Soliton-antisoliton production in particle collisions

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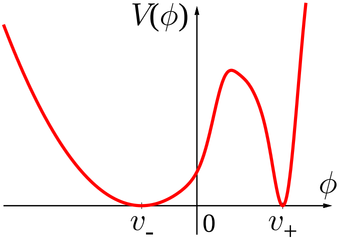
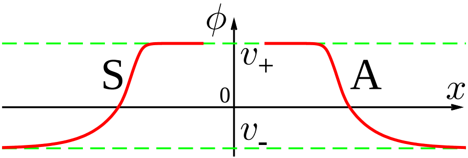
Topological solitons

$\boxed{\phi}$ $(1+1)$ $\hbar = c = 1$

$$S = \frac{1}{g^2}\int dxdt \left[(\partial_\mu\phi)^2/2 - V(\phi)\right]$$

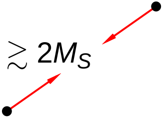
g — semiclassical parameter
and coupling constant ($\phi = g\phi'$)

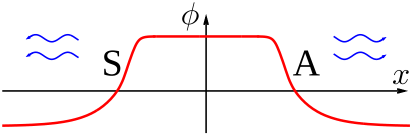




Properties: $L_S \sim m^{-1}$
$M_S \sim m/g^2$
m — mass scale of $V(\phi)$

$E \gtrsim 2M_S$





$\lambda \sim 1/E \sim g^2/m$ $\Downarrow$

$L_S \sim 1/m$

Exponential suppression!

$$\boxed{\mathcal{P}(E) \approx A(E) \cdot \mathrm{e}^{-F(E)/g^2}}$$

- Coherent–state "estimate"

  $$\bar{n}_S \sim M_S/m \sim 1/g^2$$

  $$\langle 2|SA\rangle \sim \frac{\bar{n}_S^2}{2!}\mathrm{e}^{-\bar{n}_S} \sim \mathrm{e}^{-\mathrm{c}/g^2}$$

  Drukier, Nussinov (1982)

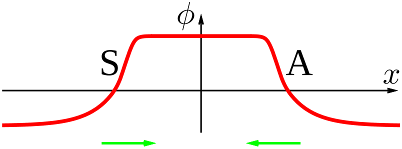
  Banks at al. (1990)

- Unitarity arguments for multiparticle production *Zakharov (1991)*

No reliable estimate of $\mathcal{P}(E)$ so far!

Aim: calculate semiclassically $F(E)$.

NOT tunneling?!

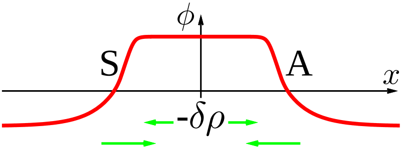


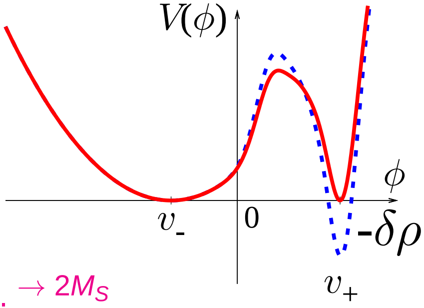
$\Rightarrow$

Many particles

Attraction!

Not a tunneling process? Introduce a potential barrier:





Critical bubble $\Leftrightarrow$ Barrier top

$\delta\rho \to 0$: cr. bubble $\longrightarrow$ SA; $E_{\text{cr.b.}} \to 2M_S$

In–state

Rubakov, Son, Tinyakov, 1992

Not semiclassical!

$E \sim m/g^2$

RST conjecture: $F(E)$ universal

Does not depend on details of the in–state

$$\mathcal{P}(E,N) = \sum_{i,f} \left| \langle i | \underbrace{\hat{P}_E \hat{P}_N}_{\text{Projectors}} \hat{S} | f \rangle \right|^2$$

- $N \gg 1 \Rightarrow$ semiclassical in–states
- $N \ll 1/g^2 \Rightarrow F(E,N) \approx F(E)$

Checks of universality:

- Field theory

  Tinyakov, 1991

  Mueller, 1992

- Toy QM models

  Bonini et al, 1999

  Levkov et al, 2009

$$\boxed{F(E) = \lim_{g^2 N \to 0} F(E,N)}$$

Semiclassical method

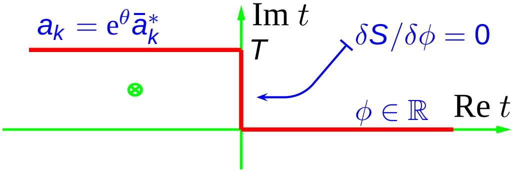
Rubakov, Son, Tinyakov, 1992

$$\mathcal{P}(E,N) = \sum_{i,f} \left| \underbrace{\langle i | \hat{P}_E \hat{P}_N}\, \underbrace{\hat{S} | f \rangle} \right|^2 = \int d\phi_i d\phi_f \left| \int d\phi_i' [d\phi]\, \mathrm{e}^{i(S+B)} \right|^2$$

$$\langle \phi_i | \hat{P}_E \hat{P}_N | \phi_i' \rangle = \mathrm{e}^{iB(\phi_i, \phi_i')} \qquad \langle \phi_i' | \hat{S} | \phi_f \rangle = \int [d\phi]\, \mathrm{e}^{iS[\phi]}$$

$$S \propto 1/g^2 \Rightarrow \text{Saddle–point method!}$$

$$\boxed{\phi(x,t) \in \mathbb{C}}$$

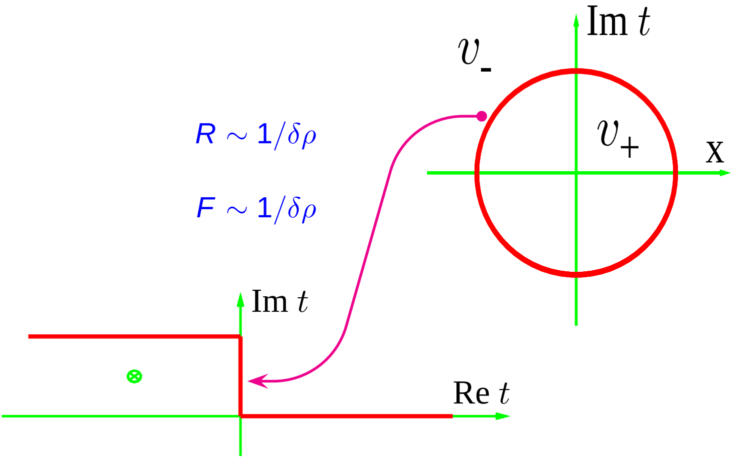


$$\mathcal{P} = A \cdot \mathrm{e}^{-F/g^2}$$
$$F(E,N) = 2g^2 \mathrm{Im}(S+B)$$

$E = N = 0$: Bounce

$R \sim 1/\delta\rho$

$F \sim 1/\delta\rho$



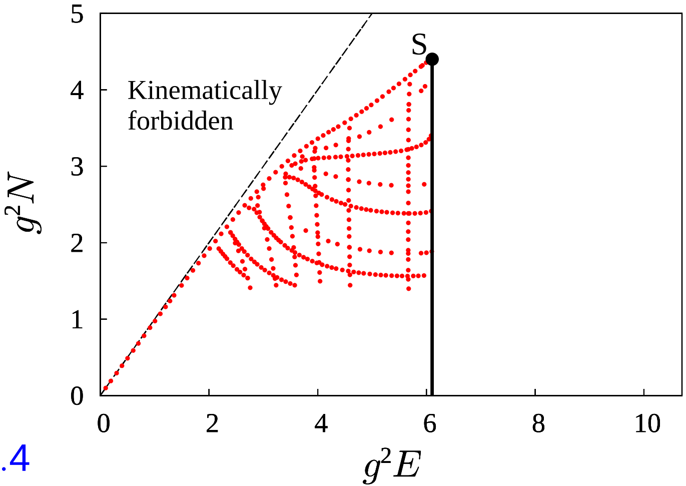
$\phi(x, t)$

Voloshin, Kobzarev, Okun, 1974

Coleman, 1977

Numerical solutions

$$V(\phi) = \frac{1}{2}(\phi+1)^2\left[1 - v\cdot f\left(\frac{\phi-1}{a}\right)\right], \qquad f(x) = \mathrm{e}^{-x^2}\left(1+x^3+x^5\right)$$

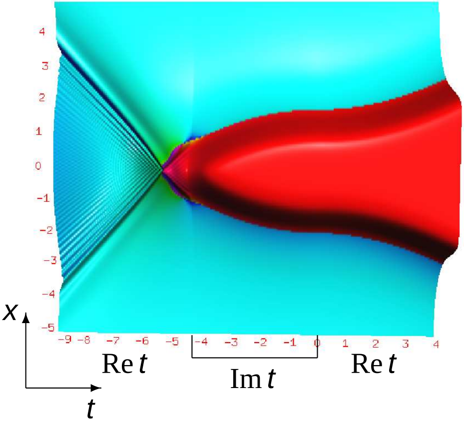


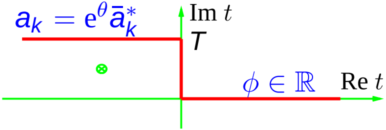
$\delta\rho = 0.4$

$E < 2M_S$, direct tunneling

$E \approx 5.48$ $(2M_S \approx 6.23)$

$N \approx 2.39$, $\delta\rho = 0.4$





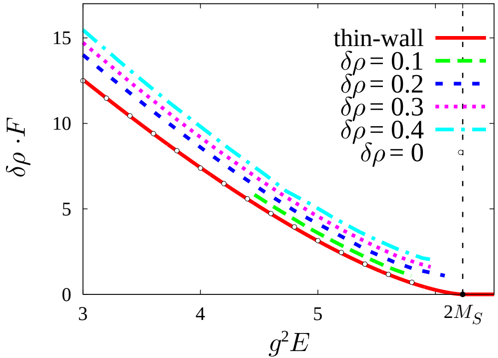
$\delta\rho \to 0$: thin–wall limit!

$$F(\delta\rho) = F_{-1}/\delta\rho + F_0 + O(\delta\rho)$$

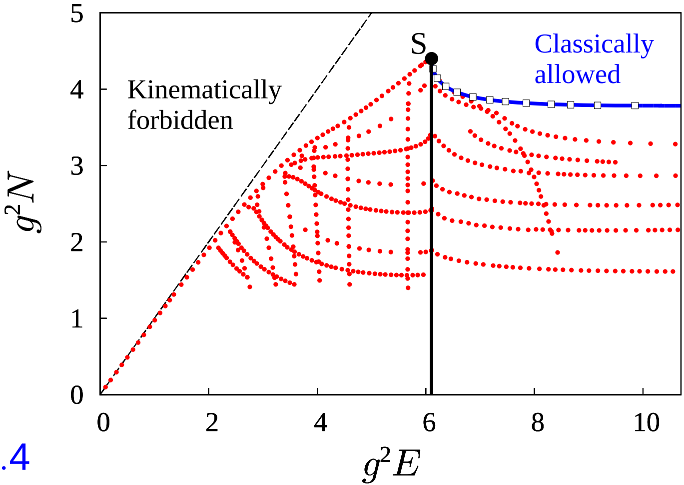
$$F_{-1}(E) = E_S^2 \left(\pi - 2\arcsin\frac{E}{2E_S} - \frac{E}{E_S}\sqrt{1 - \frac{E^2}{4E_S^2}} \right)$$

Voloshin, Selivanov, 1986

Rubakov et al, 1991



Going to $E > 2M_S$

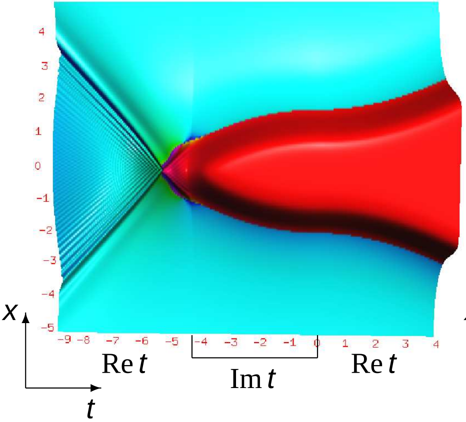


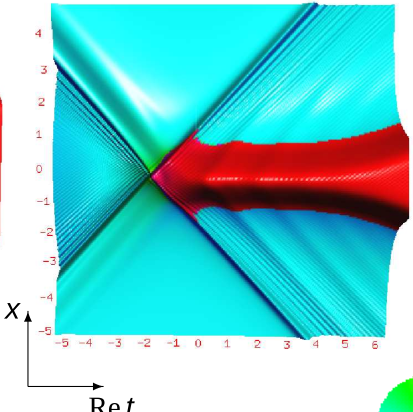
$\delta\rho = 0.4$

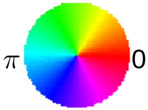
Going to $E > 2M_S$

$E \approx 5.48$ $(2M_S \approx 6.23)$ $E \approx 9.06$

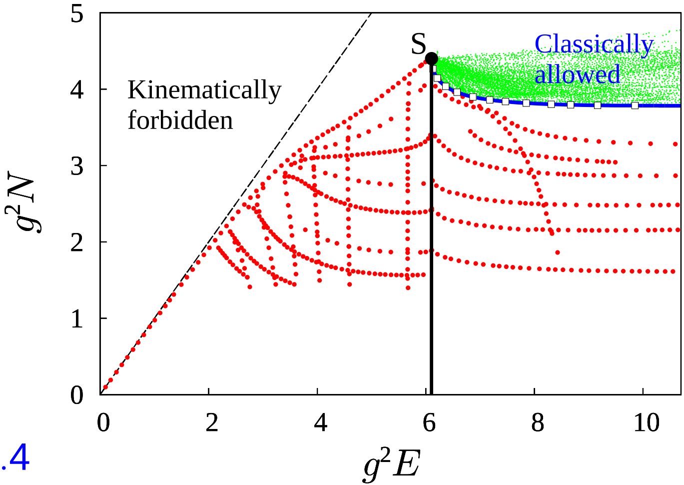
$N \approx 2.39$, $\delta\rho = 0.4$ $N \approx 2.47$, $\delta\rho = 0.4$





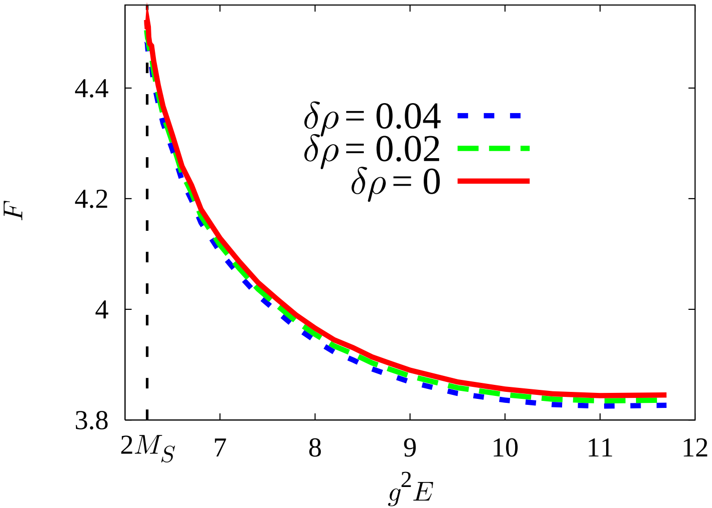


Classically allowed solutions



$\delta\rho = 0.4$

Result



$\delta\rho = 0.04$
$\delta\rho = 0.02$
$\delta\rho = 0$
F
4.4
4.2
4
3.8
$2M_S$
7
8
9
10
11
12
g^2E

Conclusions

- Method is applicable in $2D$ scalar field models.
- The probability of SA creation in high–energy collisions is

$$\mathcal{P}(E) \approx \mathrm{e}^{-F(E)/g^2}$$

Generalizations to other models?