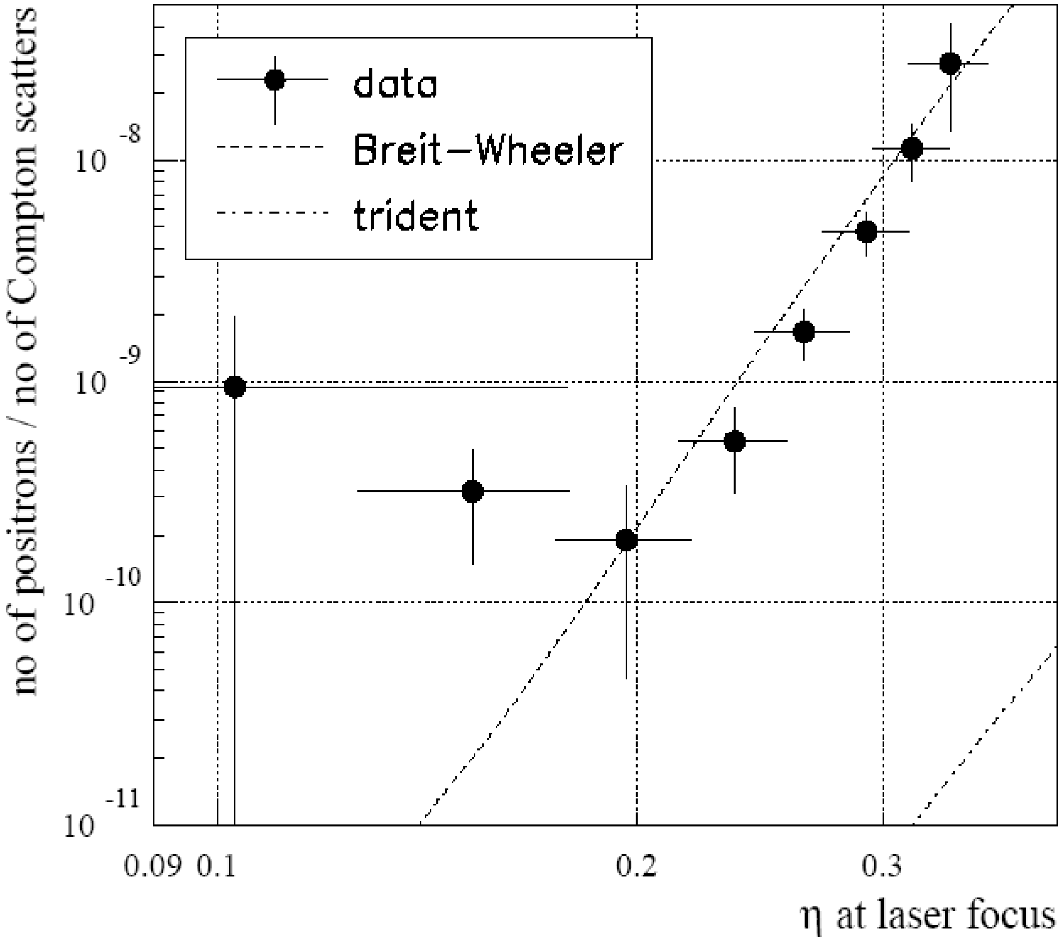
Multiphoton pair creation by light can arise from a two-step process in which a high-energy photon from the previous reaction interacts with the laser beam

$$\omega + n\omega_0 \to e^+e^-$$

This process is the strong-field variant of Breit-Wheeler pair creation.



(a) Laser-on and -off spectra of positrons from reaction (7).
(b) Subtracted spectrum. The solid line is a model calculation.



$$\eta = \frac{e\sqrt{\langle A_\mu A^\mu \rangle}}{mc^2} = \frac{eE_{\mathrm{rms}}}{m\omega_0 c} = \frac{eE_{\mathrm{rms}}\lambda\!\!\!^{-}_0}{mc^2}$$

governs the importance of multiple photons in the initial state

$$\Upsilon = \frac{\sqrt{\langle (F^{\mu\nu} p_\nu)^2 \rangle}}{mc^2 E_{\rm crit}} = \frac{2p_0}{mc^2} \frac{E_{\rm rms}}{E_{\rm crit}} = \frac{2p_0}{mc^2} \frac{\lambdabar_C}{\lambdabar_0} \eta$$

governs the importance of "spontaneous" pair creation

$$P \propto \exp\left(-\frac{d}{\lambda_C}\right) = \exp\left(-\frac{2m^2c^3}{e\hbar E}\right) = \exp\left(-\frac{2E_{\rm crit}}{E}\right) = \exp\left(-\frac{2}{\Upsilon}\right)$$