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A five-dimensional effective model for excited light mesons

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A brief reminder

AdS/CFT correspondence – the conjectured equivalence between a string theory defined on one space and a CFT without gravity defined on conformal boundary of this space.

Maldacena example (1997):

Type IIB string theory on $AdS_5 \times S^5$ in low-energy (i.e. supergravity) approximation $\Longleftrightarrow$ $\mathrm{N}=4$ YM theory on AdS boundary in the limit $g_{YM} N_C >> 1$

AdS/QCD correspondence – a program to implement such a duality for QCD following the principles of AdS/CFT correspondence

String theory

Up down

Bottom up

We will discuss

QCD

$SO(4,2)$: Equivalence of energy scales $\Longrightarrow$ The 5-th coordinate – (inverse) energy scale

A typical model (Erlich et al., PRL 95, 261602 (2005))

$$S = \int d^5x\,\sqrt{g}\,\mathrm{Tr}\left\{|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right\}$$

$D_\mu X = \partial_\mu X - iA_{L\mu}X + iXA_{R\mu}$, $A_{L,R} = A^a_{L,R}t^a$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$.

For $N_f = 2$ $\quad t^a = \sigma^a/2$

Hard wall model: AdS_5 space with the metric $\boxed{ds^2 = \frac{R^2}{z^2}(-dz^2 + dx^\mu dx_\mu)}$

where R is the AdS curvature radius, cut at z coordinate: $0 < z \leq z_m$

The fifth coordinate corresponds to the energy scale: $Q \sim 1/z$

Because of the conformal isometry of the AdS space, the running of the QCD gauge coupling is neglected until an infrared scale $Q_m \sim 1/z_m$. At $z = z_m$ one imposes certain gauge invariant boundary conditions on the fields.

Equation of motion for the scalar field

$$\frac{1}{z^5}3X = \frac{1}{z^3}\partial_\mu\partial^\mu X - \partial_z\frac{1}{z^3}\partial_z X$$

Solution independent of usual 4 space-time coordinates

$$X_0(z) = \frac{1}{2}Mz + \frac{1}{2}\Sigma z^3$$

where ***M*** is identified with the quark mass matrix and ***Σ*** with the condensate.

Denoting

$$X_0(z) = \frac{1}{2} v(z) \mathbf{1}, \qquad v(z) = mz + \sigma z^3$$

the equations of motion for the vector fields are (in the axial gauge)

$$\left[\partial_z \left(\frac{1}{z} \partial_z V_\mu^a(q,z) \right) + \frac{q^2}{z} V_\mu^a(q,z) \right]_\perp = 0$$

where $V(q,z) = \int d^4x \;\; e^{iqx} V(x,z)$

$$\left[\partial_z \left(\frac{1}{z} \partial_z A_\mu^a \right) + \frac{q^2}{z} A_\mu^a - \frac{g_5^2 v^2}{z^3} A_\mu^a \right]_\perp = 0$$

Due to chiral symmetry breaking

They have normalizable solutions only for discrete values of 4d momentum $q^2 = m_n^2$

Some conceptual problems:

1. Matching to QCD is implemented in the UV region (small z) where QCD is weakly coupled.

2. The spectrum is drastically shaped by the UV region contrary to what we expect from QCD. In addition, it does not have Regge form. A popular phenomenological solution – soft wall model,

$$S = \int d^5x \sqrt{g}\, e^{\Phi(z)} L$$

However, such a 5D background cannot follow from a solution of 5D Einstein equations.

Alternative mechanism? – Higgs one

Vacuum sector

$$S_{\rm vac} = \int d^4x dz \left(\frac{1}{2}\partial_A\varphi\partial^A\varphi + \frac{1}{2}m^2\varphi^2 - \frac{1}{4}\lambda\varphi^4 \right)$$

$$\eta_{AB} = (1,-1,-1,-1,-1), \qquad A = 0,1,2,3,4$$

By assumption, $\varphi(x_\mu, z)$ is dual to the gluonic field strength tensor square $G^2_{\mu\nu}$ whose vev causes the anomaly in the trace of energy-momentum tensor,

$$4\varepsilon_{\rm vac} = \langle \Theta^\mu_\mu \rangle_{\rm n.p.} = \frac{\beta(\alpha_s)}{4\alpha_s}\langle G^2_{\mu\nu}\rangle_{\rm n.p.} + \mathcal{O}(\alpha) + \cdots$$

Making use of the scaling

$$x \to \frac{x}{m}, \qquad \varphi \to \frac{m}{\sqrt{\lambda}}\varphi,$$

the action can be rewritten as

$$S_{\rm vac} = \frac{1}{\lambda m}\int d^4x dz \left(\frac{1}{2}\partial_A\varphi\partial^A\varphi + \frac{1}{2}\varphi^2 - \frac{1}{4}\varphi^4 \right)$$

By assumption the selfinteraction is weak $\qquad \lambda m \ll 1$

The classical equation of motion is

$$\partial_\mu^2\varphi - \partial_z^2\varphi - \varphi(1-\varphi^2) = 0$$

We assume that the vacuum solution does not depend on the usual space-time coordinates,

$$\varphi(x_\mu, z) = \varphi(z)$$

The equation above has then a kink solution

$$\varphi_{\mathrm{kink}} = \pm\tanh(z/\sqrt{2})$$

Translational invariance along the *z*-direction is broken!

Thus, different energy scales are now not equivalent. The effect is essential at large *z* (small energies) but disappears at small *z* (high energies)

Consider the particle-like excitations by varying $\varphi = \varphi + \varepsilon$

Assuming $\varepsilon(x_\mu, z) = e^{ipx}\varepsilon(z)$ with $p^2 = M^2$ and retaining only linear part,

$$\left(-\partial_z^2 + 3\tanh^2(z/\sqrt{2}) - 1\right)\varepsilon_n = M_n^2 \varepsilon_n$$

There are two normalizable discrete states,

$$\varepsilon_0 = \frac{1}{\cosh^2(z/\sqrt{2})}, \qquad M_0^2 = 0;$$

$$\varepsilon_1 = \frac{\tanh(z/\sqrt{2})}{\cosh(z/\sqrt{2})}, \qquad M_1^2 = \frac{3}{2}.$$

Continuum begins at $p^2 = 2$

"glueball"?

Coupling to bosons

For simplicity, we consider the scalar case only

$$S_{\rm bos} = \int d^4x dz \left(\frac{1}{2}\partial_A\Phi\partial^A\Phi - \frac{G}{2}\varphi^2\Phi^2\right)$$

Making the rescaling above and $\Phi \longrightarrow m^{3/2}\Phi$ the corresponding Lagrangian reads

$$\mathcal{L}_{\rm bos} = \frac{1}{2}\left(\partial_A\Phi\partial^A\Phi - \frac{G}{\lambda}\varphi^2\Phi^2\right)$$

Consider the particle-like excitations $\Phi(x_\mu, z) = e^{ipx}f(z), p^2 = M^2$

$$\left(-\partial_z^2 + \frac{G}{\lambda}\tanh^2(z/\sqrt{2})\right) f_n = M_n^2 f_n$$

The discrete spectrum is

$$M_n^2 = \frac{1}{2}\left[\sqrt{1+\frac{8G}{\lambda}}\left(n+\frac{1}{2}\right) - \left(n+\frac{1}{2}\right)^2 - \frac{1}{4}\right]$$

$$f_n = \cosh^{n-s}(z/\sqrt{2})\times$$
$$F\left[-n, 2s+1-n, s+1-n, \frac{1-\tanh(z/\sqrt{2})}{2}\right]$$

where *F* is hypergeometric function and

$$s = \frac{1}{2}\left(\sqrt{1+\frac{8G}{\lambda}}-1\right),$$

$$n = 0, 1, 2, \ldots, \qquad n < s.$$

The continuum sets in at $n = s$

An example of numerical fit

Fitting the masses of ρ-mesons by the curve

$$m_n^2 = An^2 + Bn + C$$

one obtains

$$m_n^2 \approx (-0.09 \pm 0.02)n^2 + (1.30 \pm 0.08)n + (0.71 \pm 0.02)$$

which gives

$$(C/B)_{\rm exp} = 0.55 \pm 0.05$$

while in the model

$$(C/B)_{\rm theor} = 0.53 \pm 0.01$$

Coupling to fermions

$$S_{\rm ferm} = \int d^4x dz \left(i\bar{\Psi}\Gamma^A \partial_A \Psi - h\varphi \bar{\Psi}\Psi \right)$$

Here $\Gamma^\mu = \gamma^\mu,\ \Gamma^4 = -i\gamma^5$ After our rescaling and $\psi \to m^2 \psi$

$$\mathcal{L}_{\rm ferm} = i\bar{\Psi}\Gamma^A \partial_A \Psi - \frac{h}{\sqrt{\lambda}} \varphi \bar{\Psi}\Psi$$

Let us find particle-like excitations $\Psi_{L,R}(x_\mu, z) = e^{ipx} U_{L,R}(z)$ for the left and right components $\gamma_5 \Psi_{L,R} = \pm \Psi_{L,R}$

$$\left(\pm \partial_z + \frac{h}{\sqrt{\lambda}} \tanh(z/\sqrt{2}) \right) U_{L,R} = M U_{L,R}$$

The equation is known to possess a normalizable zero-mode solution

$$M = 0, \qquad U_L = \cosh^{-\frac{\sqrt{2}\,h}{\sqrt{\lambda}}}(z/\sqrt{2}), \qquad U_R = 0$$

This mode is located near $z = 0$ There is also an asymptotic solution

$$z \to \infty : \qquad M = \frac{h}{\sqrt{\lambda}}, \qquad U_{L,R} = C_{L,R}$$

Conclusions

We have proposed a new way for description of meson spectrum within holographic models for QCD based on the Higgs mechanism

Next steps – extension of the presented scheme to the AdS background and simultaneous description of the chiral symmetry breaking with all related physics.