Bogomolnyi equations and a new exact solution of charge and Higgs bosons of SU(2)xU(1) gauge theory in superstrong magnetic fields.

A.M.Agalarov[1*] , A.K.Rabazanov[2]

[1]DSC RAN,Institute of Physics, Department of Theoretical Physics,

[2]Dagestan State University, Makhachkala, Russia

*E-mail: aglarow@mail.ru

Abstract

Bogomolnyi multicomponent equations and a new exact solution - mixed vacuum solitons, describing in magnetic field the behavior of bosonic condensates $SU(2)\otimes U(1)$-non-Abelian gauge theory are obtained. It is shown that in $2D$-condensate of mixed charged vector (W) and (ϕ) neutral scalar bosons the nontrivial effects of magnetic catalysis: phase separation at $H=H_{c1}$ and conversion of topological charge at $H=H_{c2}$ take place. The points of degeneracy (H_{c1}) and conversion (H_{c2}) in condensed mixture depend on the masses of vector W-and Higgs ϕ-bosons.

1.Introduction

The first collisions have been observed at the Large Hadron Collider(LHC),which is now the world`s highest energy accelerator. Two of the central goals of the LHC are to find the Higgs boson and to look for physics beyond the standard $SU(2)\otimes U(1)$-Weinberg-Salam theory. Very recently, many scenarios of Higgs boson with mass $m_H>2m_Z$ was considered[1,2]. Neverthelles, in the D-reduced supersymmetric theories (SUSY) it is also possible the Higgs boson with mass $m_H<2m_Z$[1]. Recently a such model in (1+1)D was considered in context of ultracold boson-fermion mixture of the Bose-Einstein condensate (BEC) in the laboratory magnetic and optical traps [3]. In this connection increasing interest is caused by the unusual phenomena and phase transitions induced by external magnetic field in (non-) Abelian condensates, in particular, effects of magnetic catalysis in the nonlinear field models [4,5] and in gauge theories with spontaneously broken symmetry (SBS) [6-10].

In this context, the unified $SU(2)\otimes U(1)$-Weinberg-Salam theory in an external magnetic field is of interest, where on the basis of constant solutions of the theory the features of vacuum [7] and phase transition are studied [6]. In the bosonic sector of Weinberg-Salam theory are also known topological solutions - sfaleronies and multisfaleronies [9,10] describing transitions of the type "vacuum-vacuum" and are of interest in the cosmological problem of baryon asymmetry in the Universe [10-12].

In the Grand unified theories (GUT) , as well as in the electroweak $SU(2)\otimes U(1)$-theory, the generation of masses of physical particles (fermions and bosons) occurs through the mechanism of spontaneous breaking of gauge symmetry [13,14]. Despite the "universality" in the nature of Higgs mechanism of the SBS, till now, remains opened a question about the mass of Higgs particle, about the topology of the vacuum states of unified theories, etc. Theories, where in the role of Higgs elementary scalar the compound scalars are suggested [14]. Such examples are known in condensed matter physics, in particular, in the Bardeen-Cooper-Schrieffer theory of superconductivity (SC), where the Cooper pair (boson) is a composite scalar (spin $s=0$), and the mechanism of SC also serves as a Higgs phenomenon that occurs due to the interaction of electrons with phonons of the lattice. If there is an analogy with the SC and ferromagnetism (FM), then symmetry, which at low magnetic fields and temperatures ($H,T<H_C,T_C$) has been spontaneously broken, should be restored at high H and T $(>H_c,T_c)$. In [7] the $2D$-reduction of $SU(2)\otimes U(1)$-Lagrangian is considered and for the first time, by the method of trial orbits [8], some partial solutions (vacuum solitons) of field equations of the 2-nd order are found.

In this paper, the behavior of bosonic Weinberg-Salam-Higgs condensates in a magnetic field is studied by the generalized method of Bogomolnyi [17] and instead of field equations of the 2-nd order equations of the 1-st order are obtained. It is shown that the equations are isomorphic on the class of solutions of mixed vector solitons [18], stable on Bogomolnyi bounder. Significant physical result of a new approach lies in the fact that in the $2D$-condensate of mixed charged vector ($W^{\pm}$) and a neutral scalar (ϕ) of bosons there are nontrivial effects induced by external magnetic field H: separation of phases at

$H = H_{c1}$ and conversion of the topological charge of solitons at $H = H_{c2}$. Points of degeneracy H_{c1} and conversion H_{c2} in the condensate mixture depend on the masses of the W-vector and ϕ-Higgs bosons. It is shown that the proposed method is regular and allows you to find stable soliton solutions in multicomponent systems at known superpotential of interacting nonlinear fields.

2. $(1+1)$D-reduction SU(2)⊗U(1) theory in a magnetic field

Bosonic sector of the electroweak $SU(2)\otimes U(1)$-theory in $(3+1)D$ space-time is described by the action [9,12]

$$S = \int d^4x \left\{ -\frac{1}{4} Tr\, G^2 + \frac{1}{2}|D\varphi|^2 - V(\varphi) \right\}. \tag{1}$$

Here $V(\varphi) = \lambda \left(|\varphi|^2 - \varphi_0^2 \right)^2$ -potential of scalar fields

$$G^2 = g^{\mu\nu} g^{\rho\sigma} G^A_{\mu\rho} G^A_{\nu\sigma}, \qquad A = (0, \alpha) = 0,1,2,3,$$

$$G_{\mu\nu}{}^0 = \partial_\mu A^\alpha_\nu - \partial_\nu A^0_\mu \ \left(= F_{\mu\nu}\right); \quad G^\alpha_{\mu\nu} = \partial_\mu V^\alpha_\nu - \partial_\nu V^\alpha_\mu + g\varepsilon^{\alpha\beta\gamma} V^\beta_\mu V^\gamma_\nu$$

where $D_\mu \varphi = \partial_\mu \varphi - \frac{1}{2}\left(g\sigma^\alpha V^\alpha_\mu + g' A^0_\mu \right)\varphi$ - covariant derivative, σ^α - Pauli matrices; V^α_μ / A^0_μ and g/g' is a $SU(2)\otimes U(1)$-gauge field and coupling constant, respectively; $g \sin\theta_W = e$ - electric charge, θ_W-Weinberg angle; $g^{\mu\nu}$-Minkowski tensor with signature $(1,-1,-1,,-1)$. Gauge-equivalent minima of the potential $V(\varphi)$ with vacuum average $\langle\varphi\rangle = \phi_0 (0.1)^t$ lead to the Higgs mechanism for the SBS: $SU(2) \to U(1)$. In the unitary gauge, in the representation of physical fields $W^\pm_\mu = \left(V^1_\mu \pm i V^2_\mu \right)/\sqrt{2}$, $\left(A_\mu, Z_\mu \right)^t = U(\theta_\mu)\left(A_\mu{}^0, U_\mu{}^3 \right)^t$, when the external magnetic field $A^{\mathbf{ext}}_\mu = A_\mu$, and a field of a neutral Z^0 - boson $Z_\mu = 0$, action (1) takes the form

$$S = \int d^4x \left\{ -\frac{1}{2} H^*_{\mu\nu} H^{\mu\nu} + m^2(\phi) W^*_\mu W^\mu + \frac{1}{2} e f^{\mu\nu} d_{\mu\nu} - \frac{1}{4} g^2 d_{\mu\nu} d^{\mu\nu} + \frac{1}{2} \partial_\mu \phi\, \partial^\mu \phi - V(\phi) \right\}. \tag{2}$$

Here

$$H_{\mu\nu} = D_\mu W_\nu - D_\nu W_\mu\ , \quad D_\mu = \partial_\mu + ieA_\mu, \quad f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

$$d_{\mu\nu} = i\left(W_\mu^* W_\nu - W_\nu^* W_\mu\right), \quad V(\phi) = \lambda\left(\phi^2 - \phi_0^2\right)^2, \quad m(\phi) = g\phi/\sqrt{2};$$

where $A_\mu \left(= A_\mu^{\mathbf{ext}}\right)$-vector potential of the external field, which we will then choose in a gauge $A_\mu = (0, 0, -Hx, 0)$; $W_\mu(x)$– -vector field, $\phi(x)$- a scalar Higgs field; g, λ, ϕ_0-constants determining in the theory of SBS mass W- boson $m_W = g\phi_0/\sqrt{2}$ and the Higgs particle $m_H = 2\sqrt{\lambda}\phi_0$.

We consider, that the field ϕ is not dependent on the coordinates of x and y, and the components of the wave function of W-boson $W_0 = W_3 = 0$, $W_2 = iW_1$. It is easy to show that such a set of components W_μ in the linearized theory corresponds to the ground state of W-boson in a constant magnetic field H. The wave function W_μ meets the conditions

$$D_\mu W^\mu = 0 \tag{3}$$

$$\left[\partial_x + i\left(\partial_y - ieHx\right)\right]W^1 = 0. \tag{4}$$

Equation (4) is equivalent to $\alpha W_1 = 0$, where α-annihilation operator. Thus, solution (4) actually corresponds to the ground state of W-boson, and the spatial dependence of the wave function has the form $W_1 \equiv \psi$,

$$\psi = (eH/\pi)^{1/4} L^{-1} \exp\left[ip_3 z + ip_2 y - \left(eH\left(x - p_2/eH\right)^2\right)/2\right], \tag{5}$$

where L- normalization length. Solution (5) represents localized in coordinate x gauss with center in a point $x_0 = p_2/eH$ and width of order $(eH)^{-1/2}$.

Taking into account (3) and (4) averages $|W_1|^2$, $|W_1|^4$ for the x, y-coordinates can be represented as [5,7]

$$\int dxdy\, |W_1|^2 = \mu\theta\, |\tilde{\psi}(t, z)|^2, \qquad \int dxdy\, |W_1|^4 = \mu^2\theta\, |\tilde{\psi}(t, z)|^4, \tag{6}$$

where $\mu = H/2e$ and θ– the positive parameter whose value will be defined below.

Averaging in the transverse to the magnetic field plane of coordinates x, y, we get $(1+1)D$-reduction of action (2) for a mixture of electrically charged ψ and neutral ϕ -bosons in an external magnetic field H:

$$S_M = \int dt\, dz \left[|\dot{\psi}|^2 - |\psi'|^2 + \frac{1}{2}\left(\dot{\phi}^2 - \phi'^2\right) - U(\phi, \psi)\right], \tag{7}$$

$$2U = g^2\theta^{-1}|\psi|^4 + \left(g^2\phi^2 - 2eH\right)|\psi|^2 + 2\lambda\left(\phi^2 - \phi_0{}^2\right)^2 .$$

Here the *dot/dot* denotes differentiation with respect to t/z, and $|\psi|^2 = eH\theta g^{-2}\left|\tilde{\psi}\ (t,z)\right|^2$.

Variational principle $\delta S_M = 0$ leads to the Lorentz-invariant equations of motion ($\Box_2 = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}$):

$$\left[\Box_2 + 4\lambda\left(\phi^2 - \phi_0^2\right) + |g\psi|^2\right]\phi = 0,$$

$$\left[\Box_2 - eH + \left(g^2/2\right)\phi^2 + \theta^{-1}|g\psi|^2\right]\psi = 0. \qquad (8)$$

It should be noted that the energy of a massive vector boson W with a charge (e) in a constant homogeneous magnetic field, H [19] $E^* = \left[m_W^2 + (2n + 1 - 2\sigma)|e|H + p_z^2\right]^{1/2}$, $\sigma = -1, 0, +1$, in the ground state $(n = 0 = p_z,\ \sigma = 1)$ at $H > m_W^2/e$ becomes purely imaginary and corresponds to an unstable mode of W-boson. It is in these circumstances a consistent nonperturbative approach (without the perturbation theory) is required.

Multicomponent system of nonlinear field equations (8), in general position, is not integrable and for arbitrary values of the parameters of the system the regular analytical method of the exact solution does not exist [18, 20]. The peculiarity of system (8) lies in the fact that along with a discrete P-symmetry $(\phi = -\phi)$, it admits a global $U(1)$-symmetry ($\psi \to \psi \exp(i\,\alpha)$). Hereafter it will be shown that the invariance of the action (7) about discrete P- and charge $U(1)$- symmetries allows us to construct fields superpotential, which leads to Bogomolnyi equations, isomorphic system of nonlinear equations of the 2-nd order (8).

3. P-odd superpotentials and Bogomolnyi equation

By virtue of Lorentz invariance the dynamic solutions of system (8) are associated with the static - coordinate transformation $z \to (z - vt)/\left(1 - v^2\right)^{1/2}$, where v-speed of soliton ($c = 1$). Since the static solutions of $(1+1)D$-system (8) with finite energy (E) are the instantons of corresponding $1D$-Euclidean theory of field, we obtain the energy $E \equiv S_E$ by analytic continuation in the Minkowski space-time $S_E = -i\, S_M(t \to -i\,\tau)$. Besides we will consider, that due to global symmetry $U(1)$: $\psi \to e^{i\,\alpha}\psi$ there is a current $\partial_\mu j^\mu = 0$ and the corresponding invariant - electric charge $q = \int j_0 dz$ in the system, where $j_\mu = i\left(\psi^*\psi,_\mu - \psi\psi^*,_\mu\right)$. Since the charge ($q$) depends linearly on the $\psi,_t$, then the nontrivial solutions with finite energy and with $q \neq 0$ should also be functions of time t. For the field configurations lying in the vicinity of $\min\{E = S_E\}$ it can be shown that $\psi \sim \exp(-i\Omega t)$. Certainly, that $U(1)$-rotation (Ω) is "internal" and occurs in charging q-space. Thus accounting of $U(1)$-symmetry of the system (8), along with Lorentz invariance allows to choose solutions and scale of fields in the form: $\phi(t,z) = \phi_0\phi_1(\tilde z)$, $\psi(t,z) = \left(\phi_0/\sqrt{2}\right)\phi_2(\tilde z)\,e^{-i\,\Omega t}$, where ϕ_1 and ϕ_2 - real-valued functions of the dimensionless variable $\tilde z = z(2\lambda)^{1/2}\phi_0$.

Euclidean action for a mixture of neutral (ϕ_1) and charged (ϕ_2) bosons in a magnetic field (H) takes the form (further the tilde sign is omitted):

$$S_E = C_W \int_{-\infty}^{\infty} dz \left\{ \frac{1}{2}\sum_j \left(\phi_{j,z}\right)^2 + U_{\text{eff}}\left(\phi_1, \phi_2\right) \right\}, \tag{9a}$$

$$U_{\text{eff}} = \frac{1}{2}\sum_j \left[-2c_j\phi_j^2 + \left(\delta_{jk} + \sum_{k\neq j}\right)\alpha_{jk}\phi_j^{\,2}\phi_k^{\,2} + 1 \right]. \tag{9b}$$

Here

$$C_W = \left(\frac{m_H^2 m_W}{2e\sqrt{\lambda}}\right)\sin\theta_W\,, \quad c_1 = 1, \quad c_2 = m_H^{-2}\left(eH + \Omega^2\right)$$

$$\alpha_{11} = 1, \quad \alpha_{12} = m_H^{-2} m_W^2 = \alpha_{21}, \quad \alpha_{22} = \theta^{-1}\alpha_{12}\,,$$

where $U(1)$ - frequency Ω is associated with the charge $q = \int j_0 dz$ by ratio:

$$\Omega^{-1} q = \left(m_W \sin\theta_W / e\sqrt{\lambda}\right)\int dz\,\phi_2^2(z)\,. \tag{10}$$

From (9) we obtain a system of coupled quasi-static nonlinear equations of the 2-nd order [20]:

$$\phi_{j,zz} + 2\left(c_j - \sum_k \alpha_{jk}\phi_k^2\right)\phi_j = 0\,; \qquad j,k = 1,2\,. \tag{11}$$

It should be noted that the quasi-static nature of the system (11) is due to the charge degrees of freedom of the initial model (7), whose contribution to the kinetic energy of $\sim \Omega^2$ leads to deformation of potential $U_{\text{eff}} = U - \frac{1}{2}\Omega^2\phi_2^2(z)$. The field equations arising from (8) with $\Box_2 \to -\partial_{zz}$ and at the actual field $\psi(t,z)$ (i.e. in the absence of $U(1)$ - symmetry) are static and equivalent to system (11) with $\Omega = 0$.

For arbitrary values of the parameters c_j and α_{jk} system (11), as well as system (8), does not allow exact integration [18]. However, analysis of the problem of extremal of the deformed potential $U_{\text{eff}}(\phi_1, \phi_2)$ shows that in the points of a local minimum:

$$A\left(\phi_{1a}^1, \phi_{1a}^2\right) = \left(0, c_2\alpha_{22}^{-1}\right), \text{ with } eH > \theta^{-1}m_H^2 - \Omega^2;$$

$$B\left(\phi_{1b}^2, \phi_{2b}^2\right) = \left(\alpha_{12}^2 - \alpha_{22}\right)^{-1}\left(\alpha_{12}c_2 - \alpha_{22}, \alpha_{12} - c_2\right), \text{ with } eH > m_W^2 - \Omega^2.$$

potential accepts the values:

$$U_{\text{eff}}^A\left(0, \pm\phi_{2\alpha}\right) = \frac{1}{2}\left[1 - \left(H/H_c\right)^2\right], \tag{12}$$

$$U_{\text{eff}}^B\left(\pm\phi_{1b}, \pm\phi_{2b}\right) = \left[4m_W^2\, m_H^2\left(\theta_c - \theta^{-1}\right)\right]^{-1}\left(eH + \Omega^2 - m_W^2\right)^2. \tag{13}$$

As it can be seen in the neighborhood of points A and B, interaction of fields ϕ_1 and ϕ_2 (order parameters) in the presence of an external magnetic field H has variable character (attraction$\leftrightarrow$repulsion) with $H \gtrless H_c$ and $\theta \gtrless \theta_c$, where $H_c = \left(\theta^{-1/2}m_W m_H - \Omega^2\right)e^{-1}$, $\theta_c = m_W^2 m_H^{-2}$. Further, it will be shown that with $\theta \cdot \theta_c \neq 1$ such class of solutions does exist if a constant of interaction of fields ϕ_1 and ϕ_2 has the form $\alpha_{12} = c_1c_2 + \varepsilon_m$, where values $\varepsilon_m(m = 1, 2)$ strictly depend on the conditions of spontaneous violations of P- symmetry of phases: $\phi_j = -\phi_j$, $j = 1, 2$..

We introduce P - odd superpotential

$$P^m\left(\phi_1, \phi_2\right) = \sum_{k=1}\phi_m^{2k-1}\left(\alpha_m\phi_1^{2k} + \beta_m\phi_2^{2k} - 1\right), \tag{15}$$

with the obvious property of $P^m\left(\phi_j, -\phi_m\right) = -P^m\left(\phi_j, \phi_m\right)$ and represent the Euclidean action (9) in the form

$$S_E = S_E^P + S_E^B, \tag{16}$$

$$S_E^P = \frac{1}{2} C_W \int_{-\infty}^{\infty} dz \left\{ \sum_j \left(\phi_{j,z} \pm P,_{\phi_j}^m \right)^2 + 2U_{\text{eff}}^{(m)} - \sum_j \left(P,_{\phi_j}^m \right)^2 \right\}, \tag{17}$$

$$S_E^B = C_W \int_{-\infty}^{\infty} dz \sum_j \phi_{j,z} P,_{\phi_j}^m , \qquad j = 1, 2 . \tag{18}$$

It follows that the Euclidean action (16) satisfies the inequality

$$S_E \ge S_E^B , \tag{19}$$

if the effective potential (9) admits the representation

$$U_{\text{eff}}^{(m)}(\phi_1 \phi_2) = \frac{1}{2} \sum_j \left(P,_{\phi_j}^m \right)^2 . \tag{20}$$

Obviously, in these conditions the Bogomolnyi boundary

$$S_E^B = C_W \left| \pm P^m (\phi_1, \phi_2) \Big|_{-\infty}^{\infty} \right| \equiv \min\{E\} \tag{21}$$

is achieved on the field configurations $\phi_j(z)$, satisfying the system of the differential equations of the 1-st order:

$$\phi_{j,z} = \pm P,_{\phi_j}^m (\phi_1, \phi_2), \qquad j = 1, 2 . \tag{22}$$

Since $U_{\text{eff}}^{(m)}$ - is a polynom<u>ial</u> not higher than the fourth order of ϕ_1 and ϕ_2, then in (15) it is sufficient to limit to members of $k = 1$. In the representation (16-20) it is remarkable that the equation of the connection (20) allows to determine not only the parameters of α_m and β_m of superpotential (15), but at the same time, to obtain get strict ratios between the coefficients of c_j and α_{jk}, found above on the physical level.

From (16), (20) taking into account (15) we find that the following P - odd superpotentials are admissible:

$$P^m(\phi_1, \phi_2) = \begin{cases} \frac{1}{3} c_1 \phi_1^3 + c_2 \phi_1 \phi_2^2 - \phi_1, & m = 1 \\ c_1 \phi_1^2 \phi_2 + \frac{1}{3} c_2 \phi_2^3 - \phi_2, & m = 2 \end{cases} . \tag{23}$$

It is easy to see from (23) and (20) that the violation of P- symmetry in the phases $\phi_1(m = 1)$ and $\phi_2(m = 2)$ takes place at values of $\varepsilon_1 = 2c_2{}^2$ and $\varepsilon_2 = 2c_1{}^2$, respectively. Consequently, the system of Bogomolnyi equations (22) with P-odd superpotential (23), for the found above relationships of the parameters c_j, α_{jk}, is isomorphic to the system of quasi-static nonlinear equations of the 2-nd order (11) on the class of exact solutions with finite action.

5.Conversion of solitons and magnetic separation of phases in the condensate mixture of charged vector and neutral Higgs bosons.

Define the possible manifestation of non-trivial effects in system (9) as a change of the character of exact solutions of Bogomolnyi equations at some critical value $H = H_{cj}$. It is easy to notice that the effect of the $U(1)$-charge $q \sim \Omega$ presence already manifests itself in the system at the level of constant (vacuum) solutions (12-13). It consists, of the fact that the phase transition occurs in the field of $H = H_c$, certainly less ($H = H_c^* - \Omega^2/e$), than in the absence of charge degrees of freedom ($H_c^* = m_W m_H / e\sqrt{\theta}$ [6,7]). Effects of such kind of "catalysis" in the nonperturbative sector of solitons are non-trivial and their non-linear character essentially depends on the coupling constant of fields ϕ_1 and ϕ_2.

In the space of controlling parameters $\{eH,\ \Omega^2,\ m_W^2,\ m_H^2,\ \theta\}$ we will assume mass of W - boson (m_W) and $U(1)$-frequency Ω to be fixed, and the free parameter θ we will define as the function of $\theta = \theta(\theta_c)$, where $\theta_c = m_W^2 / m_H^2$. The behavior of the condensate mixture of neutral ϕ_1 - and charged ϕ_2 - bosons now depends on the value of magnetic field H and the condition on θ.

P- odd superpotential (23) with $m = 1$ corresponds to the case of constant cross-modulation $\alpha_{12} = c_1 \cdot c_2 + 2c_2^2$ of fields ϕ_1 and ϕ_2. The condition on θ takes the form $\theta = 2\left(1 - \theta_c^{-1}\theta^*\right)^{-1}$, where $4\theta^* = (1 + 8\theta_c)^{1/2} - 1$, and the critical value of magnetic field equals $H_{c0} = \left(m_H^2/2\right)\left[\theta^* - (\Omega/m_H)^2\right]$. Configuration of fields ϕ_1 and ϕ_2:

$$\phi_1(z) = \mathrm{th}\left[2^{1/2} m_H^{-2}\left(eH + \Omega^2\right)(z - z_0)\right],$$

$$\phi_2(z) = \left(\left(eH + \Omega^2\right)^{-1} m_H^2 - 2\right)^{1/2} \mathrm{sech}\left[2^{1/2} m_H^{-2}\left(eH + \Omega^2\right)(z - z_0)\right], \qquad (24)$$

is an exact solution of system of Bogomolnyi equations (22). The constant of integration Z_0 - is an arbitrary point in space, determining the center of the soliton. In the condensate mixture (24) ϕ_1 is a phase of broken P-symmetry with topological charge

$$Q_{(1)} = \int_{-\infty}^{\infty} dz\, J^0_{(1)} = \left[\phi_1(\infty) - \phi_1(-\infty)\right], \tag{25}$$

where the density of the topological current is

$$J^{\mu}_{(1)} = \varepsilon^{\mu\nu}\partial_\nu \phi_1 \quad , \quad \partial_\mu J^{\mu}_{(1)} = 0\,. \tag{26}$$

From (21) it follows, that the energy of the system (9) on the configuration of fields (24) is proportional to the topological charge $Q_{(1)}$ and is equal to $E_{(1)} = \frac{2}{3} C_W \left|Q_{(1)}\right| = \frac{4}{3} C_W$. At the same time from (24) and (10) it is clear that the phase of ϕ_2 with $U(1)$ - charge:

$$q = \frac{2\sqrt{2} C_W \Omega \left[m_H^2 - 2\left(eH + \Omega^2\right)\right]}{m_H \left(eH + \Omega^2\right)^2} \tag{27}$$

disappears at values of the magnetic field $H \le \left(m_H^2/e\right)\cdot\left(\left(0{,}5 - \Omega^2\right)/m_H^2\right)$.

In the case of superpotential (23) with $m = 2$ the constant of interaction of fields ϕ_1 and ϕ_2 is $\alpha_{12} = c_1 \cdot c_2 + 2c_2^2$. The condition on θ has the form $\theta = \left(\theta_c - 2\right)^{-2}\theta_c$. The critical value of the magnetic field is $H_{c2} = \left(m_H^2/2\right)\cdot\left[\theta_c - \left(\Omega^2/m_H^2\right) - 2\right]$. Exact solutions of the system of Bogomolnyi equations (22) now correspond to the new configuration of fields:

$$\phi_1(z) = \left(1 - 2m_H^2\left(eH + \Omega^2\right)^{-1}\right)^{1/2} \operatorname{sech}\left[\sqrt{2} m_H^2 \left(eH + \Omega^2\right)^{-1/2}\left(z - z_0\right)\right],$$

$$\phi_2(z) = \left(eH + \Omega^2\right)^{-1/2} m_H \operatorname{th}\left[\sqrt{2} m_H^2 \left(eH + \Omega^2\right)^{-1/2}\left(z - z_0\right)\right]. \tag{28}$$

From (24) and (28) it is clear that the magnetic field value $H = H_{c2}$ is critical. At the point $H = H_{c2}$ the topological and charging phases change "roles": $\phi_1 \leftrightarrow \phi_2$. This nontrivial effect of "conversion" of solitons can be interpreted differently - as the transformation "of the electric charge into topological" and vice versa. Since the energy difference in the solutions (24) and (28) is equal to $\Delta E = E_{(1)} - E_{(2)} = \frac{4}{3} C_W \left(1 - \left(eH + \Omega^2\right)^{-1/2} m_H\right)$, then with $H < \left(m_H^2/e\right)\left(1 - \Omega^2/m_H^2\right)$ the configuration of fields (24) is energetically more favourable in the system (9). Actually, steady solutions (28) exist in the region of more stronger magnetic fields, $H > \left(m_H^2/e\right)\left(1-\Omega^2/m_H^2\right)$, than the solutions (24).

Let’s consider the behavior of the phases ϕ_1 and ϕ_2 in the special case when the parameters of P - odd-potential (23) are equal to $c_1 = c_2 = 1$. The constant of interaction of the fields ϕ_1 and ϕ_2 accepts the value $\alpha_{12} = 3$. The condition on the θ is $\theta = \theta_c$ and critical value of the magnetic field is $H_{c1} = e^{-1}\left(m_H^2 - \Omega^2\right)$. In the plane of the coordinate of fields $\phi^{\pm} = \phi_1 \pm \phi_2$ at $c_1 = c_2 = 1$ potentials (23) and (9) appear to be degenerated:

$$P^{1,2}\left(\phi^+,\phi^-\right) = \frac{1}{2}\left(P^+ \pm P^-\right), \qquad P^{\pm} = \phi^{\pm}\left[\frac{1}{3}\left(\phi^{\pm}\right)^2 - 1\right]; \tag{29}$$

$$U_{\text{eff}}\left(\phi^+,\phi^-\right) = U^+_{\text{eff}} + U^-_{\text{eff}}, \qquad U^{\pm}_{\text{eff}} = \frac{1}{4}\left[\left(\phi^{\pm}\right)^2 - 1\right]^2, \tag{30}$$

and describe two independent phases of ϕ^+ and ϕ^-. A consequence of the degeneracy of the condensate mixture is that Bogomolnyi equations (22) factorize and take the form

$$\phi,^{\pm}_z = 1 - \left(\phi^{\pm}\right)^2, \tag{31}$$

and the system of coupled nonlinear equations of the 2-nd order (11) splits into two independent subsystems (nonlinear Schrodinger equations [20])

$$\phi,^{\pm}_{zz} + 2\left[1 - \left(\phi^{\pm}\right)^2\right]\phi^{\pm} = 0\,. \tag{32}$$

By direct substitution of (31) to (32) **we** can verify the equivalence of them. Hopf-Cole transformation $\phi^{\pm} = \partial_z\left[\ln \mathrm{f}(z)\right]$ is linearizing for nonlinear equations of the first order (31), where function $f(z) = \mathrm{ch}\, z$ is the solution of linear equations $f,_{zz} = f$. Hence it follows that the configuration of fields

$$\phi^{\pm}\left(z^{\sim}\right) = \pm\,\mathrm{th}\left[m_H\left(z^{\sim} - z_0^{\sim}\right)/\sqrt{2}\right], \tag{33}$$

is the exact solution (topological solitons) of nonlinear Bogomolnyi equations (31) and at the same time, "dark" soliton of stationary nonlinear Schrödinger equations (32).

6. Conclusion

1). The multi- component Bogomolnyi equations (22) on the class of exact solutions of mixed solitons (24), (28) are isomorphic to the system of quasi-static nonlinear Schrödinger equations (11) and the reverse does not always take place.

2). The solution (33) describes the state of phase separation arising in the condensate mixture of charged and neutral bosons at $H = H_{c1}$. Phase separation in condensate mixture (7) can be defined as a state of coexistence of two independent symmetric topological phases $\phi_1 = \phi_2 = 1/2\,\phi^{\pm}$ of broken P-symmetry.

3). The found in [18] nontrivial class of exact solutions of vector solitons of integrable compact $U(m)$ - nonlinear Schrödinger model possesses the same switching-effects, as mixed solitons (24) and (28) of Bogomolnyi equations (22). The difference is that in the first case they are true solitons and switching effects are due to their dynamic-topological nature, in the latter - they are "solitary waves" and the effect of conversion $\phi_1 \leftrightarrow \phi_2$ (charge vector bosonic |Soliton> $\Leftrightarrow$ neutral Higgs bosonic |Soliton>) is induced by an external superstrong magnetic field.

The author (A.M.) express their gratitude to A.V. Borisov, D.V. Gal'tsov, V. Ch. Zhukovsky, V.R. Khalilov (Dept. of TP, Lomonosov Moscow State University), V.N. Rodionov (JINR, Dubna), V.E.Zakharov (Landay ITP RAN), I.K. Kamilov (IP, Dagestan Scientific Center of RAN) for the discussion and useful comments. This work was supported by Grant Hill -2027.2009.

References

1.V.Barger, P.Langacker and H.S.Lee, Phys.Rev.Lett.**103**, 251802 (2009).

2. P.Langacker, Rev.Mod.Phys. **81**, 1199 (2009).

3. L.Pitaevski, UFN,**176**, 345 (2006).

4. V.Zhukovski et.al. JETP Lett.**74**, 595 (2001).

5. H.Nielson and P.Olesen, Nuclear Physics **B160**, 380 (1979).

6. V.Scalozub, Sov.Nucl.Phys. **37**, 474 1983

7. V.Khalilov, Sov.Nucl.Phys. **47**, 1336, 1988.

8. R.Rajaraman, Solitons and Instantons in QFT, M.,Mir,(1985)

9. B.Kleinhaus and J.Kunz, Phys.Rev.D, 35 (1994).

10. M.Volkov and D.Gal`tsov, Phys.Rep. **319**, 1 (1999).

11. V.Rubakov and M.Schaposnikov,UFN,**166**,493(1996).

12.A.Agalarov, Phil.doc.tes., Lomonosov M S U,12(1984)

13.ATLAS Collaboration//CERN-OPEN-2009-056, 1(2009)

14.A.Slavnov,Theor.Math.Phys.**146**, 1159 (2006)

15.S.Weinberg, QFT, M.,FizMatLit., **P.3**,1-528 (2005)

16.D.Kirzhnits and A.Linde, Phys.Lett.**42B**,471(1972)

17.E.Bogomolni, Sov.Nucl.Phys.**24**, 449(1976)

18.A.Agalarov et al., JETP Lett.**76**, 488(2002)

19.V.Khalilov, Electrons in Strong E-M Fields//Gordon&Breach Sci.Publ(1996)

20.V.Zakharov et al., Theory of Solitons, M.,Nauka,1-320 (1980)