Asymptotic behavior of form factor of the composite system at large momentum transfer. [1]

Gamzova E.S., Krutov A.F., Troitsky V.E.

Samara State University
D.V. Skobeltsyn Institute of Nuclear Physics

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[1] Gamzova E.S., Krutov A.F., Troitsky V.E., Tsirova N.A. Relativistic Constituent Quark Model and Experiments at JLab. Physics of Atomic Nuclei. 2010. **73**. p. 1063–1068.

Motivations

- The relativistic description of composite systems, on example the pion, as quark-antiquark system;
- Modernization of JLab will allow for measurement of the pion form factor at the large momentum transfer and check the predictions of pQCD and CQM;
- Comparison of predictions CQM and pQCD for the pion in the region of future experiments.

Instant form of relativistic quantum mechanics

The electromagnetic current matrix element for pion:

$$\langle p_\pi |\, j_\mu \,| p'_\pi \rangle = (p_\pi + p'_\pi)_\mu \, F_\pi(Q^2)$$

$F_\pi(Q^2)$ – the electromagnetic form factor of the pion, p_π, p'_π – the four-momentum of the pion.

In RQM the Hilbert space of composite particle states is:

$$\mathcal{H}_{q\bar{q}} \equiv \mathcal{H}_q \otimes \mathcal{H}_{\bar{q}}$$

As a basis in $\mathcal{H}_{q\bar{q}}$: one can choose the following set of vectors:

$$|\,\vec{p}_1\,, m_1;\, \vec{p}_2\,, m_2\rangle = |\,\vec{p}_1\,, m_1\rangle \otimes |\,\vec{p}_2\,, m_2\,\rangle \;,$$

$$\langle \vec{p}\,, m\,|\,\vec{p}\,'\,, m'\rangle = 2p_0\,\delta(\vec{p} - \vec{p}\,')\,\delta_{mm'} \;,$$

Here $\vec{p}_1\,,\ \vec{p}_2$ — are particle momenta, $m_1\,,\ m_2$ — spin projections.

Instant form of relativistic quantum mechanics

- The natural basis is one with separated center-of-mass motion:

$$|\,\vec{P},\ \sqrt{s},\ J,\ L,\ S,\ m_J\,\rangle\ ,$$

with $P_\mu = (p_1 + p_2)_\mu$, $P_\mu^2 = s$, $\sqrt{s}$ — the invariant mass of two-particle system , L — the angular momentum in the center-of-mass frame, S — the total spin, J — the total angular momentum, m_J — the projection of the total angular momentum.

- Wave function of the composite system in RQM:

$$\langle \vec{P}\,',\ \sqrt{s'},\ J',\ l',\ S',\ m'_J|\,p_\pi\rangle = N_\pi \delta(\vec{P}\,' - \vec{p}_c)\delta_{JJ'}\delta_{m_J m'_J}\delta_{ll'}\delta_{SS'}\,\varphi^J_{lS}(k)$$

$s = 4(k^2 + M^2)$, M is the quark mass , N_π, N_{CG} are factors due to normalization.

Electromagnetic structure of the pion within the instant form of RQM

- The pion electromagnetic form factor:

$$F_\pi(Q^2) = \int \mathrm{d}\sqrt{s}\,\mathrm{d}\sqrt{s'}\,\varphi(k) g_0(s, Q^2, s')\varphi(k')$$

$\varphi(k)$ – the pion wave function within relativistic quantum mechanics (RQM)
$g_0(s, Q^2, s')$ – free two-particle form factor.

Electromagnetic structure of the pion within the instant form of RQM

$$g_0(s, Q^2, s') = a(s, Q^2, s')(G_E^u(Q^2) + G_E^{\bar{d}}(Q^2)) + b(s, Q^2, s')$$
$$(G_M^u(Q^2) + G_M^{\bar{d}}(Q^2))$$

where $G_E^q(Q^2)$ and $G_M^q(Q^2)$ – the constituent quark electric and magnetic form factors [2]

$$G_E^q(Q^2) = e_q f_q(Q^2), \quad G_M^q(Q^2) = (e_q + \kappa_q) f_q(Q^2)$$

e_q — the quark charge, κ_q — the constituent quark anomalous magnetic moment

[2] A. F. Krutov, V. E. Troitsky, Relativistic instant–form approach to the structure of two–body composite systems // Phys. Rev. C **65**, 045501 (2002)

Electromagnetic structure of the pion within the instant form of RQM

$$f_q(Q^2) = \frac{1}{1 + \ln(1 + \langle r_q^2 \rangle Q^2/6)}$$

where $\langle r_q^2 \rangle$ – the root-mean-square radius of the quark.
Prediction pQCD quark counting rules: [3]

$$F_\pi(Q^2) \sim Q^{-2}$$

[3] V. A. Matveev, R. M. Muradyan, and A. N. Tavkhelidze,Lett. Nuovo Cimento **7**, 719 (1973); **15**, 907 (1973); S. Brodsky and G. Farrar, Phys. Rev. Lett. **31**, 1153 (1973).

Asymptotic estimation of some multiple integrals

In the following we will consider integrals of the kind:

$$F(\lambda) = \int_\Omega f(\lambda, x) e^{S(\lambda,x)} \mathrm{d}x$$

where Ω is a domain in $\mathbf{R}^n$, $x = (x_1; ...; x_n)$, λ is a large positive parameter.
Then at $\lambda \to \infty$ the following asymptotic expansion is valid:[4]

$$F(\lambda) \sim \exp[S(\lambda, x^0)] \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} h_{km}(\lambda).$$

$h_{km}(\lambda)$ – known function.

[4] A. F. Krutov, V. E. Troitsky, and N. A. Tsirova, J. Phys. A **41**, 255401 (2008) [nucl-th/0709.2312].

Asymptotic behavior of the pion form factor for $Q^2 \to \infty$

- Pointlike quarks:

$$G_E^u(Q^2) + G_E^{\bar d}(Q^2) = 1,\; G_M^u(Q^2) + G_M^{\bar d}(Q^2) = 1$$

$$F_\pi(Q^2) \sim \frac{2^{5/2}M}{Q}\mathrm{e}^{-\frac{QM}{4b^2}}\left(1 + \frac{7b^2}{2MQ}\right).$$

- In case of the electromagnetic structure of the constituent quarks:

$$F_\pi(Q^2) \sim \frac{2^{5/2}M}{Q}\mathrm{e}^{-\frac{QM}{4b^2}}\left(G_E^u(Q^2) + G_E^{\bar d}(Q^2)\right) \times$$
$$\times\left(1 + \frac{b^2}{2MQ}\frac{16\left(G_M^u(Q^2)+G_M^{\bar d}(Q^2)\right)-9\left(G_E^u(Q^2)+G_E^{\bar d}(Q^2)\right)}{\left(G_E^u(Q^2)+G_E^{\bar d}(Q^2)\right)}\right)$$

Asymptotic behavior of the pion form factor for $Q^2 \to \infty$

In the limit case $M/b \to 0$

- Pointlike quarks:

$$F_\pi(Q^2) \sim \frac{14\sqrt{2}b^2}{Q^2}$$

- In case of the electromagnetic structure of the constituent quarks:

$$F_\pi(Q^2) \sim \tfrac{2^{3/2}b^2}{Q^2} \times$$
$$\times \left(16\left(G_M^u(Q^2) + G_M^{\bar{d}}(Q^2)\right) - 9\left(G_E^u(Q^2) + G_E^{\bar{d}}(Q^2)\right)\right)$$

Asymptotic behavior of the pion form factor and relevant present-day experiments

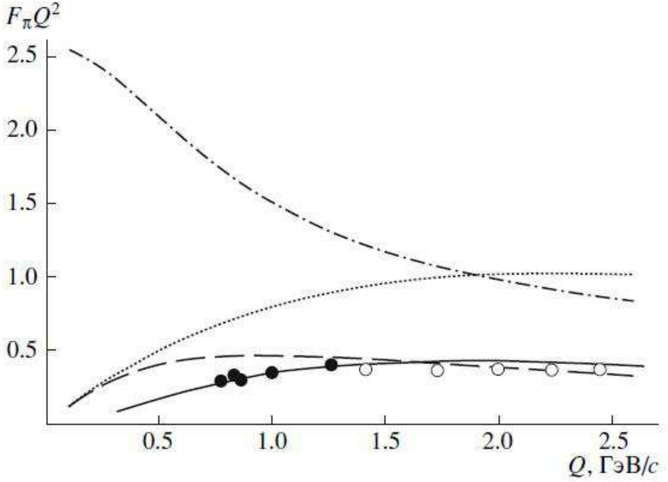

Figure: Results of asymptotic calculations for the pion form factor.

Conclusions

- The result obtained in limit $M/b \to 0$ in the constituent quark model for the pion coincides with the $F_\pi(Q^2)Q^2 = const$ behavior predicted by pQCD.
- The region of experiments at JLab is asymptotic for the pion in the relativistic constituent quark model.