Gravitational models with nonlocal scalar fields

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based on the following papers

I.Ya. Aref'eva, L.V. Joukovskaya, S.V.,
J. Phys A **41** (2008) 304003, arXiv:0711.1364
S.V., Class. Quant. Grav. **27** (2010) 035006, arXiv:0907.0468
S.V., arXiv:1005.0372

To specify different types of cosmic fluids one uses a relation between the pressure p and the energy density ϱ

$$p = w\varrho, \qquad p = E_k - V, \qquad \varrho = E_k + V$$

where w is the state parameter.

The spatially flat Friedmann–Robertson–Walker metric:

$$ds^2 = \, - dt^2 + a^2(t)\left(dx_1^2 + dx_2^2 + dx_3^2\right), \tag{1}$$

where $a(t)$ is the scale factor, the Hubble parameter $H \equiv \dot{a}/a$.

$$w(t) = \, - 1 - \frac{2}{3}\frac{\dot{H}}{H^2} = \, - 1 + \frac{2E_k}{\varrho}. \tag{2}$$

Contemporary experiments give strong support that

$w > 0$ — **Atoms. (4%)**

$w = 0$ — **the Cold Dark Matter. (23%)**

$w < 0$ — **the Dark Energy. (73%)**

$$w_{DE} = -1 \pm 0.2. \tag{3}$$

We consider the case $w_{DE} < -1$. Null energy condition (NEC) is violated and there are problems of instability.

Papers about cosmological models with nonlocal fields:

I.Ya. Aref'eva, Nonlocal String Tachyon as a Model for Cosmological Dark Energy, astro-ph/0410443, 2004.

I.Ya. Aref'eva and L.V. Joukovskaya, 2005;

I.Ya. Aref'eva and A.S. Koshelev, 2006; 2008;

I.Ya. Aref'eva and I.V. Volovich, 2006; 2007;

I.Ya. Aref'eva, 2007; A.S. Koshelev, 2007;

L.V. Joukovskaya, 2007; 2008; 2009

I.Ya. Aref'eva, L.V. Joukovskaya, S.Yu.V., 2007

J.E. Lidsey, 2007;

G. Calcagni, 2006; G. Calcagni, M. Montobbio and G. Nardelli, 2007;

G. Calcagni and G. Nardelli, 2007; 2009; 2010

N. Barnaby, T. Biswas and J.M. Cline, 2006; N. Barnaby and J.M. Cline, 2007; N. Barnaby and N. Kamran, 2007; 2008; N. Barnaby, 2008; 2010;

D.J. Mulryne, N.J. Nunes, 2008;

B. Dragovich, 2008;

A.S. Koshelev, S.Yu.V., 2009; 2010

The SFT inspired nonlocal cosmological models

From the Witten action of bosonic cubic string field theory, considering only tachyon scalar field $\phi(x)$ one obtains:

$$S = \frac{1}{g_o^2}\int d^{26}x\left[\frac{\alpha'}{2}\phi(x)\Box\phi(x) + \frac{1}{2}\phi^2(x) - \frac{1}{3}\gamma^3\Phi^3(x) - \tilde{\Lambda}\right], \qquad (4)$$

where

$$\Phi = e^{k\Box}\phi, \qquad k = \alpha'\ln(\gamma), \qquad \gamma = \frac{4}{3\sqrt{3}}. \qquad (5)$$

g_o **is the open string coupling constant,** α' **is the string length squared and** $\tilde{\Lambda} = \frac{1}{6}\gamma^{-6}$ **is added to the potential to set the local minimum of the potential to zero. The action (4) leads to equation of motion**

$$(\alpha'\Box + 1)e^{-2k\Box}\Phi = \gamma^3\Phi^2. \qquad (6)$$

In the majority of the SFT inspired nonlocal gravitation models the action is introduced by hand as a sum of the SFT action of tachyon field and gravity part of the action:

$$S = \frac{1}{g_o^2}\int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R + \frac{1}{2}\phi\Box_g\phi + \frac{1}{2}\phi^2 - \frac{1}{3}\gamma^3\Phi^3 - \Lambda\right), \tag{7}$$

Action (7) includes a nonlocal potential. Using a suitable redefinition of the fields, one can made the potential local, at that the kinetic term becomes nonlocal.

This nonstandard kinetic term leads to a nonlocal field behavior similar to the behavior of a phantom field, and it can be approximated with a phantom kinetic term.

The behavior of an open string tachyon can be effectively simulated by a scalar field with a phantom kinetic term.

Another type of the SFT inspired models includes nonlocal modification of gravity.

Recently G. Calcagni and G. Nardelli have considered nonlocal gravity with nonlocal scalar field (arXiv: 1004.5144).

Nonlocal action in the general form

We consider a general class of gravitational models with a non-local scalar field, which are described by the following action:

$$S=\int d^4x\sqrt{-g}\alpha'\left(\frac{R}{16\pi G_N}+\frac{1}{g_o^2}\left(\frac{1}{2}\phi\mathcal{F}(\Box_g)\phi-V(\phi)\right)-\Lambda\right), \qquad (8)$$

G_N **is the Newtonian constant:** $8\pi G_N=1/M_P^2$**,**
M_P **is the Planck mass.**
We use the signature $(-,+,+,+)$**,**
$g_{\mu\nu}$ **is the metric tensor,**
R **is the scalar curvature,**
Λ **is the cosmological constant.**

Hereafter the d'Alembertian $\Box_g$ **is applied to scalar functions and can be written as follows**

$$\Box_g=\frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu\,. \qquad (9)$$

The function $\mathcal{F}(\Box_g)$ is assumed to be an analytic function:

$$\mathcal{F}(\Box_g) = \sum_{n=0}^{\infty} f_n \Box_g^{\,n}. \tag{10}$$

Note that the term $\phi\mathcal{F}(\Box_g)\phi$ include not only terms with derivatives, but also $f_0\phi^2$.

In an arbitrary metric the energy-momentum tensor

$$T_{\mu\nu} = \, -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = \frac{1}{g_o^2}\Big(E_{\mu\nu} + E_{\nu\mu} - g_{\mu\nu}\left(g^{\rho\sigma}E_{\rho\sigma} + W\right)\Big), \tag{11}$$

$$E_{\mu\nu} \equiv \frac{1}{2}\sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \partial_\mu \Box_g^l \phi \partial_\nu \Box_g^{n-1-l}\phi, \tag{12}$$

$$W \equiv \frac{1}{2}\sum_{n=2}^{\infty} f_n \sum_{l=1}^{n-1} \Box_g^l \phi \Box_g^{n-l}\phi - \frac{f_0}{2}\phi^2 + V(\phi). \tag{13}$$

From action (8) we obtain the following equations

$$G_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} - \Lambda g_{\mu\nu}\right), \tag{14}$$

$$\mathcal{F}(\Box_g)\phi = \frac{dV}{d\phi}, \tag{15}$$

where $G_{\mu\nu}$ is the Einstein tensor.

From action (8) we obtain the following equations

$$G_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} - \Lambda g_{\mu\nu}\right), \tag{16}$$

$$\mathcal{F}(\Box_g)\phi = \frac{dV}{d\phi}, \tag{17}$$

where $G_{\mu\nu}$ is the Einstein tensor.

It is a system of nonlocal nonlinear equations !!!

HOW CAN WE FIND A SOLUTION?

The Ostrogradski representation.

- M. Ostrogradski, *Mémoire sur les équations differentielles relatives aux problèmes des isoperimétres*, Mem. St. Petersbourg VI Series, V. 4 (1850) 385–517
- A. Pais and G.E. Uhlenbeck, *On Field Theories with Non-localized Action*, Phys. Rev. 79 (1950) 145–165

$$\mathcal{F}(\Box) = \mathcal{F}_1(\Box) \equiv \prod_{j=1}^{N} \left(1 + \frac{\Box}{\omega_j^2}\right), \tag{18}$$

all roots, which are equal to $-\omega_j^2$, are simple.

We want to get for

$$\mathcal{L}_F = \phi \mathcal{F}_1(\Box) \phi. \tag{19}$$

the Ostragradski representation: find such numbers c_j, that

$$\mathcal{L}_F \cong L_l = \sum_{j=1}^{N} c_j \phi_j (\Box + \omega_j^2) \phi_j. \tag{20}$$

We define

$$\phi_j = \prod_{k=1,k\neq j}^{N} \left(1 + \frac{1}{\omega_k^2}\Box\right)\phi, \qquad \Rightarrow \qquad \left(\Box + \omega_j^2\right)\phi_j = 0. \tag{21}$$

Substituting ϕ_j in L_l, we get

$$L_l = \phi \mathcal{F}_1^2(\Box)\left(\sum_{k=1}^{N} \frac{c_k \omega_k^4}{\omega_k^2 + \Box}\right)\phi. \tag{22}$$

$$L_l = \mathcal{L}_F \quad \Leftrightarrow \quad \sum_{k=1}^{N} \frac{c_k \omega_k^4}{\omega_k^2 + \Box} = \frac{1}{\mathcal{F}_1(\Box)}. \tag{23}$$

All roots of $\mathcal{F}_1(\Box)$ are simple, hence, we can perform a partial fraction decomposition of $1/\mathcal{F}_1(\Box)$.

$$c_k = \frac{\mathcal{F}_1'(-\omega_k^2)}{\omega_k^4}, \quad \textbf{where} \quad \mathcal{F}_1(-\omega_k^2)' \equiv \frac{d\mathcal{F}_1}{d\Box}|_{\Box=-\omega_k^2}. \tag{24}$$

Let $\mathcal{F}_1(\Box)$ has two real simple roots. $\mathcal{F}_1' > 0$ in one and only one root. We get model with one phantom and one standard field.

The Ostrogradski representation and an algorithm of localization in the case of gravitational models with an arbitrary quadratic potential

Generalization:

- **Gravitation**
- $\mathcal{F}(\Box)$ **is an analytic functions**
- $\mathcal{F}(\Box)$ **has both simple and double roots.**
- **The potential** $V(\phi) = C_2\phi^2 + C_1\phi + C_0$**.**

$$V_{eff} = \left(C_2 - \frac{f_0}{2}\right)\phi^2 + C_1\phi + C_0 + \Lambda. \tag{25}$$

We can change values of f_0 **and** Λ **such that the potential takes the form** $V(\phi) = C_1\phi$**. So, we put** $C_2 = 0$ **and** $C_0 = 0$**.**

Let us start with the case $C_1 = 0$ and the equation

$$\mathcal{F}(\Box_g)\phi = 0. \tag{26}$$

We seek a particular solution of (26) in the following form

$$\phi_0 = \sum_{i=1}^{N_1} \phi_i + \sum_{k=1}^{N_2} \tilde{\phi}_k. \tag{27}$$

$$(\Box_g - J_i)\phi_i = 0, \qquad (\Box - \tilde{J}_k)^2 \tilde{\phi}_k = 0 \tag{28}$$

J_i **are simple roots of the characteristic equation** $\mathcal{F}(J) = 0$**.**
$\tilde{J}_k$ **are double roots.**

Energy–momentum tensor for special solutions

If we have *one simple root* ϕ_1 such that $\Box_g\phi_1 = J_1\phi_1$, then

$$E_{\mu\nu}(\phi_1) = \frac{1}{2}\sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} J_1^{n-1}\partial_\mu\phi_1\partial_\nu\phi_1 = \frac{\mathcal{F}'(J_1)}{2}\partial_\mu\phi_1\partial_\nu\phi_1.$$

$$W(\phi_1) = \frac{1}{2}\sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} J_1^{n}\phi_1^2 = \frac{J_1}{2}\sum_{n=1}^{\infty} f_n n J_1^{n-1}\phi_1^2 = \frac{J_1\mathcal{F}'(J_1)}{2}\phi_1^2.$$

In the case of *two simple roots* ϕ_1 and ϕ_2 we have

$$E_{\mu\nu}(\phi_1+\phi_2) = E_{\mu\nu}(\phi_1) + E_{\mu\nu}(\phi_2) + E_{\mu\nu}^{cr}(\phi_1,\phi_2), \tag{29}$$

where the cross term

$$E_{\mu\nu}^{cr}(\phi_1,\phi_2) = A_1\partial_\mu\phi_1\partial_\nu\phi_2 + A_2\partial_\mu\phi_2\partial_\nu\phi_1. \tag{30}$$

$$A_1 = \frac{1}{2}\sum_{n=1}^{\infty} f_n J_1^{n-1}\sum_{l=0}^{n-1}\left(\frac{J_2}{J_1}\right)^l = \frac{\mathcal{F}(J_1)-\mathcal{F}(J_2)}{2(J_1-J_2)} = 0, \tag{31}$$

$$A_2 = 0. \tag{32}$$

So, the cross term $E^{cr}_{\mu\nu}(\phi_1, \phi_2) = 0$ and

$$E_{\mu\nu}(\phi_1 + \phi_2) = E_{\mu\nu}(\phi_1) + E_{\mu\nu}(\phi_2) \tag{33}$$

Similar calculations shows

$$W(\phi_1 + \phi_2) = W(\phi_1) + W(\phi_2). \tag{34}$$

In the case of N *simple roots* the following formula has been obtained:

$$T_{\mu\nu} = \sum_{k=1}^{N} \mathcal{F}'(J_k) \left(\partial_\mu \phi_k \partial_\nu \phi_k - \frac{1}{2} g_{\mu\nu} \left(g^{\rho\sigma} \partial_\rho \phi_k \partial_\sigma \phi_k + J_k \phi_k^2 \right) \right). \tag{35}$$

Note that the last formula is exactly the energy-momentum tensor of many free massive scalar fields. If $\mathcal{F}(J)$ has simple real roots, then positive and negative values of $\mathcal{F}'(J_i)$ alternate, so we can obtain phantom fields.

Let $\tilde{J}_1$ is a double root. The fourth order differential equation $(\Box - \tilde{J}_1)^2\tilde{\phi}_1 = 0$ is equivalent to the following system of equations:

$$(\Box - \tilde{J}_1)\tilde{\phi}_1 = \varphi_1, \qquad (\Box - \tilde{J}_1)\varphi_1 = 0. \tag{36}$$

It is convenient to write $\Box^l\tilde{\phi}_1$ in terms of the $\tilde{\phi}_1$ and φ_1:

$$\Box^l\tilde{\phi}_1 = \tilde{J}_1^l\tilde{\phi}_1 + l\tilde{J}_1^{l-1}\varphi_1. \tag{37}$$

For one double root we obtain the following result:

$$E_{\mu\nu}(\tilde{\phi}_1) = \frac{\mathcal{F}''(\tilde{J}_1)}{4}\left(\partial_\mu\tilde{\phi}_1\partial_\nu\varphi_1 + \partial_\mu\phi_1\partial_\nu\tilde{\varphi}_1\right) + \frac{\mathcal{F}'''(\tilde{J}_1)}{12}\partial_\mu\varphi_1\partial_\nu\varphi_1.$$

Similar calculations gives

$$W(\tilde{\phi}_1) = \frac{\tilde{J}_1\mathcal{F}''(\tilde{J}_1)}{2}\tilde{\phi}_1\varphi_1 + \left(\frac{\tilde{J}_1\mathcal{F}'''(\tilde{J}_1)}{12} + \frac{\mathcal{F}''(\tilde{J}_1)}{4}\right)\varphi_1^2.$$

For any analytical function $\mathcal{F}(J)$, which has simple roots J_i and double roots $\tilde{J}_k$, the energy–momentum tensor

$$T_{\mu\nu}\left(\phi_0\right) = T_{\mu\nu}\left(\sum_{i=1}^{N_1}\phi_i + \sum_{k=1}^{N_2}\tilde{\phi}_k\right) = \sum_{i=1}^{N_1}T_{\mu\nu}(\phi_i) + \sum_{k=1}^{N_2}T_{\mu\nu}(\tilde{\phi}_k), \tag{38}$$

where

$$T_{\mu\nu} = \frac{1}{g_o^2}\Big(E_{\mu\nu} + E_{\nu\mu} - g_{\mu\nu}\left(g^{\rho\sigma}E_{\rho\sigma} + W\right)\Big), \tag{39}$$

$$E_{\mu\nu}(\phi_i) = \frac{\mathcal{F}'(J_i)}{2}\partial_\mu\phi_i\partial_\nu\phi_i, \qquad W(\phi_i) = \frac{J_i\mathcal{F}'(J_i)}{2}\phi_i^2, \qquad \mathcal{F}' \equiv \frac{d\mathcal{F}}{dJ} \tag{40}$$

$$E_{\mu\nu}(\tilde{\phi}_k) = \frac{\mathcal{F}''(\tilde{J}_k)}{4}\left(\partial_\mu\tilde{\phi}_k\partial_\nu\varphi_k + \partial_\nu\tilde{\phi}_k\partial_\mu\varphi_k\right) + \frac{\mathcal{F}'''(\tilde{J}_k)}{12}\partial_\mu\varphi_k\partial_\nu\varphi_k, \tag{41}$$

$$W(\tilde{\phi}_k) = \frac{\tilde{J}_k\mathcal{F}''(\tilde{J}_k)}{2}\tilde{\phi}_k\varphi_k + \left(\frac{\tilde{J}_k\mathcal{F}'''(\tilde{J}_k)}{12} + \frac{\mathcal{F}''(\tilde{J}_k)}{4}\right)\varphi_k^2. \tag{42}$$

Consider the following local action

$$S_{loc} = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N} - \Lambda\right) + \sum_{i=1}^{N_1} S_i + \sum_{k=1}^{N_2} \tilde{S}_k, \qquad (43)$$

where

$$S_i = -\frac{1}{g_o^2}\int d^4x\sqrt{-g}\frac{\mathcal{F}'(J_i)}{2}\left(g^{\mu\nu}\partial_\mu\phi_i\partial_\nu\phi_i + J_i\phi_i^2\right),$$

$$\tilde{S}_k = -\frac{1}{g_o^2}\int d^4x\sqrt{-g}\left(g^{\mu\nu}\left(\frac{\mathcal{F}''(\tilde{J}_k)}{4}\left(\partial_\mu\tilde{\phi}_k\partial_\nu\varphi_k + \partial_\nu\tilde{\phi}_k\partial_\mu\varphi_k\right) + \right.\right.$$
$$\left.\left. + \frac{\mathcal{F}'''(\tilde{J}_k)}{12}\partial_\mu\varphi_k\partial_\nu\varphi_k\right) + \frac{\tilde{J}_k\mathcal{F}''(\tilde{J}_k)}{2}\tilde{\phi}_k\varphi_k + \left(\frac{\tilde{J}_k\mathcal{F}'''(\tilde{J}_k)}{12} + \frac{\mathcal{F}''(\tilde{J}_k)}{4}\right)\varphi_k^2\right).$$

Remark 1. If $\mathcal{F}(J)$ has an infinity number of roots then one nonlocal model corresponds to infinity number of different local models. In this case the initial nonlocal action (8) generates infinity number of local actions (43).

Remark 2. We should prove that the way of localization is self-consistent. To construct local action (43) we assume that equations (28) are satisfied. Therefore, the method of localization is correct only if these equations can be obtained from the local action S_{loc}. The straightforward calculations show that

$$\frac{\delta S_{loc}}{\delta \phi_i} = 0 \;\Leftrightarrow\; \Box_g \phi_i = J_i \phi_i; \qquad \frac{\delta S_{loc}}{\delta \tilde{\phi}_k} = 0 \;\Leftrightarrow\; \Box_g \varphi_k = \tilde{J}_k \varphi_k. \tag{44}$$

$$\frac{\delta S_{loc}}{\delta \varphi_k} = 0 \qquad \Leftrightarrow \qquad \Box_g \tilde{\phi}_k = \tilde{J}_k \tilde{\phi}_k + \varphi_k. \tag{45}$$

We obtain from S_{loc} the Einstein equations as well:

$$G_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu}(\phi_0) - \Lambda g_{\mu\nu}\right), \tag{46}$$

where ϕ_0 is given by (27) and $T_{\mu\nu}(\phi_0)$ can be calculated by (38).

Any solutions of system (44)–(46) are particular solutions of the initial nonlocal system (14)–(15).

To clarify physical interpretation of local fields $\tilde{\phi}_k$ and φ_k, we diagonalize the kinetic terms of these scalar fields in S_{loc}.

Expressing $\tilde{\phi}_k$ and φ_k in terms of new fields:

$$\tilde{\phi}_k = \frac{1}{2\mathcal{F}''(\tilde{J}_k)}\left(\left(\mathcal{F}''(\tilde{J}_k) - \frac{1}{3}\mathcal{F}'''(\tilde{J}_k)\right)\xi_k - \left(\mathcal{F}''(\tilde{J}_k) + \frac{1}{3}\mathcal{F}'''(\tilde{J}_k)\right)\chi_k\right),$$

$$\varphi_k = \xi_k + \chi_k,$$

we obtain the corresponding $\tilde{S}_k$ in the following form:

$$\tilde{S}_k = -\frac{1}{g_o^2}\int d^4x\sqrt{-g}\left(g^{\mu\nu}\frac{\mathcal{F}''(\tilde{J}_k)}{4}\Big(\partial_\mu\xi_k\partial_\nu\xi_k - \partial_\nu\chi_k\partial_\mu\chi_k\Big) + \right.$$
$$+\frac{\tilde{J}_k}{4}\left((\mathcal{F}''(\tilde{J}_k) - \frac{1}{3}\mathcal{F}'''(\tilde{J}_k))\xi_k - (\mathcal{F}''(\tilde{J}_k) + \frac{1}{3}\mathcal{F}'''(\tilde{J}_k))\chi_k\right)(\xi_k + \chi_k) +$$
$$\left. + \left(\frac{\tilde{J}_k\mathcal{F}'''(\tilde{J}_k)}{12} + \frac{\mathcal{F}''(\tilde{J}_k)}{4}\right)(\xi_k + \chi_k)^2\right).$$

Let us consider the model with action (8) in the case $C_1 \neq 0$. If $f_0 \neq 0$, then we introduce a new scalar field

$$\chi = \phi - \frac{C_1}{f_0} \tag{47}$$

and get the previous case in terms of new field χ.

$$\mathcal{F}(\Box_g)\phi = C_1 \qquad \Longleftrightarrow \qquad \mathcal{F}(\Box_g)\chi = 0. \tag{48}$$

If $f_0 = 0$, then $J = 0$ is a root of the characteristic equation $\mathcal{F}(J) = 0$. It is easy to show, that the function

$$\tilde{\chi} = \phi_0 + \psi, \tag{49}$$

where ϕ_0 and ψ are solutions of the following equations

$$\mathcal{F}(\Box_g)\phi_0 = 0, \qquad \Box_g\psi = \frac{C_1}{f_1}. \tag{50}$$

is a solution for

$$\mathcal{F}(\Box_g)\tilde{\chi} = C_1. \tag{51}$$

The function ϕ_0 is given by (27), but the sum do not include ϕ_{i_0}, which corresponds to the root $J = 0$, because this function can not be separated from ψ.

For a quadratic potential $V(\phi) = C_2\phi^2 + C_1\phi + C_0$ ***there exists the following algorithm of localization*:**

- **Change values of** f_0 **and** Λ **such that the potential takes the form** $V(\phi) = C_1\phi$**.**
- **Find roots of the function** $\mathcal{F}(J)$ **and calculate orders of them.**
- **Select an finite number of simple and double roots.**
- **Construct the corresponding local action. In the case** $C_1 = 0$ **one should use formula (43). In the case** $C_1 \neq 0$ **and** $f_0 \neq 0$ **one should use (43) with the replacement of the scalar field** ϕ **by** χ**. In the case** $C_1 \neq 0$ **and** $f_0 = 0$ **the local action is (43) plus**

$$S_\psi = -\frac{1}{2g_o^2}\int d^4x\sqrt{-g}\left(f_1 g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi + 2C_1\psi + \frac{f_2C_1^2}{f_1^2}\right).$$

- **Vary the obtained local action and get a system of the Einstein equations and equations of motion.**
- **Seek solutions of the obtained local system.**

Conclusions

For gravitational models with minimally coupling SFT inspired nonlocal scalar fields and quadratic potentials we obtain:

- **The Ostrogradski representations for nonlocal Lagrangians in an arbitrary metric.**
- **The algorithm of localization.**
- **Local and nonlocal Einstein equations have one and the same solutions.**
- **Nonlocality arises in the case of $\mathcal{F}(\Box_g)$ with an infinite number of roots.**
- **One system of nonlocal Einstein equations $\Leftrightarrow$ Infinity number of systems of local Einstein equations.**