

# Soliton-antisoliton production in particle collisions

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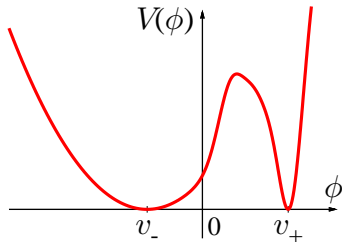
# Kink-like solitons in (1+1) scalar field theories

$\phi$  (1 + 1)

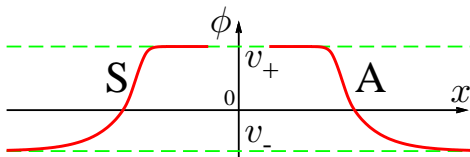
$\hbar = c = 1$

$$S = \frac{1}{g^2} \int dx dt \left[ (\partial_\mu \phi)^2 / 2 - V(\phi) \right]$$

$g$  — semiclassical parameter  
and coupling constant ( $\phi = g\phi'$ )



Soliton and Antisoliton



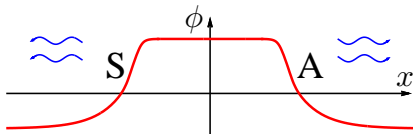
Properties:  $L_S \sim m^{-1}$   
 $M_S \sim m/g^2$   
 $m$  — mass scale of  $V(\phi)$

# Production of soliton-antisoliton pair

$$E \gtrsim 2M_S$$

$$\lambda \sim 1/E \sim g^2/m$$

Exponential suppression!



$$L_S \sim 1/m$$

$$\mathcal{P}(E) \approx A(E) \cdot e^{-F(E)/g^2}$$

- Coherent-state “estimate”

$$\bar{n}_S \sim M_S/m \sim 1/g^2$$

$$\langle 2|SA \rangle \sim \frac{\bar{n}_S^2}{2!} e^{-\bar{n}_S} \sim e^{-c/g^2}$$

*Drukier, Nussinov (1982)*

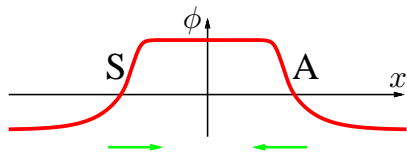
*Banks et al. (1990)*

- Unitarity arguments for multiparticle production

*Zakharov (1991)*

No reliable estimate of  $\mathcal{P}(E)$  so far!  
Aim: calculate semiclassically  $F(E)$ .

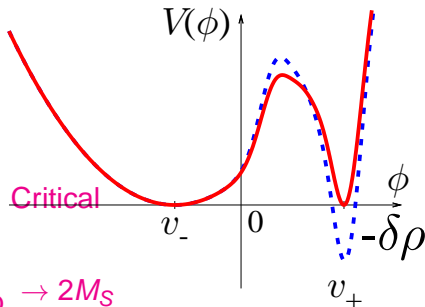
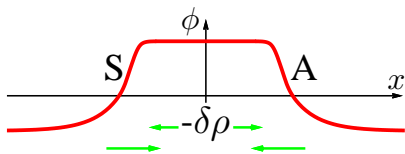
# HOWTO calculate: NOT tunneling?!



Many particles

Attraction!

Not a tunneling process? Introduce a potential barrier:



bubble  $\Leftrightarrow$  Barrier top

$\delta\rho \rightarrow 0$  : cr. bubble  $\rightarrow$  SA;  $E_{\text{cr.b.}} \rightarrow 2M_S$

# HOWTO calculate: In-state

Not semiclassical!

RST conjecture:  $F(E)$  universal

Does not depend on details of the in-state

$$\mathcal{P}(E, N) = \sum_{i,f} \left| \langle i | \underbrace{\hat{P}_E \hat{P}_N}_{\text{Projectors}} \hat{S} | f \rangle \right|^2$$

- $N \gg 1 \Rightarrow$  semiclassical in-states
- $N \ll 1/g^2 \Rightarrow F(E, N) \approx F(E)$

$$F(E) = \lim_{g^2 N \rightarrow 0} F(E, N)$$

Rubakov, Son, Tinyakov, 1992

$$E \sim m/g^2$$

Checks of universality:

- Field theory

*Tinyakov, 1991*

*Mueller, 1992*

- Toy QM models

*Bonini et al, 1999*

*Levkov et al, 2009*

## HOWTO calculate: Semiclassical method

Rubakov, Son, Tinyakov, 1992

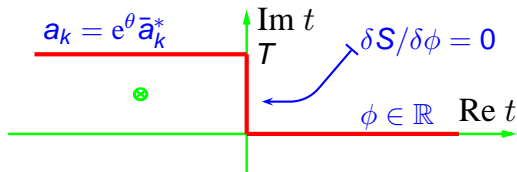
$$\mathcal{P}(E, N) = \sum_{i,f} \left| \langle i | \hat{P}_E \hat{P}_N \hat{S} | f \rangle \right|^2$$

$$\langle i | \hat{P}_E \hat{P}_N \hat{S} | f \rangle = \sum_{i'} \langle i | \hat{P}_E \hat{P}_N | i' \rangle \langle i' | \hat{S} | f \rangle = \int d\phi_{i'} e^{iB(\phi_i, \phi_{i'})} \int [d\phi] e^{iS[\phi]}$$

$$\mathcal{P}(E, N) = \int [d\phi][d\phi'] e^{iW}$$

$W \propto 1/g^2 \Rightarrow$  **Saddle-point method!**

Boundary value problem



$$\phi(\mathbf{x}, t) \in \mathbb{C}$$

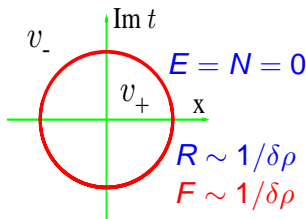
$$\mathcal{P} = A \cdot e^{-F/g^2}$$

$$\frac{1}{g^2} F(E, N) = 2\text{Im}S[\phi] - TE - \theta N$$

# Outline of the procedure

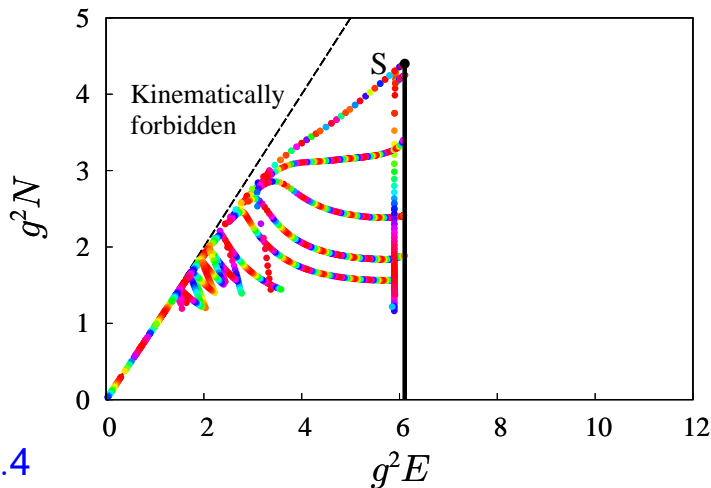
- 1 Fix small value  $\delta\rho$
- 2 Solve boundary value problem for fixed  $E$  and  $N$  and find  $F(E, N)$
- 3 Take the limit  $\lim_{g^2 N \rightarrow 0} F(E, N)$  at nonzero  $\delta\rho$
- 4 Take the limit  $\delta\rho \rightarrow 0$

Starting point, solution at  $E = 0$  and  $N = 0$  — bounce



# Numerical solutions

$$V(\phi) = \frac{1}{2}(\phi + 1)^2 \left[ 1 - v \cdot f\left(\frac{\phi-1}{a}\right) \right], \quad f(x) = e^{-x^2} (1 + x^3 + x^5)$$



$$\delta\rho = 0.4$$

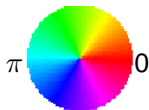
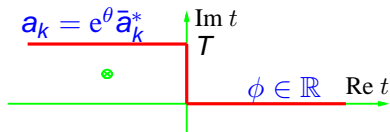
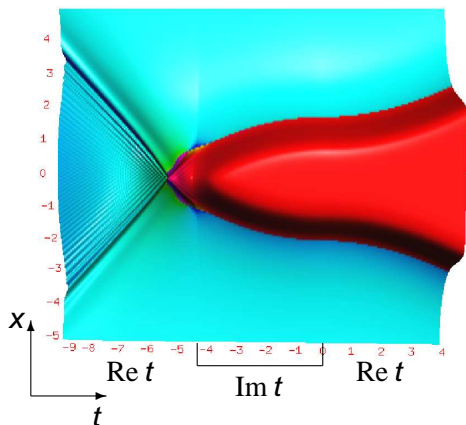


$E < 2M_S$ , direct tunneling

$E \approx 5.48$

$(2M_S \approx 6.23)$

$N \approx 2.39, \delta\rho = 0.4$



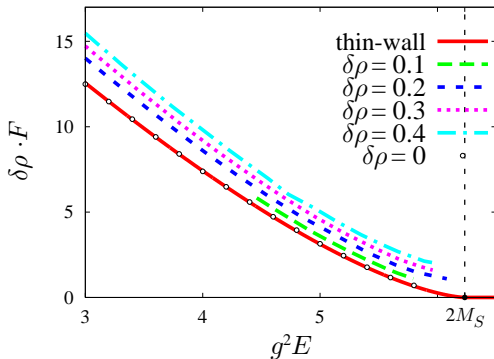
$\delta\rho \rightarrow 0$ : thin-wall limit!

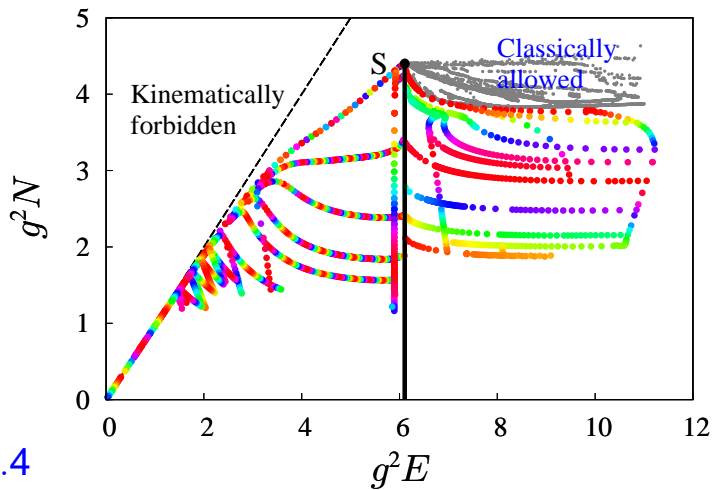
$$F(\delta\rho) = F_{-1}/\delta\rho + F_0 + O(\delta\rho)$$

$$F_{-1}(E, N) = E_S^2 \left( \pi - 2\arcsin\frac{E}{2E_S} - \frac{E}{E_S} \sqrt{1 - \frac{E^2}{4E_S^2}} \right)$$

*Voloshin, Selivanov, 1986*

*Rubakov et al, 1991*



Going to  $E > 2M_S$ 

$$\delta\rho = 0.4$$

Going to  $E > 2M_S$ 

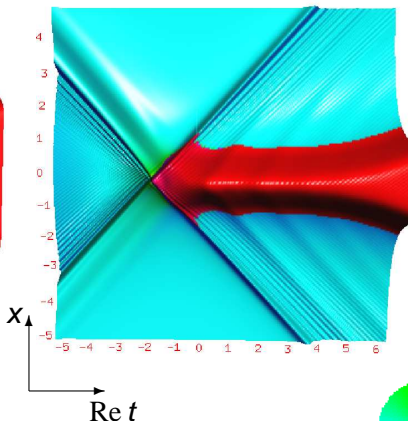
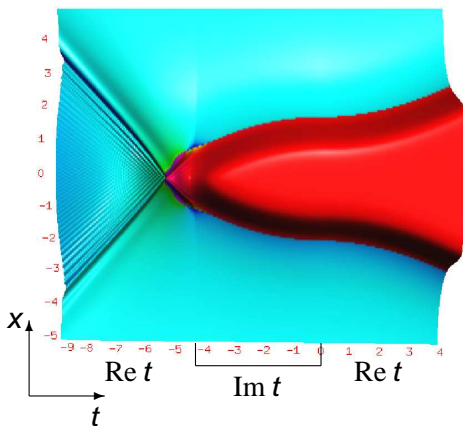
$E \approx 5.48$

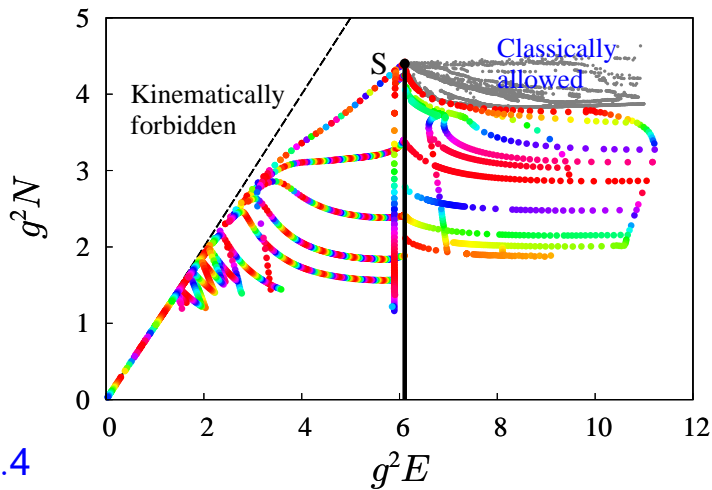
$N \approx 2.39, \delta\rho = 0.4$

$(2M_S \approx 6.23)$

$E \approx 9.06$

$N \approx 2.47, \delta\rho = 0.4$

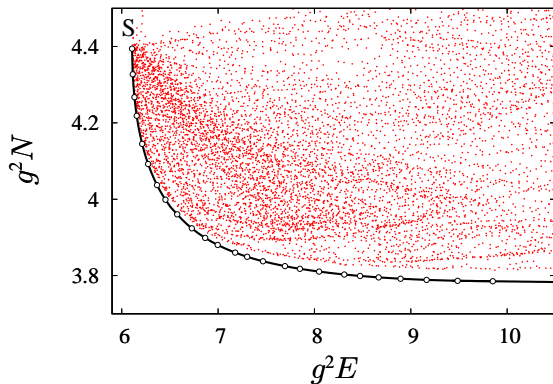


Extrapolating to  $N \rightarrow 0$ 

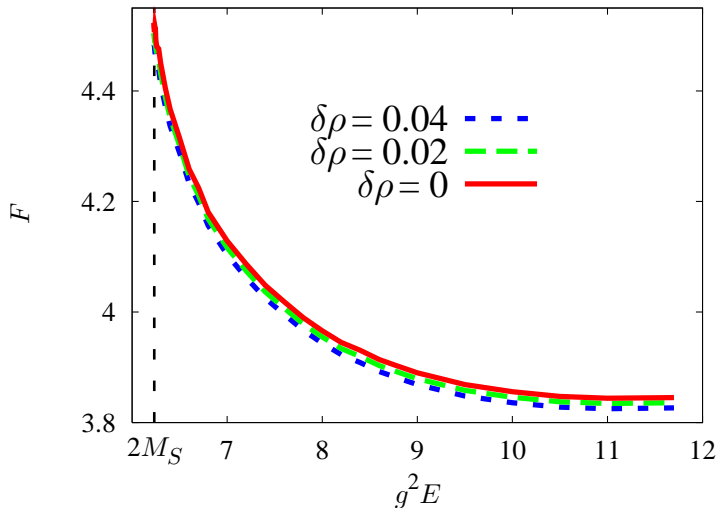
$$\delta\rho = 0.4$$

# Classically allowed region

- 1  $\phi_0(\mathbf{x}) = \phi_{cr.bubble}(\mathbf{x}) + \delta\phi(\mathbf{x})$
- 2 Evolution according to classical field equations
- 3 Find  $N$  in asymptotic future



## Result



# Conclusions

- Method is applicable in  $2D$  scalar field models.
- The probability of  $SA$  creation in high-energy collisions is

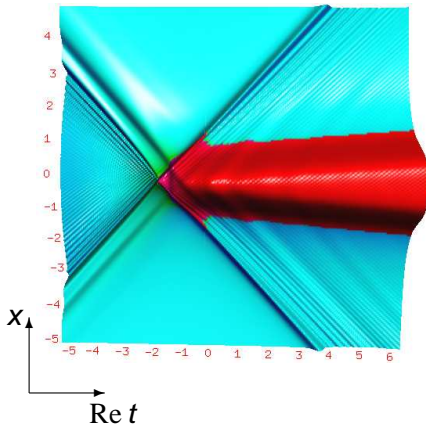
$$\mathcal{P}(E) \approx e^{-F(E)/g^2}$$

Generalizations to other models?



# Limit $\delta\rho \rightarrow 0$

$E \approx 8.95$   
 $N \approx 2.42, \delta\rho = 0.02$



$E \approx 9.06$   
 $N \approx 2.47, \delta\rho = 0.4$

