THE RELATIVISTIC ELECTROMAGNETIC STRUCTURE OF THE THREE-BODY SYSTEM

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Problem

1. *Composite quark model.*
2. *Non Rezenblut behavior $F_e(Q^2)/F_m(Q^2)$.*
3. *Construction of conserving current.*

These problems will be considered in the framework of the instant form of the relativistic quantum mechanics. This approach was developed for two body systems by Krutov A.F. and Troitsky V.E. and gives good description of the deuteron and different two-quark systems.[1]

[1] A.F. Krutov V.E. Troitsky, Phys.Rev.C, 65,04501(2002).

Instant form of dynamics

Complete set of commuting operators:

$$\hat{M}_I, \hat{J}^2, \hat{J}_3, \hat{\mathbf{P}} \ . \tag{1}$$

In the instant form of dynamics $\hat{J}^2$, $\hat{J}_3$, $\hat{\vec{P}}$ operators are coincided with non-interaction system operators, therefore their diagonalization is reduced to appropriate basis choice.

$$\hat{M}_I = \hat{M}_0 + \hat{V} \ , \tag{2}$$

where $\hat{M}_0$ - non-interactions system invariant mass operator , $\hat{V}$ - interaction operator.

Additional constraints on mass operator:

$$\hat{M}_I = \hat{M}_I^{+}, \hat{M}_I > 0; \tag{3}$$

$$[\hat{\vec{P}}, \hat{V}] = [\hat{\vec{J}}, \hat{V}] = [\hat{\vec{\nabla}}_P, \hat{V}] = 0. \tag{4}$$

$$\hat{M}_I|\psi(1,2,3)\rangle \equiv (\hat{M}_0 + \hat{V})|\psi(1,2,3)\rangle = \lambda|\psi(1,2,3)\rangle. \tag{5}$$

where λ - eigenvalue of mass operator.

Three body basis

$$|\vec{p}_1, m_1; \vec{P}_{23}, \sqrt{s_{23}}, m_{J_{23}}, L_{23}, S_{23}, J_{23}\rangle =$$
$$= |\vec{p}_1, m_1;\rangle \otimes |\vec{P}_{23}, \sqrt{s_{23}}, m_{J_{23}}, L_{23}, S_{23}, J_{23}\rangle\ , \tag{6}$$

where

$$|\vec{P}_{23}, \sqrt{s_{23}}, m_{J_{23}}, L_{23}, S_{23}, J_{23}\rangle =$$
$$= \sum \int d\hat{k}_{23} |\vec{p}_2, m_2; \vec{p}_3, m_3\rangle Y_{L_{23} m_{L_{23}}}(\hat{k}_{23})$$
$$\langle \frac{1}{2}\frac{1}{2}\tilde{m}_2\tilde{m}_3|S_{23} m_{S_{23}}\rangle\langle S_{23} L_{23} m_{S_{23}} m_{L_{23}}|J_{23} m_{J_{23}}\rangle$$
$$D^{\frac{1}{2}}_{m_2' m_2}(P_{23} p_2) D^{\frac{1}{2}}_{m_3' m_3}(P_{23} p_3)\ . \tag{7}$$

Three body basis

Normalization.

$$\langle \vec{p}'_1 , m'_1 | \vec{p}_1 , m_1 \rangle = 2 p_{10} \delta(\vec{p}_1 - \vec{p}\,'_1) \delta_{m_1 m'_1} \ , \tag{8}$$

$$\langle \vec{P}'_{23} , \sqrt{s'_{23}} , m'_{J_{23}} , L'_{23} , S'_{23} , J'_{23} | \vec{P}_{23} , \sqrt{s_{23}} , m_{J_{23}} , L_{23} , S_{23} , J_{23} \rangle =$$
$$= N^2 2 P^0_{23} \delta(\vec{P}_{23} - \vec{P}'_{23}) \delta(\sqrt{s_{23}} - \sqrt{s'_{23}}) \delta_{m_{J_{23}} m'_{J_{23}}} \delta_{S_{23} S'_{23}} \delta_{L_{23} L'_{23}} \delta_{J_{23} J'_{23}} , \tag{9}$$

$$|\vec{P}, m_J; \sqrt{s}, \sqrt{s_{23}}, L, S, J, (L, S, J)_{23}\rangle\ . \tag{10}$$

$$\langle\vec{P}', m'_J; \sqrt{s'}, \sqrt{s'_{23}}, L', S', J', (L', S', J')_2|\vec{P}, m_J; \sqrt{s}, \sqrt{s_{23}}, L, S, J, (L, S, J)_2$$

$$= N_1^2 \frac{2P_0}{kk_{23}} \delta(\vec{P} - \vec{P}')\delta(\sqrt{s} - \sqrt{s'})\delta(\sqrt{s_{23}} - \sqrt{s'_{23}})$$

$$\delta_{m_J m'_J}\delta_{S'S}\delta_{L'L}\delta_{L'_{23}L_{23}}\delta_{S'_{23}S_{23}}\delta_{J'J}\delta_{J'_{23}J_{23}}\ . \tag{11}$$

$$|\vec{P}, m_J; \sqrt{s}, \sqrt{s_{23}}, L, S, J, (L, S, J)_{23}\rangle =$$

$$= \sum |\vec{p}_1, m_1; \vec{P}_{23}, \sqrt{s_{23}}, m_{J_{23}}, L_{23}, S_{23}, J_{23}\rangle$$

$$Y_{Lm_L}(\hat{k})\langle \frac{1}{2} J_{23} m_1' m_{J_{23}}' | S m_S\rangle$$

$$\langle S L m_S m_L | J m_J\rangle D^{\frac{1}{2}}_{m_1{}' m_1}(P p_1) D^{J_{23}}_{m_{J_{23}}{}' m_{J_{23}}}(P P_{23}) \ . \qquad (12)$$

Vector $|\vec{P}, m_J; \sqrt{s}, \sqrt{s_{23}}, L, S, J, (L, S, J)_2\rangle$ is analogue to the one-particle one with momentum $\vec{P}$ and spin $\vec{J}$. Particulary under Lorentz-transformation $U(\Lambda)$ this vector behave as one-particle vector:

$$|\vec{P}, m_J; \sqrt{s}, \sqrt{s_{23}}, L, S, J, (L, S, J)_{23}\rangle =$$

$$= \sum_{m_J} |\Lambda\vec{P}, m_J'; \sqrt{s}, \sqrt{s_{23}}, L, S, J, (L, S, J)_{23}\rangle D^J_{m_J' m_J}(\Lambda P, P) \ . \quad (13)$$

Wave-function for three particles system with interaction may be presented as:

$$\langle \vec{P}, m_J; \sqrt{s}, \sqrt{s_{23}}, L, S, J, (L, S, J)_{23} | \vec{p}_c, \mu_c \rangle =$$

$$N_c\, \delta(\vec{P} - \vec{p}_c)\delta_{m_J m_c} \varphi^J_\gamma(\sqrt{s}, \sqrt{s_{23}}) \ , \qquad (14)$$

where $\gamma \equiv \{L, S, (L, S, J)_{23}\}$, N_c - normalization factor.

Parametrization of current operator

Electromagnetic current operator for one-particle system:

$$\langle\vec{p}, M, j, m|J_\mu(0)|\vec{p'}, M, j, m'\rangle = \sum_{m''}\langle m|D^j(pp')|m''\rangle$$

$$\langle m''|F_1 K'_\mu + F_2\Gamma_\mu(p') + F_3 R_\mu + F_4 K_\mu|m'\rangle \ , \qquad (15)$$

$$F_i = \sum_{n=0}^{2j} f_{in}(Q^2)(ip_\mu\Gamma^\mu(p'))^n \ ,$$

$$K'_\mu = (p+p')_\mu,\ K_\mu = (p-p')_\mu = Q_\mu,\ R_\mu = \epsilon_{\mu\nu\lambda\rho}p^\nu p'^\rho\Gamma^\rho(p') \ .$$

$$D^{\frac{1}{2}}(pp') = Cos\frac{\omega}{2} - 2i(\mathbf{nj})Sin\frac{\omega}{2}\ , \tag{16}$$

$$D^{1}(pp') = I - i(\mathbf{nj})Sin\omega + (\mathbf{nj})^2(Cos\omega - 1)\ , \tag{17}$$

$$\mathbf{n} = \frac{[\mathbf{pp}']}{|[\mathbf{pp}']|}\ ,\ \omega = 2arctn\frac{|[\mathbf{pp}']|}{(p_{01}+M_1)(p_{02}+M_2) - (\mathbf{pp}')}\ .$$

Additional constraints on four-vectors:

Hermitian charakter:

$$\Gamma_\mu(p') \to \Gamma_\mu(p') - \frac{K'_\mu}{K'^2}(p_\nu \Gamma^\nu(p'))\ , n = 0\ ,$$

$$F_i A_\mu \to \frac{1}{2}\{F_i, A_\mu\}_+\ , i = 2, 3\ , n \neq 0\ ,$$

$$F_i \to \mathbf{i} F_i\ , i = 3, 4\ .$$

Orthogonality:

$$\Gamma_\mu(p') \to \Gamma_\mu(p') - \left(\frac{K_\mu}{K^2} + \frac{K'_\mu}{K'^2}\right)(p_\nu \Gamma^\nu(p'))\ , i = 2\ ,$$

Additional constraints on four-vectors:

Conservation of parity:

$F_i A_\mu$ contains even factors $\Gamma_\mu(p')$.

Conditions of conservation:

$$j_\mu K^\mu = j_\mu Q^\mu = 0 \Rightarrow F_4 = 0 \ .$$

$$\langle\vec{p},M,j,m|J_\mu(0)|\vec{p'},M,j,m'\rangle=\sum_{m''}\langle m|D^j(pp')|m''\rangle$$

$$\langle m''|F_1K'_\mu+\{F_2\Big(\Gamma_\mu(p')-\Big(\frac{K_\mu}{K^2}+\frac{K'_\mu}{K'^2}\Big)(p_\nu\Gamma^\nu(p'))\Big)\}_++\mathbf{i}\{F_3R_\mu\}_+|m'\rangle\ . \tag{18}$$

In the case $j=\frac{1}{2}$ we are have:

$$\langle\vec{p},M,\frac{1}{2},m|J_\mu(0)|\vec{p'},M,\frac{1}{2},m'\rangle=\sum_{m''}\langle m|D^{\frac{1}{2}}(pp')|m''\rangle$$

$$\langle m''|f_{10}(Q^2)K'_\mu+\mathbf{i}f_{30}(Q^2)R_\mu|m'\rangle\ . \tag{19}$$

Connections with Sachs formfactors:

$$f_{10}(Q^2)=\frac{G_E(Q^2)}{\sqrt{1+\frac{Q^2}{4M^2}}}\ ,f_{30}(Q^2)=\frac{2G_M(Q^2)}{M^2\sqrt{1+\frac{Q^2}{4M^2}}}$$

Parametrization of electromagnetic current matrix element for free three-body system

Current operator of free three-particle system:

$$J^0_\mu(0) = J^1_\mu(0) \otimes I^{23} \oplus J^2_\mu(0) \otimes I^{13} \oplus J^3_\mu(0) \otimes I^{12} \ .$$

$$\langle \vec{p}_a, m_a; \vec{P}_{bc}, \sqrt{s_{bc}}, m_{J_{bc}}, \gamma_{bc}|J^0_\mu(0)|\vec{p'}_a, m'_a; \vec{P'}_{bc}, \sqrt{s'_{bc}}, m'_{J_{bc}}, \gamma'_{bc}\rangle =$$

$$= \sum^{P(123)} \langle \vec{P}_{bc}, \sqrt{s_{bc}}, m_{J_{bc}}, \gamma_{bc}|\vec{P'}_{bc}, \sqrt{s'_{bc}}, m'_{J_{bc}}, \gamma'_{bc}\rangle \langle \vec{p}_a, m_a|J^a_\mu|\vec{p'}_a, m'_a\rangle \ , \tag{20}$$

where $\gamma_{bc} \equiv \{L_{bc}, S_{bc}, J_{bc}\}$.

$$\langle \vec{P}, m_J; \sqrt{s}, \sqrt{s_{23}}, \gamma | J^0_\mu(0) | \vec{P}', m'_J; \sqrt{s'}, \sqrt{s'_{23}}, \gamma \rangle =$$

$$= \sum_{m''_J} \langle m_J | D^{\frac{1}{2}}(PP') | m''_J \rangle \langle m''_J | F_E^{\gamma\gamma'} A^1_\mu + F_M^{\gamma\gamma'} A^3_\mu | m'_J \rangle \ , \qquad (21)$$

where

$$F_i^{\gamma\gamma'} \equiv F_i^{\gamma\gamma'}(s, s_{23}, Q^2, s', s'_{23}) \ ,$$

$$\gamma \equiv \{L, S, J, (L, S, J)_{23}\} \ ,$$

$$A^1_\mu = \frac{1}{Q^2}\Big((s - s' + Q^2)P_\mu + (s' - s + Q^2)P'_\mu\Big) \ ,$$

$$A^3_\mu = \frac{i}{\sqrt{s'}} R_\mu \ .$$

Free formfactors of three body systems

$$F_E^{00} = \sum^{P(123)} \frac{3Q^2(s+s'+Q^2)}{2\lambda^{\frac{3}{2}}(s,s',-Q^2)}\frac{k_{23}\delta(\sqrt{s_{23}}-\sqrt{s'_{23}})}{kk'} \times$$
$$\times\left[Af_{10}^a(Q^2)\,Cos\frac{\omega 1+\omega 2}{2} - \frac{M}{2}Bf_{30}^a(Q^2)\,Sin\frac{\omega 1+\omega 2}{2}\right],$$

$$F_E^{11} = \sum^{P(123)} \frac{Q^2(s+s'+Q^2)}{4\lambda^{\frac{3}{2}}(s,s',-Q^2)}\frac{k_{23}\delta(\sqrt{s_{23}}-\sqrt{s'_{23}})}{kk'} \times$$
$$\times\left[2Af_{10}^a(Q^2)\left\{Cos\frac{\omega 1+\omega 2}{2} + 2Cos\frac{\omega 1+\omega 2+2\omega 3}{2}\right\} -\right.$$
$$\left.-\frac{M}{4}Bf_{30}^a(Q^2)\left\{Sin\frac{\omega 1+\omega 2}{2} + 2Sin\frac{\omega 1+\omega 2+2\omega 3}{2}\right\}\right],$$

$$A = s + s' + Q^2 + 2(M^2 - s_{23})\ ,$$

$$B = \left(-M^2\lambda(s,s',-Q^2) + ss'Q^2 - s_{23}Q^4 - (s_{23} - M^2)Q^2(s+s') + \right.$$

$$\left. + Q^2(s_{23} - M^2)^2\right)^{\frac{1}{2}}\ .$$

$$F_M^{00} = \sum^{P(123)} \frac{Q^2(s+s'+Q^2)}{8\lambda^{\frac{3}{2}}(s,s',-Q^2)} \frac{k_{23}\delta(\sqrt{s_{23}}-\sqrt{s'_{23}})}{kk'} \times$$

$$\times\left[-f_{10}^a(Q^2)K_1 Sin\frac{\omega_1+\omega_2}{2} + f_{30}^a(Q^2)(R_1 Cos\frac{\omega_1+\omega_2}{2} + R_2 Sin\frac{\omega_1+\omega_2}{2})\right.$$

$$F_M^{11} = \sum^{P(123)} \frac{(4Cos\omega_3 - 1)Q^2(s+s'+Q^2)}{4\lambda^{\frac{3}{2}}(s,s',-Q^2)} \frac{k_{23}\delta(\sqrt{s_{23}}-\sqrt{s'_{23}})}{kk'} \times$$

$$\times\left[\frac{2B_2+\lambda(s,s',-Q^2)(1-\frac{B_1}{s'\sqrt{2}})}{Q^2} f_{10}^a(Q^2)Cos\frac{\omega1+\omega2}{2} + \right.$$

$$\left. + \frac{MB_3(\lambda(s,s',-Q^2)-2s'Q^2)}{8\sqrt{s'}} f_{30}^a(Q^2)Sin\frac{\omega1+\omega2}{2}\right],$$

$$B_1 = (\lambda(s, s', -Q^2) + \lambda(s_{23}, M^2, Q^2) +$$

$$+(Q^2 + M^2)(M^2 + 2s) + 2M^2(Q^2 - s'))^{\frac{1}{2}} \ ,$$

$$B_2 = \frac{s(s + s') + (s + Q^2)(Q^2 - 4s_23 + 6M^2) + s'(3Q^2 - s')}{4} -$$

$$-\frac{s'(2s_{23} + 4M^2 - Q^2)}{4} - \frac{(s + Q^2)^3 - 2(s_{23} - M^2)(s + Q^2)^2}{4s'}$$

$$B_3 = s'(s' - 2s_{23} + 4M^2) - (s + Q^2)^2 + 2(s + Q^2)(s_{23} - M^2) \ .$$

Impulse approximation

$$G_E(Q^2) = \sum_{\gamma\gamma'} \int d\sqrt{s_{23}}\, d\sqrt{s'_{23}}\, d\sqrt{s}\, d\sqrt{s'} \times$$

$$\times \psi^{\gamma}(\sqrt{s}\sqrt{s_{23}}) F_E^{\gamma\gamma'}(s, s_{23}, Q^2, s', s'_{23}) \psi^{\gamma'}(\sqrt{s'}\sqrt{s'_{23}}) \;,$$

$$G_M(Q^2) = \sum_{\gamma\gamma'} \int d\sqrt{s_{23}}\, d\sqrt{s'_{23}}\, d\sqrt{s}\, d\sqrt{s'} \times$$

$$\times \psi^{\gamma}(\sqrt{s}\sqrt{s_{23}}) F_M^{\gamma\gamma'}(s, s_{23}, Q^2, s', s'_{23}) \psi^{\gamma'}(\sqrt{s'}\sqrt{s'_{23}}) \;.$$

Summary

1. In the framework of RQM the electromagnetic current matrix element for three-body system is constructed at conditions of conservation and Lorentz-covariance,
2. The electromagnetic form factors for free body system with $S = 1/2$ are obtained in relativistic impulse approximation.