Pion formfactor in time-like region in the framework of relativistic composite quark model.

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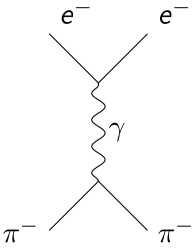
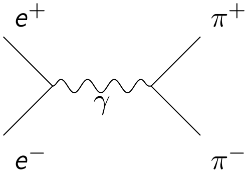
Definition of a pion formfactor.



$\pi^- e^- \to \pi^- e^-$



$e^+ e^- \to \pi^+ \pi^-$

Formfactor is a scalar function, that parametrises a current matrix element.

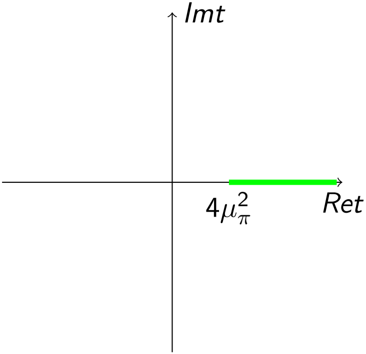
$$\langle \pi_i | \hat{j}^\mu(0) | \pi_j \rangle \sim (p_j + p_i)^\mu F_\pi \left[(p_i - p_j)^2 \right] \qquad (1)$$

$t = q^2 = -Q^2 = (p_i - p_j)^2 < 0$ - Space-like region

$$\langle \pi_i \pi_j | \hat{j}^\mu(0) | 0 \rangle \sim (p_j - p_i)^\mu F_\pi \left[(p_i + p_j)^2 \right] \qquad (2)$$

$t = q^2 = -Q^2 = (p_i + p_j)^2 > 4\mu_\pi^2$ - Time-like region

Fundamental analytical properties of F_π.



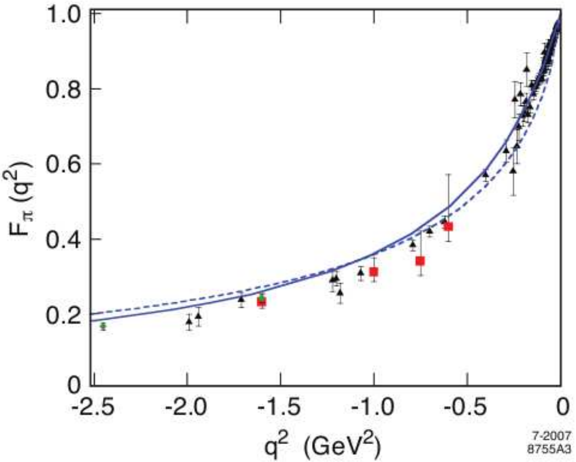
- Analiticity region.
- Normalization condition $F_\pi(0) = 1$
- Physical value of formfactor

$$F_\pi^{(phys)}(\tau) = \lim_{\varepsilon \to +0} F_\pi(\tau + i\varepsilon)$$

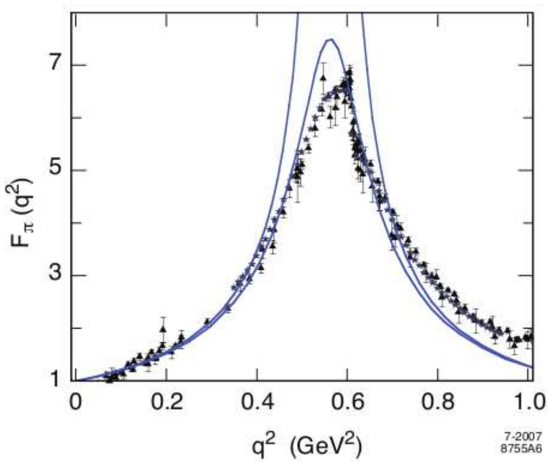
- QCD asymphtotics

$$|t| \to \infty \Rightarrow F_\pi(t) \sim \frac{1}{t}$$

Pion formfactor experimental data in space-like region.



Pion formfactor experimental data in time-like region.



Experiment gives us an absolute value of F_π.

Expression for the F_π in constituent quark model.

$$F_\pi(t) = \frac{1}{4}\int_{\Omega_{ss'}} dsds' \frac{\varphi(s)\varphi(s')}{\sqrt{(s-4M^2)(s'-4M^2)}} \frac{(-t)(s+s'-t)}{\lambda^{3/2}(s,t,s')}$$

Region of integration:

$$\Omega_{ss'} = \{(s,s')|s \in [4M^2,+\infty); s' \in [s_1(s,t), s_2(s,t)]\}$$

$$s_{1,2}(s,t) = s + t - \frac{ts}{2M^2} \pm \frac{1}{2M^2}\sqrt{(-t)(4M^2-t)s(s-4M^2)}$$

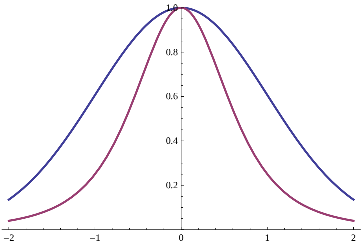
$$\lambda(s,t,s') = s^2 + t^2 + s'^2 - 2(ss' + st + s't)$$

Wavefunction of constituent in instant form of RHD:

$$\varphi(s) = \sqrt[4]{s}\frac{\sqrt{s-4M^2}}{2}u(\frac{\sqrt{s-4M^2}}{2})$$

Where $u(k)$ - wavefunction depends of a three-momentum.

Typically using wawefunctions.



- Harmonic oscillator wavefunction.

$$u(k) = N_{HO} \exp\left(\frac{-k^2}{2b^2}\right)$$

- Wavefunction with power-behaviour (QCD-motivated).

$$u(k) = N_{PL}\left(1 + \frac{k^2}{b^2}\right)^{-n}$$

- WF with linear confinement and Coulomb behaviour in a small scale.

$$u(r) = N_T \exp\left(-\alpha r^{3/2} - \beta r\right)$$

New symmetrical variables.

$$x = s + s' - t$$
$$y = \sqrt{4ss'}$$

Formfactor in new variables:

$$F_\pi(t) = \frac{(-t)}{8\sqrt{2}} \int_{x_{min}(t)}^{+\infty} \int_{y_1(x,t)}^{y_2(x,t)} \psi(x,y,t) \frac{xy^{3/2} dydx}{\sqrt{(t+x)^2 - y^2}\,(x^2 - y^2)^{3/2}}$$

where: $y_1(x,t) = x\sqrt{\frac{4M^2}{4M^2-t}}$, $y_2(x,t) = x + t$,
$x_{min}(t) = (4M^2 - t) + 2M\sqrt{4M^2 - t}$
Function $\psi(x,y,t)$ satisfies the relation:

$$u\left(\frac{\sqrt{s-4M^2}}{2}\right) u\left(\frac{\sqrt{s'-4M^2}}{2}\right) = \psi(s + s' - t, \sqrt{4ss'}, t)$$

Statement about analitical properties of a model.

Theorem

If special requirements on a ψ-function satisfied, integral representation defines an analytical function in a plane t with a cut from $4M^2$ to infinity.

Proof of analiticity.

Absolute convergence $\Rightarrow$ Uniform convergence $\Rightarrow$ Analiticity.
Integration paths in x and y planes:

$$\gamma_x = \{[x_{min}(t) + \chi] \,|\chi \in [0, +\infty)\}$$

$$\gamma_y = \{[(1-\tau)y_1(x_{min}(t) + \chi, t) + \tau y_2(x_{min}(t) + \chi, t)] \,|\tau \in [0, 1]\}$$

Absolute value of under integral expression:

$$|\psi(x, y, t)| \left| \frac{xy^{3/2}(y_2(x, t) - y_1(x, t))}{\sqrt{(t + x)^2 - y^2}(x^2 - y^2)^{3/2}} \right|_{x=x(\chi), y=y(\chi,\tau)} =$$

$$= |\psi(\chi, \tau, t)||K(\chi, \tau, t)|$$

Proof of analiticity.

$$|K(\chi,\tau,t)| \leq \frac{|x_{min}(t)+\chi|\mu(\chi)}{\sqrt{1-\tau}}$$

where $\mu(\chi) = \max_{\tau\in[0,1]}\{\frac{\sqrt{|y_2-y_1|}|y_1(1-\tau)+y_2\tau|^{3/2}}{\sqrt{|(y_2+y_1)+\tau(y_2-y_1)|}|x^2-(y_1+\tau(y_2-y_1)^2)|^{3/2}}\}$
For absolute convergence is enough that:

$$\int\limits_0^\infty \max_{\tau\in[0,1]} |\psi(\chi,\tau,t)||x_{min}(t)+\chi|\mu(\chi)d\chi < \infty$$

Asymphthotycs: $|x_{min}(t)+\chi|\mu(\chi) \sim \frac{1}{\sqrt{\chi}} \Rightarrow$ For AC is enough that

$$\chi\to\infty : \max_{\tau\in[0,1]} |\psi(\chi,\tau,t)| \sim o\left(\frac{1}{\sqrt{\chi}}\right)$$

Proof of the existence of a cut.

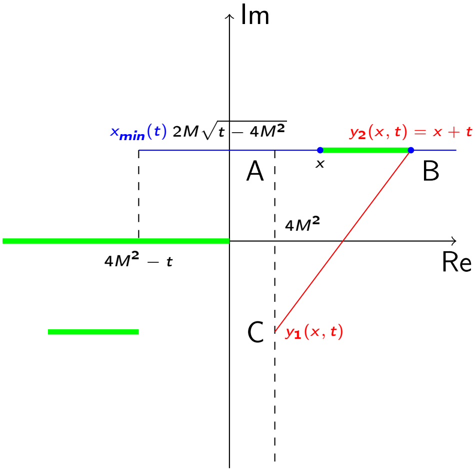
We need to prove that:
$F_\pi(\tau - i0) = (F_\pi(\tau + i0))^*$
Limits of integration:

$$x_{min}(\tau \pm i0) = (4M^2 - \tau) \mp 2Mi\sqrt{\tau - 4M^2}$$

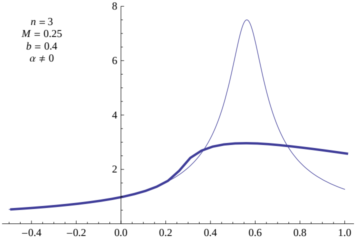
$$y_1(x, \tau \pm i0) = \pm ix\sqrt{\frac{4M^2}{\tau - 4M^2}}$$

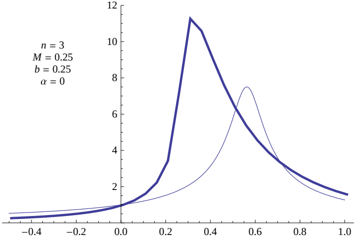
$$y_2(x, \tau \pm i0) = x + \tau$$

When we crossing the cut in a plane t, all picture of singularities and contours changing to is's complex conjugate picture.



Results of numerical calculations.





Calculations with power-behaviour WF:

$$u(k) = N_{PL}\left(1+\frac{k^2}{b^2}\right)^{-n}$$

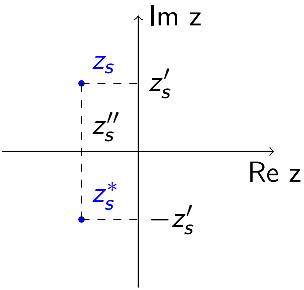
$$\psi(x,y,t) = \frac{N_{PL}^2(4b^2)^{2n}}{\left(\frac{y^2}{4}+\alpha(x+t)+\alpha^2\right)^n}$$

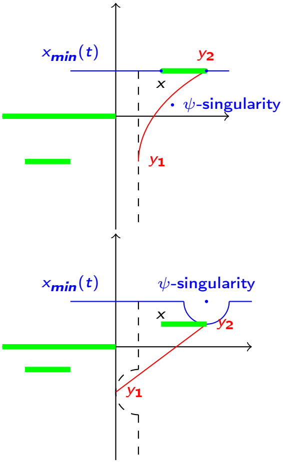
Where $\alpha = 4(b^2 - M^2)$.
Hypotesis: Behaviour of F_π in time-like region depends of analytical properties of wavefunction $u(k)$ in complex plane k.

Criteria of existence of a resonance.

New variable:

$$t = 4M^2 - z^2$$





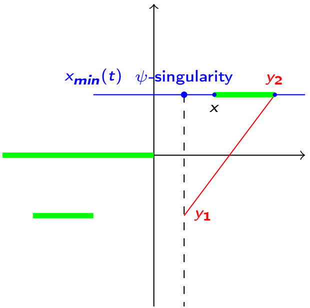
Criteria of existence of a resonance.

$$y_1(x_{min}(4M^2-z_s^2), 4M^2-z_s^2) = y_2(x_{min}(4M^2-z_s^2), 4M^2-z_s^2) = 2M(z_s+2M)$$

Criteria:

$$\psi(x_{min}(4M^2 - z_s^2), 2M(z_s + 2M)) = \infty$$

$$\psi(x_{min}(4M^2 - z_s^{*2}), 2M(z_s^* + 2M)) = \infty$$



Criteria in terms of u(k):

$$u(\sqrt{K}) = \infty$$

$$u(\sqrt{K^*}) = \infty$$

where $K = \frac{1}{4}(Mz_s - 2M^2)$

Minimal rational model

Conditions for u(k):

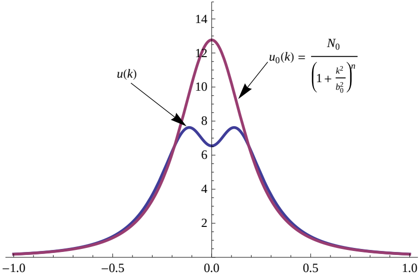
- u(k) will be even.
- Cryteria for poles in z plane.
- $k \to \infty : u(k) \sim \frac{N_0}{(1+k^2/b_0^2)^n}$

Minimal model:

$$u(k) = N \left[\frac{k^2 + a^2}{(k^2 - K)(k^2 - K^*)} \right]^n$$

Normalization condition:

$$\int\limits_{-\infty}^{+\infty} k^2 u^2(k) dk = 1$$



Definition of parameters.

Equation defines parameter a:

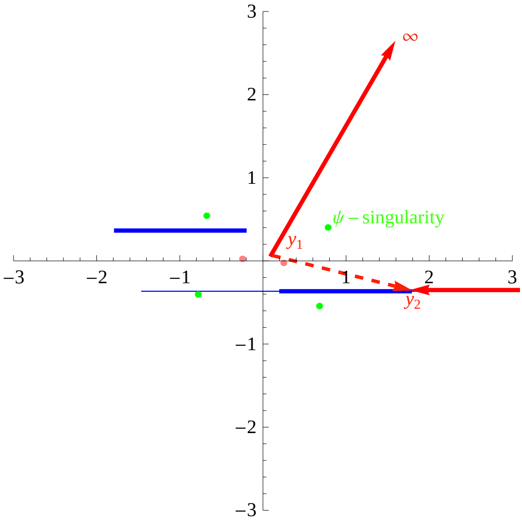
$$\int_{-\infty}^{+\infty}\left[\frac{k^2+a^2}{(k^2-K)(k^2-K^*)}\right]^n k^2dk = b_0^{3-4n}B(2n-3/2,3/2)$$

Where $K=\frac{1}{4}(iMz_s'-2b_0^2)$, and $N=(b_0^{3-4n}B(2n-3/2,3/2))^{-1/2}$.
Parameters of a model:

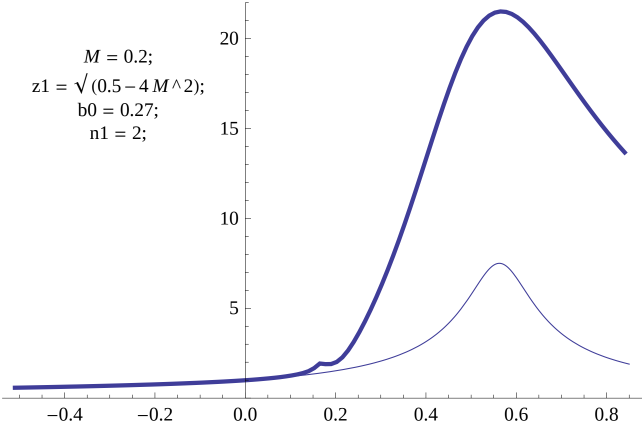
$$\underbrace{M,b_0,}\underbrace{z_s',n}$$

Spacelike timelike

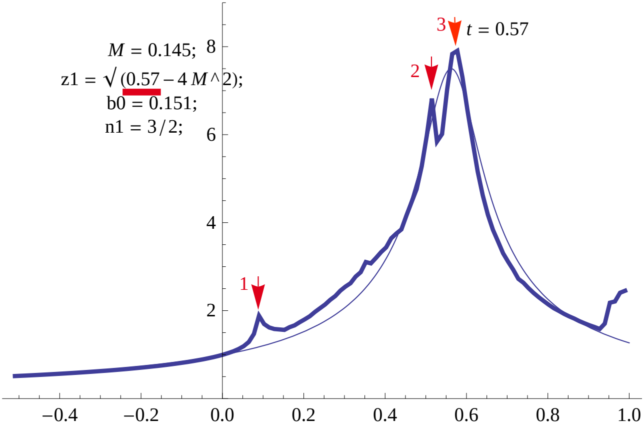
Problem of choosing of contour.



Numerical results n=2.



Numerical results n=3/2.



Thank you for your attention!