MSSM Finite-Temperature Higgs Potential for Phase Transition

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Outline

- Introduction
- **Finite-temperature corrections of squarks**
- **Thermal evolution and the critical temperature**
- Conclusions

[M. Dolgopolov, M. Dubinin, E. Rykova Threshold corrections to the MSSM finite-temperature Higgs potential. Jan 2009. 26pp. e-Print: arXiv:0901.0524v1 ; Physics of Atomic Nuclei, 2010, Vol. 73, No. 6, pp. 1032–1036]

The absence of antimatter in the Universe, a small ratio of the observed number of baryons to the observed number of photons and the absence of light CP-even Higgs boson signal at LEP2 and Tevatron energies lay a specific claims to models of particle physics

- **Two problems in the Standard Model**
  - **First order phase transition requires m_h < 50 GeV**
  - **Need new sources of CP violation**
- **Minimal Supersymmetric Standard Model**
  - **1st order phase transition is possible if** $m_{\tilde{t}_R} < 160\text{GeV}$
  - **New CP violating phases**

Light stop window: M. Carena, M. Quiros, C.E.M. Wagner, Phys.Lett. B380 (1996) 81; M. Carena, G. Nardini, M. Quiros and C.E.M.Wagner JHEP 10 (2008) 062; M. Carena, G. Nardini, M. Quiros, C.E.M. Wagner. Nucl.Phys. B812: 243-263, 2009.

M. Dolgopolov, M. Dubinin, E. Rykova Threshold corrections to the MSSM finite-temperature Higgs potential. e-Print: arXiv:0901.0524v2
In the MSSM we calculate the 1-loop FT corrections from the squarks-Higgs bosons sector, reconstruct the effective two-Higgs-doublet potential and study possibilities of the EWPT in the full MSSM ($m_{H\pm}$, $tg\beta$, $A_{t,b}$, μ, m_Q, m_U, m_D) parameter space.

MODEL [QFTHEP'2003, 2004]

THDM: Fields

Georgi: A Model Of Soft CP Violation. 1978
Lee: A Theory Of Spontaneous T Violation. 1973

$$\Phi_1 = \begin{pmatrix} \phi_1^+(x) \\ \phi_1^0(x) \end{pmatrix} = \begin{pmatrix} -i\omega_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) \end{pmatrix},$$

$$\Phi_2 = e^{i\xi} \begin{pmatrix} \phi_2^+(x) \\ \phi_2^0(x) \end{pmatrix} = e^{i\xi} \begin{pmatrix} -i\omega_2^+ \\ \frac{1}{\sqrt{2}}(v_2 e^{i\zeta} + \eta_2 + i\chi_2) \end{pmatrix}$$

$$\langle\Phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle\Phi_2\rangle = \frac{e^{i\xi}}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\zeta} \end{pmatrix} \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\theta} \end{pmatrix}.$$

$$\mathrm{tg}\,\beta = \frac{v_2}{v_1}, \qquad v^2 \equiv v_1^2 + v_2^2 = (246\ \mathrm{GeV}\ )^2.$$

Ilya F. Ginzburg, M. Krawczyk,
Symmetries of two Higgs doublet model and CP violation. Phys.Rev.D72,2005.

Akhmetzyanova E.N., *D M.V.*, Dubinin M.N.
Higgs Bosons in the Two-Doublet Model with CP Violation Phys.Rev.D71.2005.
Violation of CP invariance in the two-doublet Higgs sector of the MSSM. Phys.Part.Nucl.37,2006.

Effective THDM potential with explicit CP violation

General hermitian renormalized *SU(2)*x*U(1)* invariant potential:

$$U(\Phi_1,\Phi_2) = -\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) - \mu_{12}^2(\Phi_1^\dagger\Phi_2) - \mu_{12}^{*2}(\Phi_2^\dagger\Phi_1) +$$

$$+\frac{\lambda_1}{2}(\Phi_1^\dagger\Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) +$$

$$+\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \frac{\lambda_5^*}{2}(\Phi_2^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) +$$

$$+\lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_6^*(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) +$$

$$+\lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \lambda_7^*(\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$

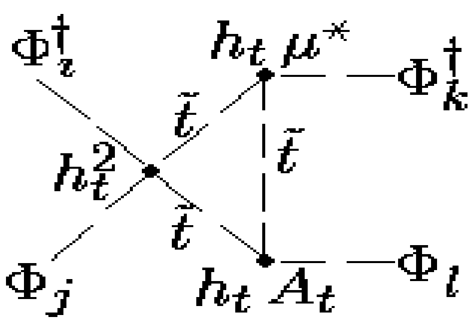
$$\Phi_1^\dagger\Phi_2 \xrightarrow{CP} \Phi_2^\dagger\Phi_1$$

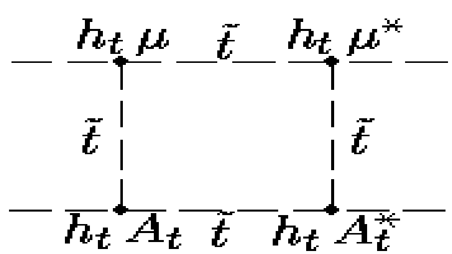
$$\boxed{\lambda_{5,6,7} \xrightarrow{CP} \lambda_{5,6,7}}$$

μ_{12}^2,
$\lambda_5, \lambda_6, \lambda_7$
complex

$\varphi = \arg(\lambda_{6,7})$

$= \arg(\lambda_5)/2$





***U* is *CP*-invariant at the M_{SUSY} scale, because $\lambda_{5,6,7}=0$**

One-loop (t, b) contributions $\cancel{CP}$ U_{eff}

m_{top} — M_{SUSY} — *μ - mass-energy scale*

Eff. potential method
or Feynman diags (temperature T)

Scalar sector for MSSM

The main contribution to self-couplings due to Yukawa 3rd generation couplings.

The corresponding potential with CPV sources

$$\mathcal{V}^0 = \mathcal{V}_M + \mathcal{V}_\Gamma + \mathcal{V}_\Lambda + \mathcal{V}_{\widetilde{Q}}\,,$$

$$\mathcal{V}_M = (-1)^{i+j} m_{ij}^2 \Phi_i^\dagger \Phi_j + M_{\widetilde{Q}}^2 \left(\widetilde{Q}^{\,\dagger}\widetilde{Q}\right) + M_{\widetilde{U}}^2 \widetilde{U}^* \widetilde{U} + M_{\widetilde{D}}^2 \widetilde{D}^* \widetilde{D}\,,$$

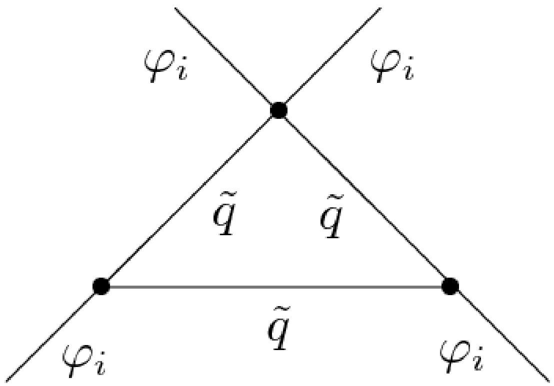
$$\mathcal{V}_\Gamma = \Gamma_i^D \left(\Phi_i^\dagger \widetilde{Q}\right) \widetilde{D} + \Gamma_i^U \left(i\Phi_i^T \sigma_2 \widetilde{Q}\right) \widetilde{U} + \overset{*}{\Gamma}{}_i^D \left(\widetilde{Q}^{\,\dagger} \Phi_i\right) \widetilde{D}^* - \overset{*}{\Gamma}{}_i^U \left(i\widetilde{Q}^{\,\dagger} \sigma_2 \Phi_i^*\right) \widetilde{U}^*$$

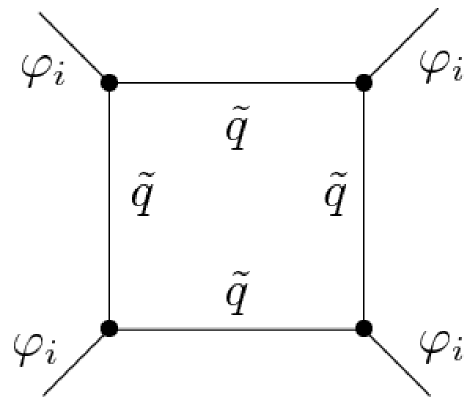
$$\mathcal{V}_\Lambda = \Lambda_{ik}^{jl} \left(\Phi_i^\dagger \Phi_j\right) \left(\Phi_k^\dagger \Phi_l\right) + \left(\Phi_i^\dagger \Phi_j\right) \left[\Lambda_{ij}^Q \left(\widetilde{Q}^{\,\dagger}\widetilde{Q}\right) + \Lambda_{ij}^U \widetilde{U}^* \widetilde{U} + \Lambda_{ij}^D \widetilde{D}^* \widetilde{D}\right] +$$

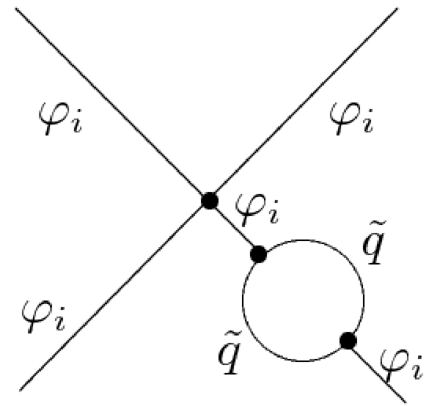
$$+\overline{\Lambda}_{ij}^Q \left(\Phi_i^\dagger \widetilde{Q}\right) \left(\widetilde{Q}^{\,\dagger} \Phi_j\right) + \frac{1}{2}\left[\Lambda \epsilon_{ij} \left(i\Phi_i^T \sigma_2 \Phi_j\right) \widetilde{D}^* \widetilde{U} + \text{э.c.}\right]\,, \quad i, j, k, l = 1, 2$$

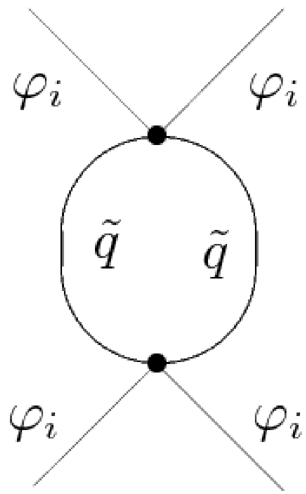
$$\Gamma^U_{\{1;\,2\}} = h_U \left\{-\mu^*; A_U\right\}, \qquad \Gamma^D_{\{1;\,2\}} = h_D \left\{A_D; -\mu^*\right\}$$

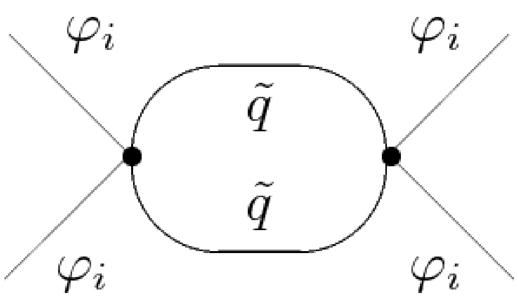
Threshold corrections (left and central diagram) and diagram contributing to the wave-function renormalization (right)

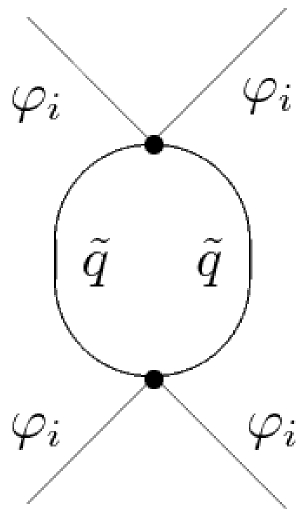






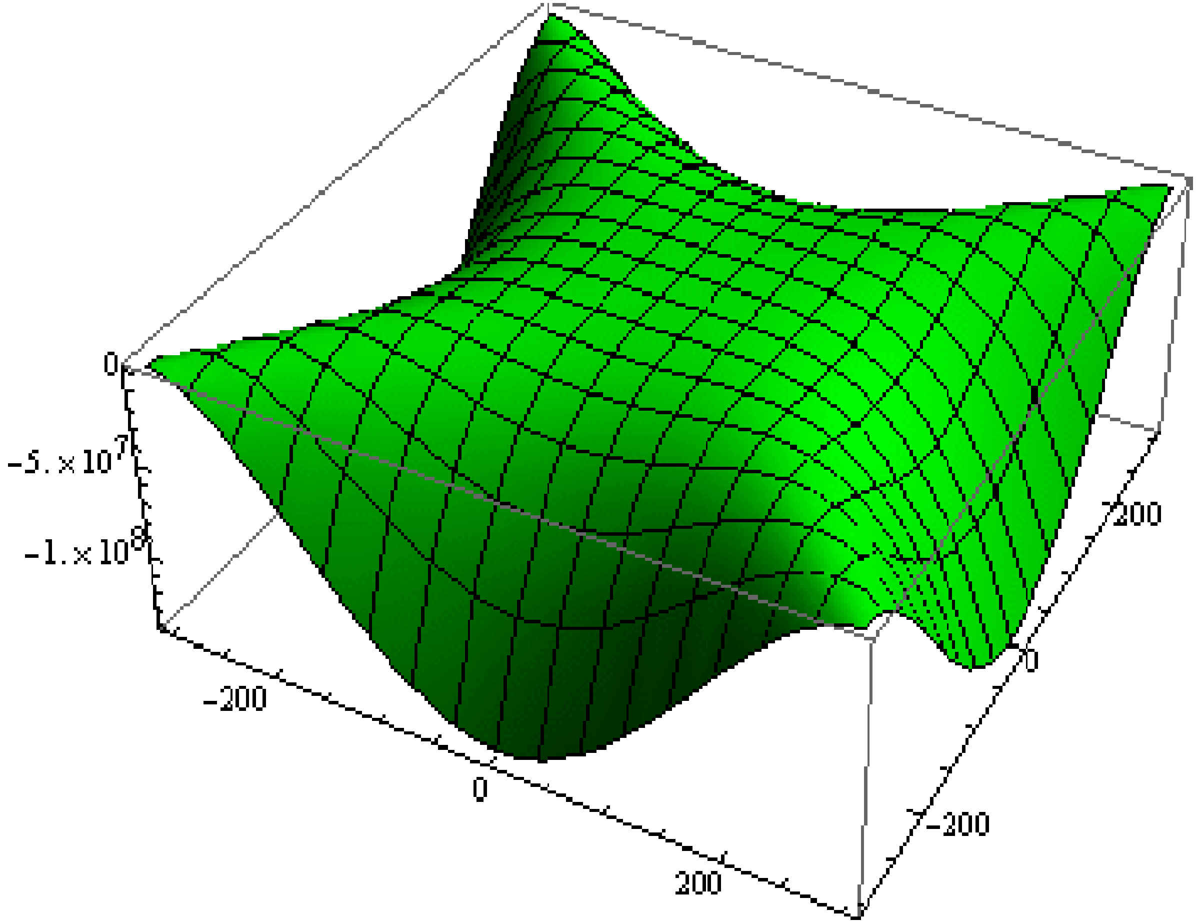






"Fish" diagrams

The *surface of minima* for zero-temperature two-doublet Higgs potential at the scale M_{SUSY}



Integration and summation method

In the finite temperature field theory Feynman diagrams with boson propagators, containing Matsubara frequencies, lead to structures of the form

$$I[m_1, m_2, ..., m_b] = T \sum_{n=-\infty}^{\infty} \int \frac{d\mathbf{k}}{(2\pi)^3} \prod_{j=1}^{b} \frac{(-1)^b}{(\mathbf{k}^2 + \omega_n^2 + m_j^2)},$$

$$\omega_n = 2\pi n T \; (n = 0, \pm 1, \pm 2, ...),$$

T - temperature

Integration and summation method

$$I[m_1, m_2, ..., m_b] \;=\; 2T\,(2\pi T)^{3-2b}\,\frac{(-1)^b \pi^{3/2}}{(2\pi)^3}\,\frac{\Gamma(b-3/2)}{\Gamma(b)}\,S(M, b-3/2),$$

$$S(M, b-3/2) \;=\; \int \{\mathrm{d}x\} \sum_{n=1}^{\infty} \frac{1}{(n^2+M^2)^{b-3/2}}, \qquad M^2 \equiv \left(\frac{m}{2\pi T}\right)^2.$$

Parameters of the effective potential
(forms of contributions)

$$\Delta\lambda_1 = 3h_t^4|\mu|^4 I_2[m_Q, m_t] + 3h_b^4|A|^4 I_2[m_Q, m_b] +$$
$$+h_t^2|\mu|^2(\frac{g_1^2 - 3g_2^2}{2} I_1[m_Q, m_t] + 2g_1^2 I_1[m_t, m_Q]) +$$
$$+h_b^2|A|^2(\frac{12h_b^2 - g_1^2 - 3g_2^2}{2} I_1[m_Q, m_b] + (6h_b^2 - g_1^2) I_1[m_b, m_Q])$$

$$\Delta\lambda_2 = 3h_t^4|A|^4 I_2[m_Q, m_t] + 3h_b^4|\mu|^4 I_2[m_Q, m_b] +$$
$$+h_b^2|\mu|^2(\frac{g_1^2 + 3g_2^2}{2} I_1[m_Q, m_b] + g_1^2 I_1[m_b, m_Q]) +$$
$$+h_t^2|A|^2(\frac{12h_t^2 + g_1^2 - 3g_2^2}{2} I_1[m_Q, m_t] + (6h_t^2 - 2g_1^2) I_1[m_t, m_Q])$$

$$\Delta\lambda_3 + \Delta\lambda_4 = 6h_t^4|\mu|^2|A|^2 I_2[m_Q, m_t] + 6h_b^4|\mu|^2|A|^2 I_2[m_Q, m_b] +$$
$$+h_t^2((|\mu|^2 \frac{12h_t^2 + g_1^2 - 3g_2^2}{4} - |A|^2 \frac{g_1^2 - 3g_2^2}{4}) I_1[m_Q, m_t] +$$
$$+(|A|^2 g_1^2 - |\mu|^2(g_1^2 - 3h_t^2)) I_1[m_t, m_Q]) +$$
$$+h_b^2((|\mu|^2 \frac{-12h_t^2 + g_1^2 + 3g_2^2}{4} - |A|^2 \frac{g_1^2 + 3g_2^2}{4}) I_1[m_Q, m_b] +$$
$$+\frac{1}{2}(|A|^2 g_1^2 - |\mu|^2(g_1^2 - 6h_b^2)) I_1[m_b, m_Q])$$

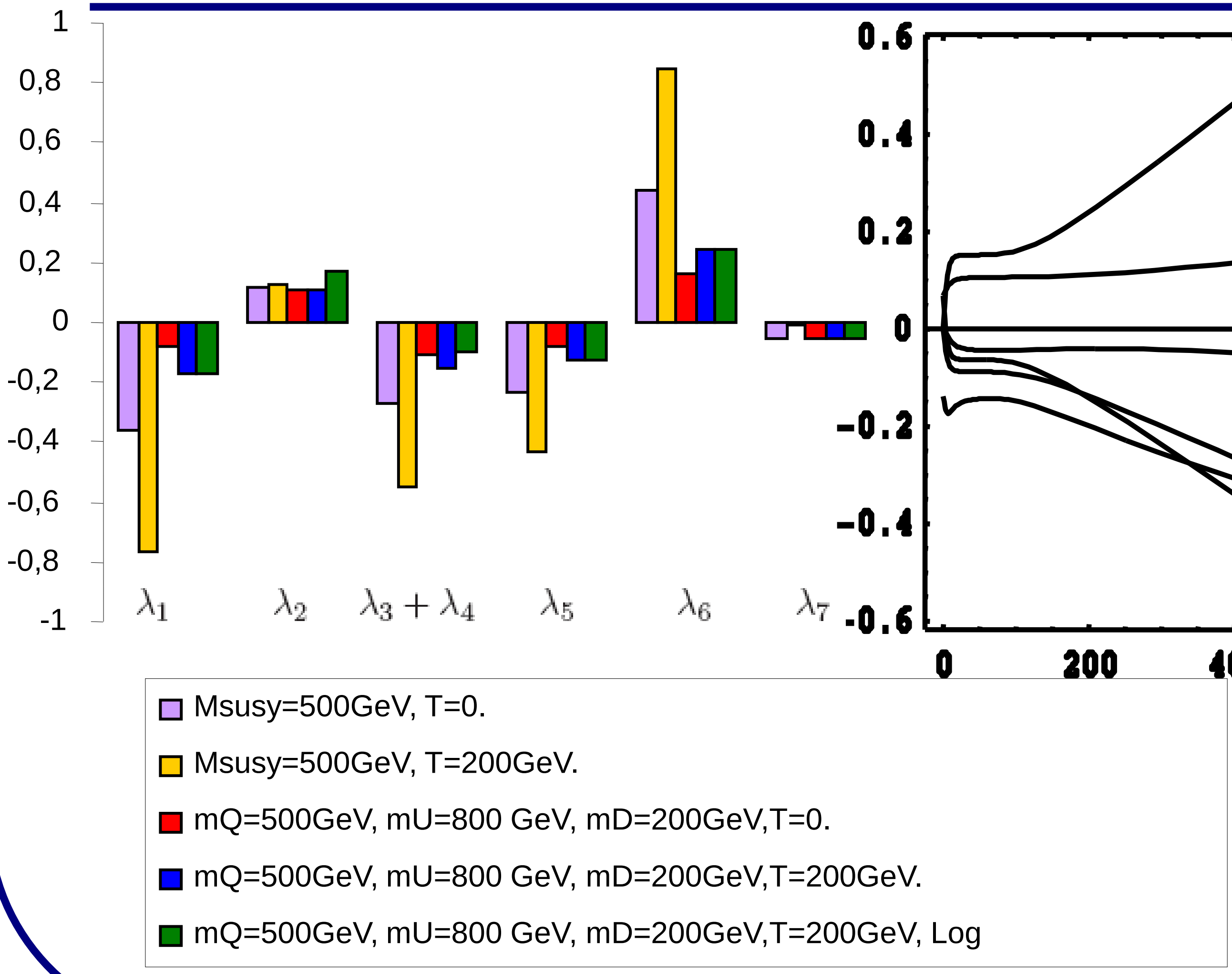
Parameters of the effective potential
(forms of contribitions)

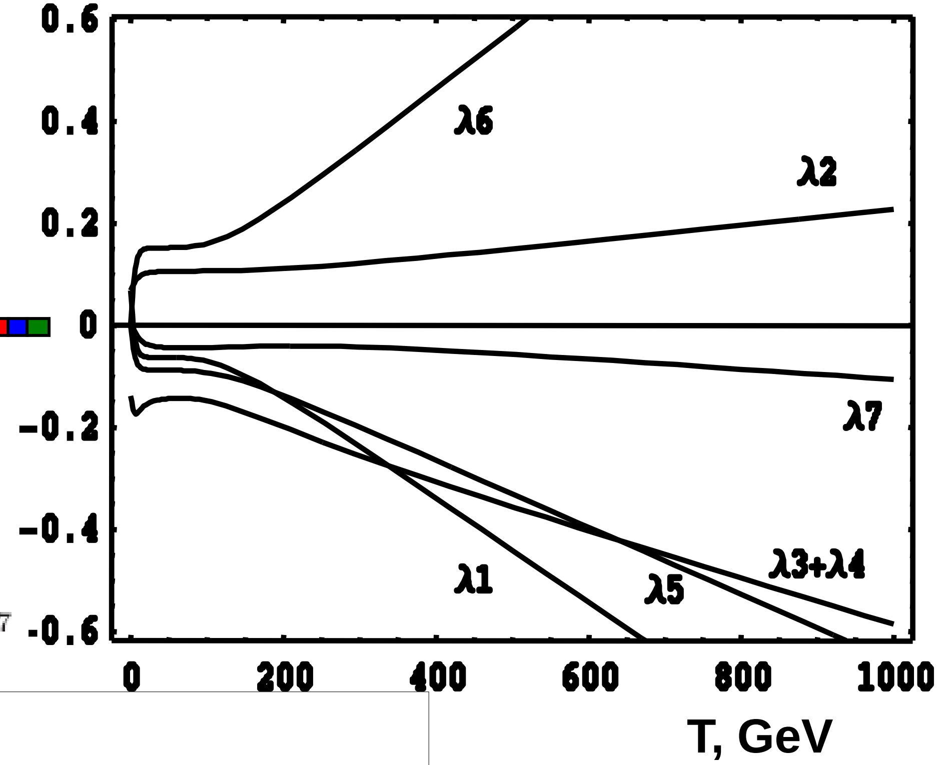
$$\Delta\lambda_5 = 3h_t^4\mu^2 A^2 I_2[m_Q, m_t] + 3h_b^4\mu^2 A^2 I_2[m_Q, m_b]$$

$$\Delta\lambda_6 = -3h_t^4\mu A|\mu|^2 I_2[m_Q, m_t] - 3h_b^4\mu A|A|^2 I_2[m_Q, m_b] +$$
$$+h_t^2\mu A(\frac{g_1^2 - 3g_2^2}{4} I_1[m_Q, m_t] - g_1^2 I_1[m_t, m_Q]) +$$
$$+h_b^2\mu A(\frac{-12h_b^2 + g_1^2 + 3g_2^2}{4} I_1[m_Q, m_b] - \frac{6h_b^2 - g_1^2}{2} I_1[m_b, m_Q])$$

$$\Delta\lambda_7 = -3h_t^4\mu A|A|^2 I_2[m_Q, m_t] - 3h_b^4\mu A|\mu|^2 I_2[m_Q, m_b] +$$
$$+h_b^2\mu A(-\frac{g_1^2 + 3g_2^2}{4} I_1[m_Q, m_b] - \frac{g_1^2}{2} I_1[m_b, m_Q]) +$$
$$+h_t^2\mu A(\frac{12h_t^2 + g_1^2 - 3g_2^2}{4} I_1[m_Q, m_t] - (3h_t^2 - g_1^2) I_1[m_t, m_Q])$$

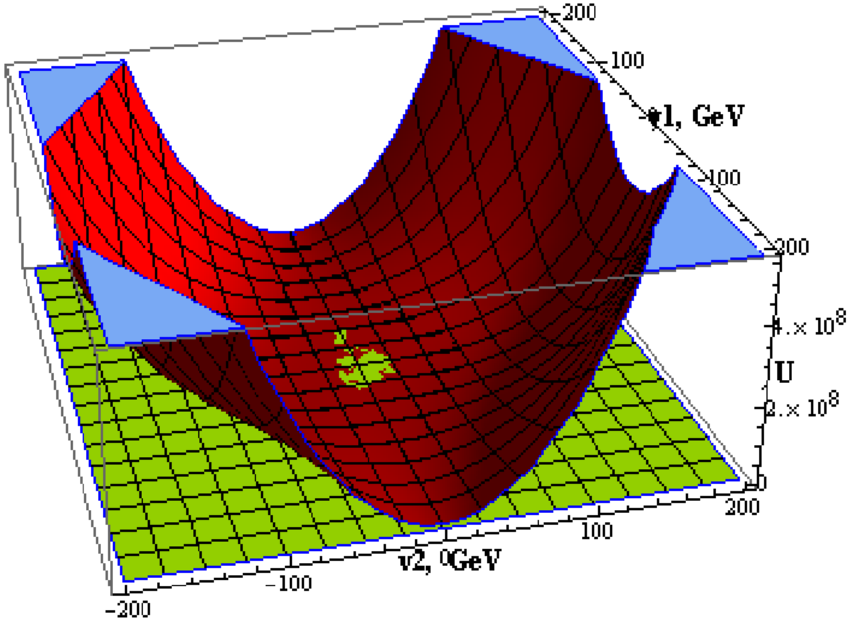
Temperature-dependent parameters with various quantum corrections in CPX-like scenario $A_t = A_b = 1000$ GeV, $\mu = 2000$ GeV

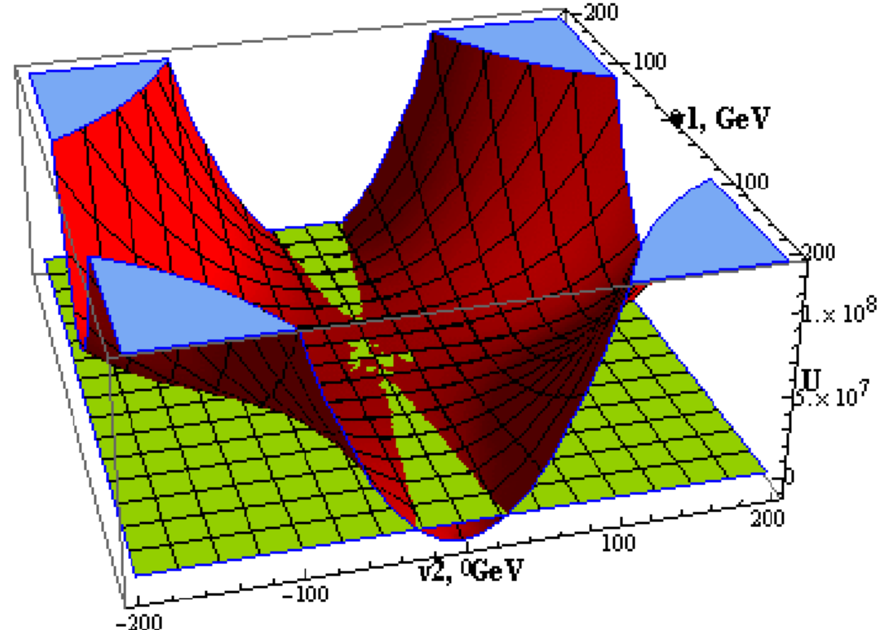




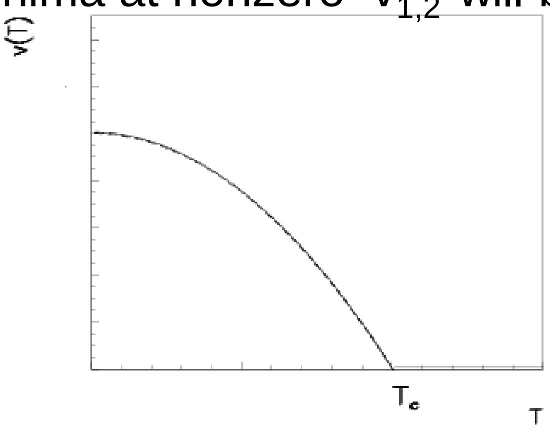
CPX: M.Carena, J.Ellis, A.Pilaftsis, C.Wagner, PL B495 (2000) 155

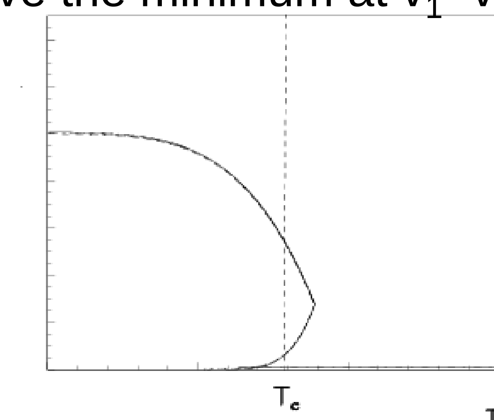
The *surfaces of minima* for effective potential U(v1,v2) at the critical temperature T=120 GeV, $\lambda_6 = \lambda_7 = 0$



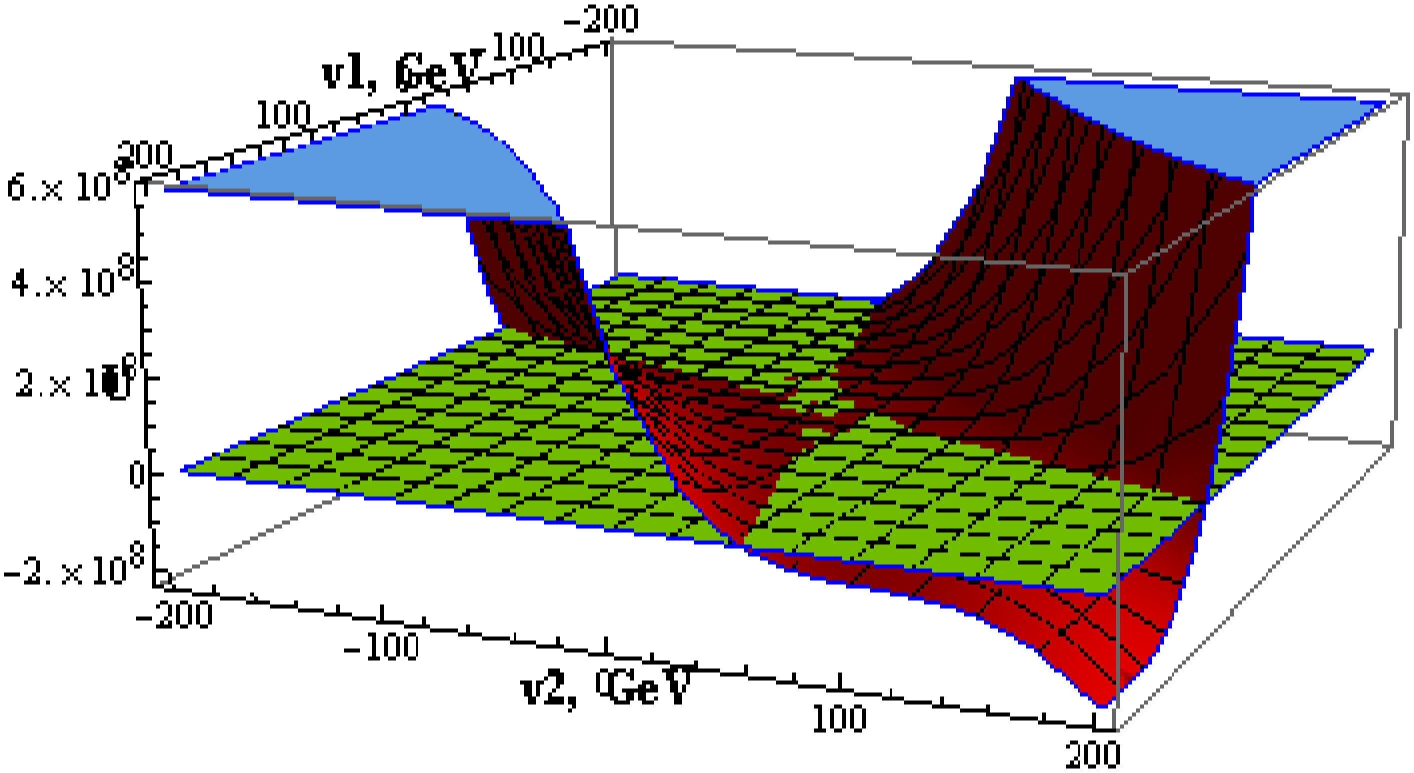


Phase transitions of the first order can occur along the going down hollow. In other directions minima at nonzero $v_{1,2}$ will be above the minimum at $v_1=v_2=0$.





The *surfaces of minima* for effective potential U(v1,v2) at the critical temperature T=120 GeV and nonzero λ_6, λ_7



At nonzero λ_6, λ_7 there are directions always along which the first order phase transition exists.

Effective potential at finite temperature

$$v_1(T) = v(T)\cos\bar{\beta}(T), \quad v_2(T) = v(T)\sin\bar{\beta}(T)$$

Mass term

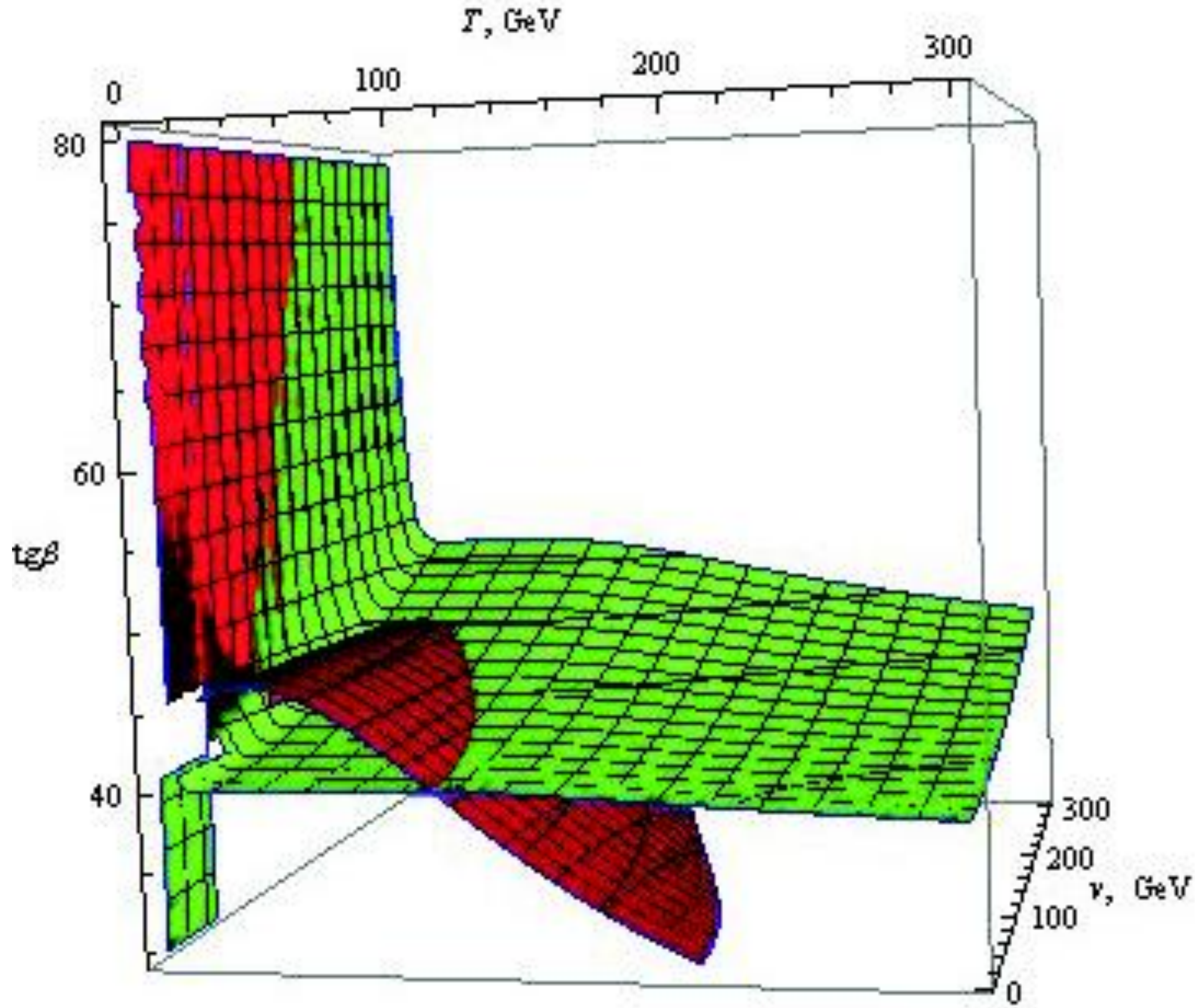
$$U_{mass}(v,\bar{\beta}) = -\frac{v^2}{2}(\mu_1^2\cos^2\bar{\beta} + \mu_2^2\sin^2\bar{\beta}) - \frac{v^2}{2}\mu_{12}^2\sin 2\bar{\beta}$$

Critical temperature determination

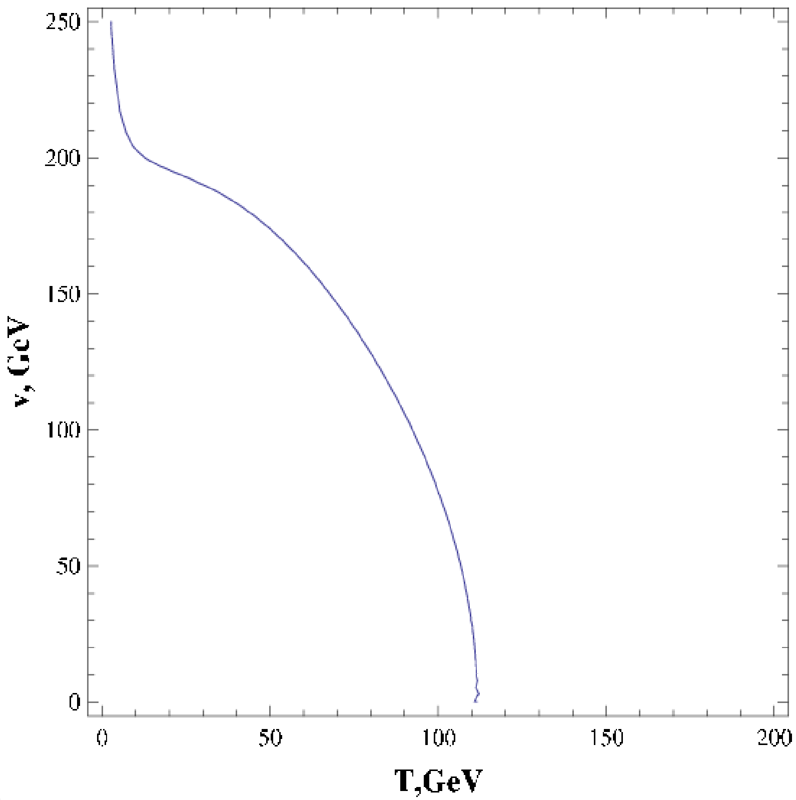
$$\partial U_{mass}/\partial v = 0 \qquad 1/v\ \partial U_{mass}/\partial\bar{\beta} = 0$$

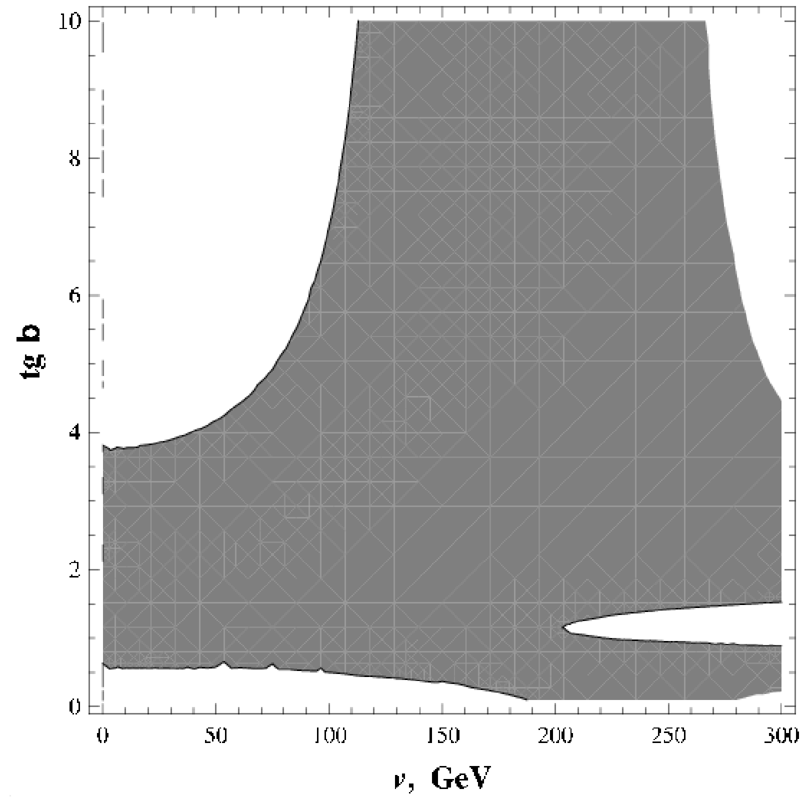
$$\mathtt{tg}2\bar{\beta} = \frac{2\mu_{12}^2}{\mu_1^2-\mu_2^2}, \qquad (\mu_1^2\mu_2^2 - \mu_{12}^4)[(\mu_1^2-\mu_2^2)^2 + 4\mu_{12}^4] = 0$$

$$\mu_1^2\mu_2^2 = \mu_{12}^4$$


T, GeV
0
100
200
300
80
60
tgβ
40
300
200
v, GeV
100
0

Evolution of the critical parameters





Effective potential U(v1,v2) at the critical temperature and nonzero λ_6 λ_7

$$\mathtt{tg}2\bar{\beta} \;=\; \mathtt{tg}2\beta\,\frac{1}{\left(\frac{v^2}{2m_A^2}-\alpha_1\right)}\,\frac{1}{\frac{2\lambda_1\cos^2\beta-2\lambda_2\sin^2\beta}{\cos 2\beta}-\lambda_{345}+\frac{2m_A^2}{v^2}+\alpha_2}$$

$$\alpha_1=\frac{\lambda_5}{2}+\frac{1}{4}(\lambda_6\mathtt{ctg}\beta+\lambda_7\mathtt{tg}\beta),$$

$$\alpha_2=\lambda_6(\mathtt{tg}2\beta-\mathtt{ctg}\beta)-\lambda_7(\mathtt{tg}\beta+\mathtt{tg}2\beta).$$

$$\Downarrow$$

$$-\frac{m_A^2}{v^2}(2\lambda_5+\lambda_6\,\mathtt{ctg}\beta+\lambda_7\,\mathtt{tg}\beta)+$$

$$+\;\frac{v^2}{m_A^2}\,\left[\frac{2\lambda_1-2\lambda_2\mathtt{tg}^2\beta+\lambda_6(3\mathtt{tg}\beta-\mathtt{ctg}\beta)+\lambda_7(\mathtt{tg}^3\beta-3\mathtt{tg}\beta)}{1-\mathtt{tg}^2\beta}-\lambda_{345}\right]=0.$$

$$\Downarrow$$

$$\lambda_1\,(2\lambda_2-\lambda_{345})^2+\lambda_2\,(2\lambda_1-\lambda_{345})^2+\lambda_{345}(2\lambda_1-\lambda_{345})(2\lambda_2-\lambda_{345})=0$$

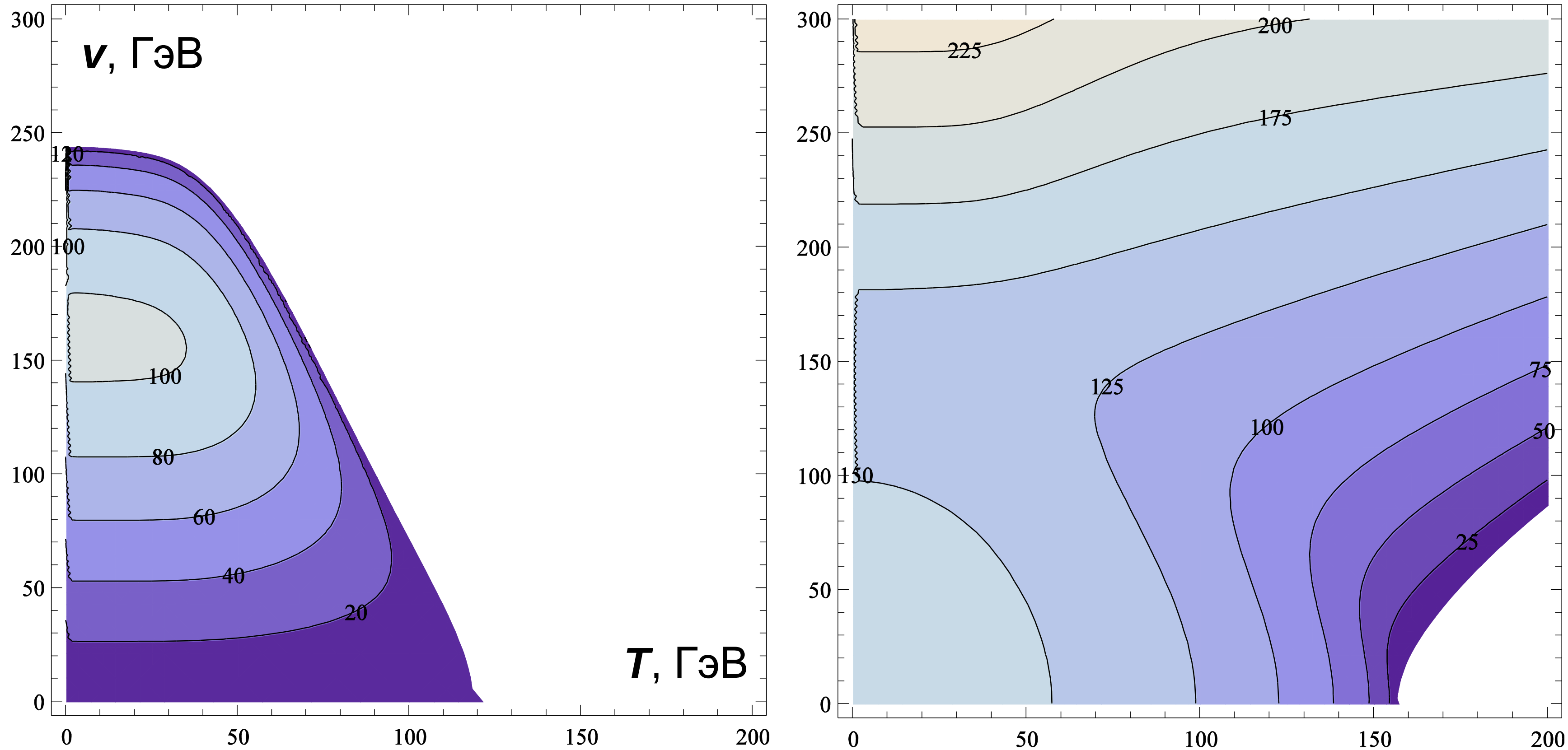
The thermal evolution of the CP-even Higgs bosons h and H is expressed by

$$
\begin{aligned}
m_h^2 = c_{\alpha-\beta}^2 m_A^2 + v^2 (2\lambda_1 s_\alpha^2 c_\beta^2 + 2\lambda_2 c_\alpha^2 s_\beta^2 - 2(\lambda_3 + \lambda_4) c_\alpha c_\beta s_\alpha s_\beta + \\
+ \mathrm{Re}\lambda_5 (s_\alpha^2 s_\beta^2 + c_\alpha^2 c_\beta^2) - 2 c_{\alpha+\beta} (\mathrm{Re}\lambda_6 s_\alpha c_\beta - \mathrm{Re}\lambda_7 c_\alpha s_\beta)), \\
m_H^2 = s_{\alpha-\beta}^2 m_A^2 + v^2 (2\lambda_1 c_\alpha^2 c_\beta^2 + 2\lambda_2 s_\alpha^2 s_\beta^2 + 2(\lambda_3 + \lambda_4) c_\alpha c_\beta s_\alpha s_\beta + \\
+ \mathrm{Re}\lambda_5 (c_\alpha^2 s_\beta^2 + s_\alpha^2 c_\beta^2) + 2 s_{\alpha+\beta} (\mathrm{Re}\lambda_6 c_\alpha c_\beta + \mathrm{Re}\lambda_7 s_\alpha s_\beta)),
\end{aligned}
$$

where *α is* the mixing angle of the CP-even states h and H.

[Akhmetzyanova E.N., Dolgopolov M.V., Dubinin M.N. Higgs Bosons in the Two-Doublet Model with CP Violation // Phys.Rev.D. V.71. N7. 2005. P.075008. (hep-ph/0405264)]

Higgs bosons masses



$\mathrm{tg}\beta = 5,\ m_{H^\pm} = 180\ \mathrm{GeV},\ A_{t,b} = 1200\ \mathrm{GeV},\ \mu = 500\ \mathrm{GeV}.$

Conclusions

1. In the MSSM we calculate the 1-loop finite-temperature corrections from the squarks-Higgs bosons sector, reconstruct the effective two-Higgs-doublet potential in the full MSSM parameter space ($m_{H\pm}$, $tg\beta$, $A_{t,b}$, μ, m_Q, m_U, m_D).

2. At large values of A and μ of around 1 TeV, favored indirectly by LEP2 and Tevatron data, the threshold finite-temperature corrections from triangle and box diagrams with intermediate third generation squarks are very substantial.

3. High sensitivity of the low-temperature evolution to the effective two-doublet and the MSSM squark sector parameters is observed, but rather extensive regions of the full MSSM parameter space allow the first-order electroweak phase transition respecting the phenomenological constraints at zero temperature.

Perspectives

- **The topology analysis of extended Higgs potentials**
- **viable models:**
  THDM, MSSM,
  Singlet models: many possibilities
- **Electroweak baryogenesis is still viable in extended Higgs sectors**
- **It would offer the possibiliy to compute the baryon asymmetry from parameters measured in collider experiments**
- **If the result would match the observations, we could claim to understand the early universe up to electroweak temperature**
- **Strong constraints on CP phases from EDM's**