Gauge model of quark-meson interactions and the Higgs status of scalar mesons

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QFT – HEP, 2010,
Moscow, Golitsyno

An exact low energy hadron theory should be nonperturbative $\longrightarrow$ effective Lagrangian approach

- Effective Lagrangians from fundamental theory (QCD) (local or nonlocal theory)

  J.Gasser,H.Leutwyler;
  M.Volkov with collaborators;
  M.Ivanov with collaborators;

- Phenomenological Lagrangians from dynamical symmetries $\longrightarrow$ LσM $\longrightarrow$ theory of meson-meson and quark-meson interactions

  M.Scadron with collaborators

- Vector mesons can be added as the gauge fields

$QCD \longrightarrow SU_L(2) \times SU_R(2)$

- Bosonization procedure (Volkov, Radzhabov, 2006)
- NJL-type model with constituent quarks, $m_q \approx 300$ MeV, gluon substructures are included
- Dynamically generated masses
- EM and strong interactions are described by the gauge (vector) fields
- Quark level σ-model (QσM) $\longrightarrow$ hadron-level (NσM)

One of the simplest gauge approach is based on the group

$$\boxed{U_0(1) \times U(1) \times SU(2)}$$

- Linear sigma model is extended by the gauge and quark-meson interactions.
- The model is renormalizable.
- EM and strong interactions are insensitive to the chirality, it should be localized only the diagonal sum of the global chiral group, SUL(2)x SUR(2)
- VDM is naturally realized in the gauge way; physical γ, ρ, ω – the mixed initial gauge fields
- Tree-level masses are produced by the Higgs mechanism
- The remained Higgs degrees of freedom can be associated with the scalar mesons (isotriplet $a_0(980)$, isosinglet $f_0(980)$)
- σ-meson ($f_0(600)$) properties followed from the model structure

The initial model Lagrangian

$$L = i\bar{q}\hat{D}q - \varkappa\bar{q}(\sigma + i\pi^a\tau_a\gamma_5)q + \frac{1}{2}(D_\mu\pi^a)^+(D_\mu\pi^a) + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\mu^2(\sigma^2 + \pi^a\pi^a)$$
$$- \frac{1}{4}\lambda(\sigma^2 + \pi^a\pi^a)^2 + (D_\mu H_A)^+(D_\mu H_A) + \mu_A^2(H_A^+H_A) - \lambda_1(H_A^+H_A)^2$$
$$- \lambda_2(H_A^+H_B)(H_B^+H_A) - h(H_A^+H_A)(\sigma^2 + \pi^a\pi^a) - \frac{1}{4}B_{\mu\nu}B^\mu - \frac{1}{4}V_{\mu\nu}V^{\mu\nu} - \frac{1}{4}V^a_{\mu\nu}V_a^{\mu\nu}.$$

Here $q = (u, d)$ - is the first generation quark doublet; $H_{1,2}$ - two scalar fields doublets with hypercharges $Y_{1,2} = \pm 1/2$, $a = 1, 2, 3$ and $A = 1, 2$.

Vacuum shifts are:

$< \sigma >= v$, $< H1 >=1\sqrt{2}(v1, 0)$, $< H2 >=1\sqrt{2}(0, v2)$.

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The gauge fields masses

In a tree approximation the vector boson physical states

$$
\boxed{\begin{aligned}
A_\mu &= \cos\theta \cdot B_\mu + \sin\theta \cdot V^3_\mu,\\
\omega_\mu &= \cos\phi \cdot V_\mu + \sin\phi \cdot (\sin\theta \cdot B_\mu - \cos\theta \cdot V^3_\mu),\\
\rho^0_\mu &= \sin\phi \cdot V_\mu + \cos\phi \cdot (-\sin\theta \cdot B_\mu + \cos\theta \cdot V^3_\mu)
\end{aligned}}
$$

Pion-quark-vector bosons interaction part of the physical Lagrangian
with the universal vector fields couplings

$$
\begin{aligned}
L_{Phys} = {} & \bar{u}\gamma^\mu u(\frac{2}{3}eA_\mu + g_{u\omega}\omega_\mu + g_{u\rho}\rho^0_\mu) + \bar{d}\gamma^\mu d(-\frac{1}{3}eA_\mu + g_{d\omega}\omega_\mu + g_{d\rho}\rho^0_\mu)\\
& + ig_2(\pi^-\pi^+_{,\mu} - \pi^+\pi^-_{,\mu})(\sin\theta\ A^\mu - \cos\theta s_\phi\ \omega^\mu + \cos\theta c_\phi\ \rho^{0\mu})\\
& - \sqrt{2}i\varkappa\pi^+\bar{u}\gamma_5 d - \sqrt{2}i\varkappa\pi^-\bar{d}\gamma_5 u - i\varkappa\pi^0(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d)\\
& + 2g_2 e\cos\theta c_\phi\ \rho^0_\mu A^\mu\pi^+\pi^- - 2g_2 e\cos\theta s_\phi\ \omega_\mu A^\mu\pi^+\pi^-\\
& + \frac{1}{\sqrt{2}}g_2\rho^+_\mu\bar{u}\gamma^\mu d + \frac{1}{\sqrt{2}}g_2\rho^-_\mu\bar{d}\gamma^\mu u + ig_2\rho^{+\mu}(\pi^0\pi^-_{,\mu} - \pi^-\pi^0_{,\mu})\\
& + ig_2\rho^{-\mu}(\pi^+\pi^0_{,\mu} - \pi^0\pi^+_{,\mu}).
\end{aligned}
$$

The set of tree mixing parameters:

$$g_{u\omega} = \frac{1}{2} g_1 c_\phi + \frac{1}{2} s_\phi \left(\frac{1}{3} g_0 \sin\theta - g_2 \cos\theta\right),$$
$$g_{u\rho} = \frac{1}{2} g_1 s_\phi - \frac{1}{2} c_\phi \left(\frac{1}{3} g_0 \sin\theta - g_2 \cos\theta\right),$$
$$g_{d\omega} = \frac{1}{2} g_1 c_\phi + \frac{1}{2} s_\phi \left(\frac{1}{3} g_0 \sin\theta + g_2 \cos\theta\right),$$
$$g_{d\rho} = \frac{1}{2} g_1 s_\phi - \frac{1}{2} c_\phi \left(\frac{1}{3} g_0 \sin\theta + g_2 \cos\theta\right),$$

c_φ, s_φ = $\cos\varphi$, $\sin\varphi$

ω-meson has a small isotriplet admixture ~ $\sin\varphi \ll 1$

This contribution can be omitted in calculations

There are basic relations in the model

$$\sin\theta = \frac{g_0}{\sqrt{g_0^2 + g_2^2}}, \quad e = g_0 \cos\theta, \quad v_1^2 + v_2^2 = 4\frac{m_{\rho^\pm}^2}{g_2^2},$$

$$\sin\phi = \frac{g_1}{g_2}\left(\frac{m_{\rho^\pm}^2 - m_\omega^2 (g_2^2/g_1^2)}{m_\omega^2 - m_{\rho^0}^2}\right)^{1/2} \quad e = g_0 \cdot g_2/(g_0^2 + g_2^2)^{1/2}$$

From vector meson decays widths

$$\Gamma(V \to \pi_1\pi_2) = \frac{g_2^2 \cdot d_{\theta\phi}}{48\pi} \cdot \frac{1}{m_V} \cdot \lambda(m_{\pi_1}, m_{\pi_2}, m_V)^{3/2}$$

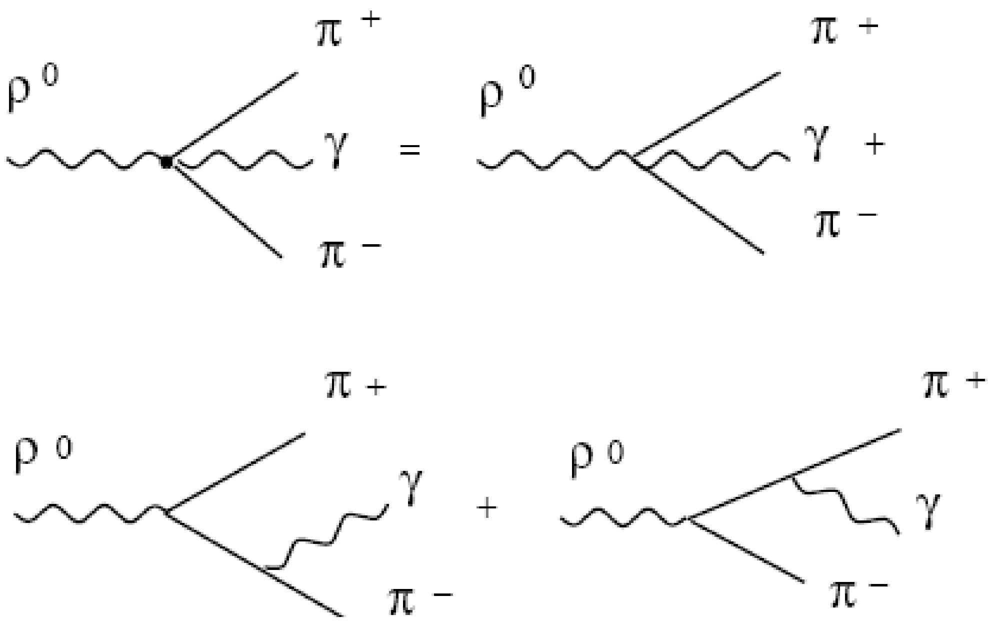
$$\Downarrow$$

$$g_0^2/4\pi = 7.32 \cdot 10^{-3}, \quad g_1^2/4\pi = 2.86, \quad g_2^2/4\pi = 2.81,$$

$$\sin\phi = 0.031, \; \sin\theta = 0.051, \; v_1^2 + v_2^2 \approx (250.7\,\mathrm{MeV})^2.$$

Radiative decays of light vector bosons at the tree level

$\boxed{\rho^0 \to \pi^+\pi^-\gamma}$ and $\boxed{\omega \to \pi^+\pi^-\gamma}$



Differential width has the form

$$d\Gamma(E_\gamma)/dE_\gamma = \frac{G}{\kappa}\left(F_1(\kappa) + F_2(\kappa)\ln F_3(\kappa)\right),$$

where:

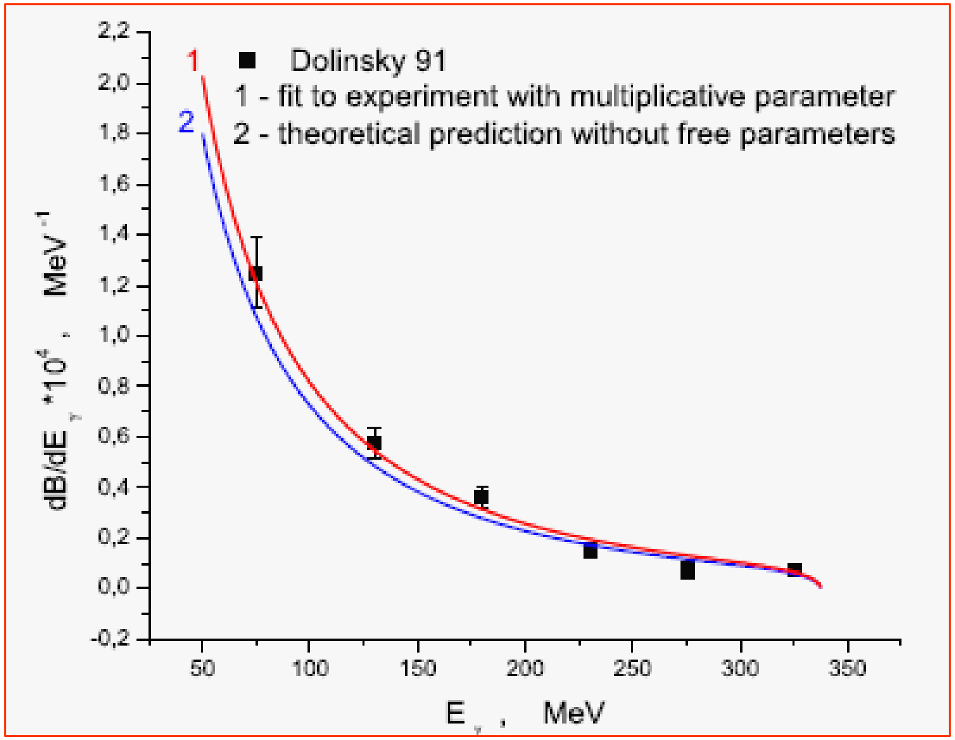
$$\kappa = E_\gamma/m_\rho, \;\; G = \alpha_{em} \cdot g_2^2 \cos^2\theta \cdot |c_\phi|^2/24\pi^2, \;\; \mu = m_\pi^2/m_\rho^2,$$

$$F_1(\kappa) = \left(\frac{1-2\kappa-4\mu}{1-2\kappa}\right)^{1/2}\left(-1+2\kappa+4\kappa^2+4\mu(1-2\kappa)\right);$$

$$F_2(\kappa) = 1-2\kappa-2\mu(3-4\kappa-4\mu);$$

$$F_3(\kappa) = \frac{1}{2\mu}\left[1-2\kappa-2\mu+\left((1-2\kappa)\cdot(1-2\kappa-4\mu)\right)^{1/2}\right].$$

Spectrum of photons in ρ ⟶ π^+π^-γ



$$B(\rho^0 \to \pi^+\pi^-\gamma) = 1.17 \cdot 10^{-2}$$

$$B^{exp}(\rho^0 \to \pi^+\pi^-\gamma) = (0.99 \pm 0.16) \cdot 10^{-2}$$

$$B^{phen}(\rho^0 \to \pi^+\pi^-\gamma) = (1.22 \pm 0.02) \cdot 10^{-2}$$

$\omega \longrightarrow \pi^+\pi^-\gamma$: with the replacement $c_\phi \longrightarrow s_\phi$

$$m_\rho \longrightarrow m_\omega$$

$$B(\omega \to \pi^+\pi^-\gamma) = 4.0 \cdot 10^{-4}$$

$$B(\omega \to \pi^+\pi^-\gamma) = 2.6 \cdot 10^{-4}$$

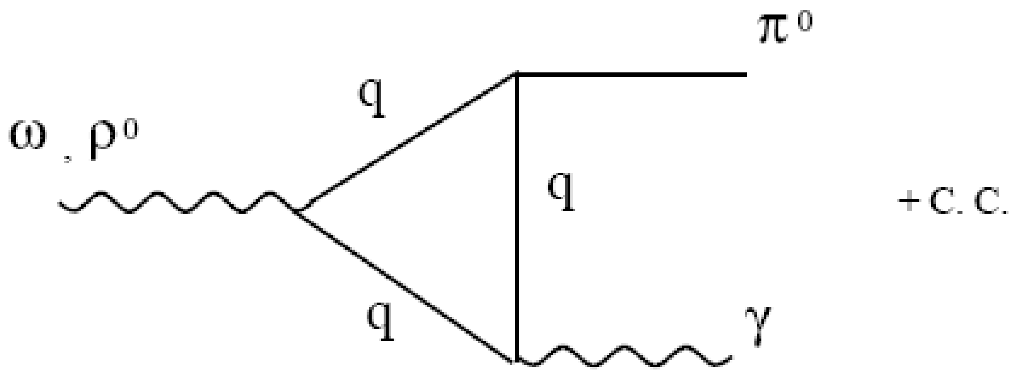
$$B^{exp}(\omega \to \pi^+\pi^-\gamma) \leq 3.6 \cdot 10^{-3}$$

Due to loop contributions

$B(\omega \to \pi^+\pi^-\gamma)$ can increase up to $(2-3) \cdot 10^{-3}$

Processes via quark loops

$$\omega,\ \rho^0 \longrightarrow \pi^0\gamma$$



$$\Gamma(\omega \to \pi^0\gamma) = \frac{3\alpha g_1^2}{2^7\pi^4}|c_\phi|^2\ m_q\frac{m_q^3}{m_\omega f_\pi^2}\left(1-\frac{m_\pi^2}{m_\omega^2}\right)|L_\omega|^2$$

$$L_\omega = Li_2\left(\frac{2}{1+\sqrt{\lambda_1}}\right) + Li_2\left(\frac{2}{1-\sqrt{\lambda_1}}\right) - Li_2\left(\frac{2}{1+\sqrt{\lambda_2}}\right) - Li_2\left(\frac{2}{1-\sqrt{\lambda_2}}\right)$$

$$\lambda_1 = 1-4m_q^2/m_\omega^2,\ \lambda_2 = 1-4m_q^2/m_\pi^2$$

$$\Gamma(\rho^0 \to \pi^0\gamma) = \frac{\alpha g_1^2}{3 \cdot 2^7 \pi^4} |c_\phi|^2 \cdot \left(\cos\theta \cdot \frac{g_2}{g_1}\right)^2 m_q \frac{m_q^3}{m_\rho f_\pi^2} \left(1 - \frac{m_\pi^2}{m_\rho^2}\right) |L_\rho|^2$$

Imaginary part of the quark loop can be:

a) kept as nonzero value – the constituent quark deconfinement occurs only in the NPT internal hadron vacuum? M_q=(170-180) MeV;

b) set to zero “by hand” (“naïve approximation” – see M.Volkov et al; M.Scadron et al;…) M_q=(280-290) MeV;

c) eliminated by special procedure to provide the quark confinement (M.Volkov et al; M.Ivanov et al, 2009;) M_q=(280-290) MeV for some fixed value of parameter λ=260 MeV

λ - is a cutoff scale for the amplitude in α-representation;
it defines the integration over the “common length” for the amplitude;
It is some universal scale for the quark propagation inside the hadron (?)

$$\int_0^\infty d\alpha \int_0^1 dx \int_0^1 dy ... F(q_i^2, m_i^2, x, y, ...\alpha) \Longrightarrow \int_0^{\frac{1}{\lambda^2}} d\alpha \int_0^1 dx \int_0^1 dy ... F(q_i^2, m_i^2, x, y, ...\alpha)$$

Due to isotopic structure of ρ-, ω-, γ- interaction with the constituent quarks

$$\Gamma(\rho^0 \to \pi^0\gamma)/\Gamma(\omega \to \pi^0\gamma) \approx 1/3^2$$

$\Gamma(\rho^0 \to \pi^0\gamma)$=0.091±0.003 MeV

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for the "infrared confinement" scheme with M_q=(280-290) MeV and λ=260 MeV;

very close result for M_q=(170-180) MeV and λ=210 MeV

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Indeed, $\lambda/m_q \approx 0.9 - 1.2$

$$\Gamma^{theor}(\omega \to \pi^0\gamma) = 0.74 \pm 0.02\,\mathrm{MeV}, \qquad \Gamma^{exp}(\omega \to \pi^0\gamma) = 0.76 \pm 0.02\,\mathrm{MeV};$$
$$\Gamma^{theor}(\rho^0 \to \pi^0\gamma) = 0.081 \pm 0.003\,\mathrm{MeV}, \qquad \Gamma^{exp}(\rho^0 \to \pi^0\gamma) = 0.090 \pm 0.012\,\mathrm{MeV}.$$

“Naïve” system of equation for characteristic hadron masses

$$m_N = 3m_q + m_G; \;\; m_\rho = 2m_q + m_G$$

Here, m_G – mass of the same “gluon” component of meson and baryon structure – in the effective models σ-meson ($f_0(600)$) can be interpreted as a scalar excitation of NPT vacuum with a mass ~ m_G.

From the system it follows

$m_q \approx 170$ MeV and $m_G \approx 435$ MeV

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{\alpha^2 \varkappa^2}{16\pi^3} m_\pi^3 m_q^2 [C_0(0,0,m_\pi^2; m_q, m_q, m_q)]^2$$

$$C_0(0,0,m_\pi^2; m_q, m_q, m_q) = \frac{1}{m_\pi^2}[Li_2(\frac{2}{1+\sqrt{\lambda}}) + Li_2(\frac{2}{1-\sqrt{\lambda}})]$$

$$\lambda = 1 - 4m_q^2/m_\pi^2$$

$$C_0(0,0,m_\pi^2; m_q, m_q, m_q) = \frac{2}{m_\pi^2}\left[\arcsin\left(\frac{m_\pi}{2m_q}\right)\right]^2$$

$\Gamma(\pi^0 \to \gamma\gamma) = 8.48\,\mathrm{eV} \Longrightarrow$ for m_q=175 MeV

In the exact chiral limit (corresponding to axial anomaly term) the width is 7.63 eV.

For m_q=300 MeV $\Gamma(\pi^0 \to \gamma\gamma)$=7.91 eV

The experimental width is in the interval

7.22 eV ≤ $\Gamma(\pi^0 \to \gamma\gamma)$ ≤ 8.33 eV.

The Goldberger-Treiman relation $\varkappa \approx m_q / f_\pi$ fixes **πqq** and **σqq** couplings $\Longrightarrow$ in the CL the width $\Gamma(\pi^0 \to \gamma\gamma)$ does not depend on the quark mass.

But:

a) the G-T relation accuracy is ~3-4%;
b) $\pi^0 - \eta$ mixing can be noticeable;
c) loop corrections can decrease the γqq-coupling – the 2% decreasing is sufficient to provide an agreement with the data.

$\Gamma(\pi^0 \to \gamma\gamma) = 7.82\,\mathrm{eV}$ for m_q =175 MeV (no imaginary part, with corrections from G-T relations)

$\Gamma(\pi^0 \to \gamma\gamma)$ =7.63 eV for m_q=175 MeV with the "infrared confinement" procedure, but λ=100 MeV only!

Effective couplings

$g_{\rho\omega\pi,\,mod}$ = 14.5 1/GeV (m_q = 175 MeV, λ=250 MeV);

$g_{\rho\omega\pi,\,mod}$ = 17.7 1/GeV (m_q = 280 MeV, λ=280 MeV);

$g_{\rho\omega\pi,\,mod}$ =16.4 1/GeV (m_q = 300 MeV, λ=280 MeV);

$g_{\rho\omega\pi,\,exp}$ = (15-17) 1/GeV

$g_{\rho\pi\gamma,\,mod}$ = 0.743 1/GeV (m_q = 280 MeV, λ=280 MeV)

$g_{\rho\pi\gamma,\,exp}$ = 0.723 ± 0.037 1/GeV

$g_{\pi\gamma\gamma,\,mod}$ = 0.278 1/GeV (m_q = 280 MeV)

$g_{\pi\gamma\gamma,\,exp}$ = 0.276 1/GeV

$\lambda/m_q = 0.9 – 1.2$

(see also M.K. Volkov et al, 1996, 2000)

For zero mixing angles, **φ** and **θ** (it corresponds to exact $v_1 = v_2 = v$), two charged scalar combinations arise after the spontaneous breaking $\Longrightarrow$ isotriplet structure (slightly broken by the small mixing with the singlet vector fields) - $a_0(980)$? Two isotriplets arise: (ρ^+, ρ^0, ρ^-) and (a^+, a^0, a^-)

$$\boxed{\begin{aligned} L_{a\omega\rho} &= \frac{1}{\sqrt{2}} v g_2 g_3\, \omega^\mu (a^- \rho^+_\mu + a^+ \rho^-_\mu + a^0 \rho^0_\mu) = \frac{1}{\sqrt{2}} v g_2 g_3\, \omega_\mu \rho^\mu_\alpha a_\alpha, \\ L_{aa\rho} &= \frac{i}{2}\, g_2\, [\rho^{0\mu}(\partial_\mu a^+ \cdot a^- - \partial_\mu a^- \cdot a^+) + \rho^{+\mu}(\partial_\mu a^- \cdot a^0 - \partial_\mu a^0 \cdot a^-) \\ &\quad + \rho^{-\mu}(\partial_\mu a^0 \cdot a^+ - \partial_\mu a^+ \cdot a^0)] = \frac{i}{2}\, g_2\, \epsilon_{\alpha\beta\gamma} \rho^\mu_\alpha a_\beta \partial_\mu a_\gamma. \end{aligned}}$$

Pion-scalar interaction part

$$L_{\pi h} = (\pi^0\pi^0 + 2\pi^+\pi^-)(g_{\sigma\pi}\sigma_0 + g_{f\pi} f_0 + g_{a\pi} a_0)$$

$g_{a\pi} = h(v_2 - v_1)/\sqrt{2}$ is close to zero for $v_2 \approxeq v_1 \Rightarrow a_0 \longrightarrow \pi\pi$ is damped

$\omega \to \pi^+\pi^-$ is suppressed but observable $\Longrightarrow v_1 \neq v_2$

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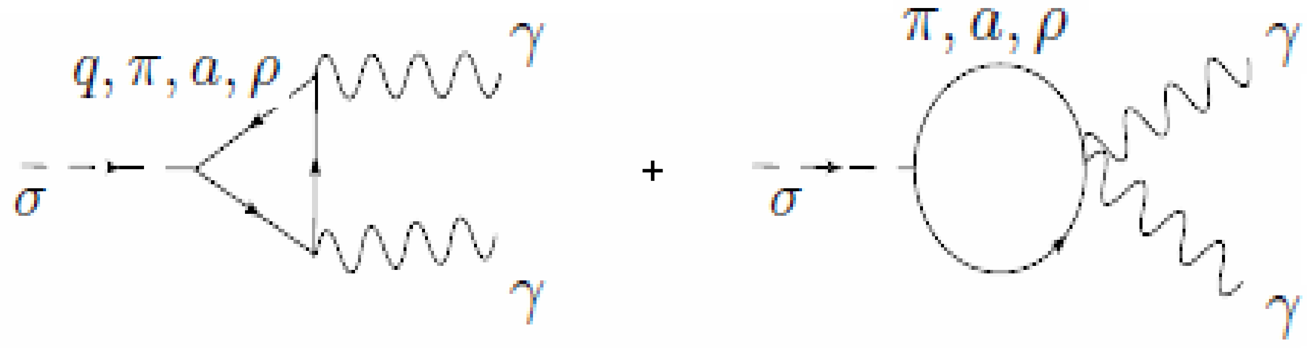
Residual global SU(2) symmetry in the scalar sector takes place approximately

$\sigma \to \gamma\gamma$ decay can be effectively considered

There are four types of loops



There is a mixing $\sigma = \sigma_0 \cdot \cos\psi + f_0 \cdot \sin\psi$,

the angle ψ defines:

- $\sigma\rho\rho$- and $f_0\rho\rho$-couplings, if $\psi=0 \Rightarrow g_{\sigma\rho\rho}=0$;
- f_0 and a_0 do not interact with quarks directly, if $\psi=0$;
- the f_0- and a_0-masses are equal, if $\psi=0$.

The decay amplitude has gauge invariance form, all divergencies are cancelled

$$M \sim [g_{\mu\nu}(k_1 k_2) - k_{1\mu} k_{2\nu}] \cdot (M_{q\text{-loop}} + M_{\pi\text{-loop}} + M_{a_0\text{-loop}})$$

Results for the decay width

$\Gamma(\sigma \to \gamma\gamma)_{exp} = (1 - 5)$ KeV

- In the "naïve" approximation (Im M=0)

$\Gamma(\sigma \to \gamma\gamma)_{mod} = (3 - 5)$ KeV for $m_q = 280$ MeV,
$g_{\sigma\pi\pi} = (m_\sigma^2 - m_\pi^2)/2f_\pi$ beyond the CL, $m_\sigma = 450 \pm 50$ MeV

- $\Gamma(\sigma \to \gamma\gamma)_{mod} = (2.5 - 5)$ KeV for $m_q = 175$ MeV,

if $g_{\sigma\pi\pi}$ a free parameter, (0.5 - 1.5) of the CL, $m_\sigma = 600 - 800$ MeV

- Im M is small for any case
- $g_{\sigma aa} \approx 1/3$ of $g_{\sigma\pi\pi}$ (CL value)

•In the approach of “infrared confinement” (when the poles of amplitude are excluded by the special integration procedure)

In this case for various m_q and m_σ values

m_q =175 MeV: for λ=(260 – 300) MeV, m_σ =500 - 800 MeV

$$\Gamma(\sigma \to \gamma\gamma)_{mod} = (4 – 6)\ \text{KeV}$$

m_q =280 MeV: for λ=260 MeV, m_σ =450 - 550 MeV

$$\Gamma(\sigma \to \gamma\gamma)_{mod} = (1.5 – 2.5)\ \text{KeV}$$

All used parameters are in agreement with $\sigma \to \pi\pi$ and $f_0 \to \pi\pi$

and $g_{\sigma\pi\pi}$ $=m_\sigma$^2 /2f_π (in the CL)

$f_0 \to \gamma\gamma$, $a_0 \to \gamma\gamma$ $\Leftrightarrow$ from two-quark structure of the mesons. ?

f_0, a_0 as qq-states from constituent quarks - do not agree with 2γ decays

M. Napsuciale, 2002

2γ decays of scalar mesons in the gauge model $\Rightarrow$ via meson loops ($f_0\pi\pi$, $f_0a_0a_0$, $f_0\rho\rho$, $a_0\rho\omega$ vertices) and quarks in two-loops – 4-quark and gluon components?

The model should be generalized to gauge SU(3) x SU(3)

$\Downarrow$

Higgs sector $\Rightarrow$ masses and couplings, scalar mesons

axial vectors, K-mesons $\Rightarrow$ scalar mesons decays

$a_0(980) \to \eta\pi$

This dominant decay can be described by the symmetry allowed term in the model Lagrangian

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$$\eta\pi_a H_A^+ \tau_a H_A$$

Conclusions

- The gauge renormalizable generalization of σ-model with the VDM at the tree level is considered.
- Higgs mechanism for meson masses, effective couplings from experiment data.
- Scalar mesons (a_0,f_0) are generated by the Higgs degrees of freedom (a vacuum nature of scalars).
- Tree-level radiative decays of light vector mesons are well described.
- Consituent quark loops define (ρ,ω) $\to \pi\gamma$ decays in a good agreement with data, various approaches to avoid deconfinement are used.
- $\pi \to \gamma\gamma, \sigma \to \gamma\gamma$ decays are considered in details: the values

  m_σ = 450-550 Mev and m_q = 280 MeV are preferred.
- Effective vertices $g_{\rho\omega\pi}$, $g_{\rho\pi\gamma}$, $g_{\pi\gamma\gamma}$ are in agreement with data.
- From the scalar sector structure it follows: the degeneracy of a_0,f_0 in mass; an observable suppression of $a_0 \to \pi\pi$, the smallness of two-quark component in a_0,f_0 mesons, $\sigma \to \pi\pi$ and $f_0 \to \pi\pi$ agree with data.

So, the gauge quantum field approach + the Higgs mechanism lead to the effective quark-meson model with scalar mesons as a vacuum fields; radiative and hadronic decay modes of vector and scalar mesons are well described.

SU(3) chiral extension will be done to include all possible scalar, vector and axial mesons in this scheme.
