New realization of Conformal Symmetry Breaking in perturbative QCD

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Plan of Presentation

Introduction – review of results of advanced QCD analytical calculations

Perturbative violation of the conformal symmetry in the quark-parton model – effects of non-zero β-function

New form of the relation between e^+e^--characteristics and DIS sum rules in Euclidean region – **power series in $\boldsymbol{\beta(a_s)/a_s}$–term**

Banks-Zaks condition $\beta_0(N_F^*) = 0$, $(N_F^* = \frac{11}{2}C_{\rm A})$ and new form of constraints for **Baikov, Chetyrkin, Kuhn (2010)** results

Conclusions

Introduction

Review of results of advanced QCD analytical calculation

Results of advanced QCD analytical calculation 1

Main quantities for e^+e^- annihilation process (Novosibirsk- Russia; Beijing- China), Adler function (nonsinglet) D_A^{NS} :

$$D_A^{NS}(Q^2) = Q^2 \int_0^\infty \frac{R(s)}{(s+Q^2)^2} ds = 3 \sum_f Q_f^2 \cdot \boldsymbol{D_A}(a_s)$$

$$\boldsymbol{D_A}(a_s) = 1 + \sum_n a_s^n \, \boldsymbol{d_n}, \quad a_s = \alpha_s(\mu^2 = Q^2)/\pi$$

Bjorken polarized sum rule B_{jp} – measured at CEBAF at intermediate/low Q^2

$$\mathrm{B_{jp}}(Q^2) = \int_0^1 \left[g_1^{lp}(x, Q^2) - g_1^{ln}(x, Q^2) \right] dx = \frac{1}{6} g_A \cdot \boldsymbol{C^{NS}}(a_s)$$

$$\boldsymbol{C^{NS}}(a_s) = 1 + \sum_n a_s^n \, \boldsymbol{c_n}$$

related with a_s^n-coefficients of pQCD series for
Gross-Llewellyn Smith sum rule - measured in νN DIS

$$\mathrm{GLS}(Q^2) = \int_0^1 F_3^{\nu N}(x, Q^2) dx$$

$d_1 = C_F \frac{3}{4}$;
d_2 – Chetyrkin,Kataev,Tkachov (1979); d_3 – Gorishny, Kataev, Larin (1988-90, presented 1990, published 1991);

$$
\begin{aligned}
d_4 \quad = \quad & \frac{d_F^{abcd} d_A^{abcd}}{d_R} \left[\frac{3}{16} - \frac{1}{4}\zeta_3 - \frac{5}{4}\zeta_5 \right] + N_F \frac{d_F^{abcd} d_F^{abcd}}{d_R} \left[-\frac{13}{16} - \zeta_3 + \frac{5}{2}\zeta_5 \right] + \\
& C_F^4 \left[\frac{4157}{2048} + \frac{3}{8}\zeta_3 \right] + C_F^3 T_F N_F \left[\frac{1001}{384} + \frac{99}{32}\zeta_3 - \frac{125}{4}\zeta_5 + \frac{105}{4}\zeta_7 \right] + \\
& C_F^2 T_F^2 N_F^2 \left[\frac{5713}{1728} - \frac{581}{24}\zeta_3 + \frac{125}{6}\zeta_5 + 3\zeta_3^2 \right] - C_F T_F^3 N_F^3 \left[\frac{6131}{972} - \frac{203}{54}\zeta_3 - \frac{5}{3}\zeta_5 \right] - \\
& C_F^3 C_A \left[\frac{253}{32} + \frac{139}{128}\zeta_3 - \frac{2255}{32}\zeta_5 + \frac{1155}{16}\zeta_7 \right] + \\
& C_F^2 T_F N_F C_A \left[\frac{32357}{13824} + \frac{10661}{96}\zeta_3 - \frac{5155}{48}\zeta_5 - \frac{33}{4}\zeta_3^2 - \frac{105}{8}\zeta_7 \right] + \\
& C_F T_F^2 N_F^2 C_A \left[\frac{340843}{5184} - \frac{10453}{288}\zeta_3 - \frac{170}{9}\zeta_5 - \frac{1}{2}\zeta_3^2 \right] + \\
& C_F^2 C_A^2 \left[-\frac{592141}{18432} - \frac{43925}{384}\zeta_3 + \frac{6505}{48}\zeta_5 + \frac{1155}{32}\zeta_7 \right] + \\
& C_F T_F N_F C_A^2 \left[-\frac{4379861}{20736} + \frac{8609}{72}\zeta_3 + \frac{18805}{288}\zeta_5 - \frac{11}{2}\zeta_3^2 + \frac{35}{16}\zeta_7 \right] + \\
& C_F C_A^3 \left[\frac{52207039}{248832} - \frac{456223}{3456}\zeta_3 - \frac{77995}{1152}\zeta_5 + \frac{605}{32}\zeta_3^2 - \frac{385}{64}\zeta_7 \right]
\end{aligned}
$$

– Baikov, Chetyrkin, Kuhn (2010), new results

$c_1 = -\mathrm{C_F}\frac{3}{4}$

c_2 – Gorishny,Larin (1986); c_3 – Larin, Vermaseren (1991)

$$
\begin{aligned}
c_4 \quad = \quad & \frac{\mathrm{d_F^{abcd}d_A^{abcd}}}{\mathrm{d_R}}\left[-\frac{3}{16}+\frac{1}{4}\zeta_3+\frac{5}{4}\zeta_5\right]+\mathrm{N_F}\frac{\mathrm{d_F^{abcd}d_F^{abcd}}}{\mathrm{d_R}}\left[\frac{13}{16}+\zeta_3-\frac{5}{2}\zeta_5\right] \\
& +\mathrm{C_F T_F^3 N_F^3}\left[\frac{605}{972}\right]+\mathrm{C_F C_A T_F^2 N_F^2}\left[-\frac{165283}{20736}-\frac{43}{144}\zeta_3+\frac{5}{12}\zeta_5-\frac{1}{6}\zeta_3^2\right] \\
& +\mathrm{C_F^2 T_F^2 N_F^2}\left[-\frac{265}{576}+\frac{29}{24}\zeta_3\right]+\mathrm{C_F^3 T_F N_F}\left[\frac{839}{2304}+\frac{451}{96}\zeta_3-\frac{145}{24}\zeta_5\right] \\
& +\mathrm{C_F^2 C_A T_F N_F}\left[-\frac{87403}{13824}-\frac{1289}{144}\zeta_3+\frac{275}{144}\zeta_5+\frac{35}{4}\zeta_7\right] \\
& +\mathrm{C_F C_A^2 T_F N_F}\left[\frac{1238827}{41472}+\frac{59}{64}\zeta_3-\frac{1855}{288}\zeta_5+\frac{11}{12}\zeta_3^2-\frac{35}{16}\zeta_7\right] \\
& +\mathrm{C_F C_A^3}\left[-\frac{8004277}{248832}+\frac{1069}{576}\zeta_3+\frac{12545}{1152}\zeta_5-\frac{121}{96}\zeta_3^2+\frac{385}{64}\zeta_7\right] \\
& +\mathrm{C_F^2 C_A^2}\left[\frac{1071641}{55296}+\frac{1591}{144}\zeta_3-\frac{1375}{144}\zeta_5-\frac{385}{16}\zeta_7\right] \\
& +\mathrm{C_F^3 C_A}\left[-\frac{3707}{4608}-\frac{971}{96}\zeta_3+\frac{1045}{48}\zeta_5\right]+\mathrm{C_F^4}\left[-\frac{4823}{2048}-\frac{3}{8}\zeta_3\right]
\end{aligned}
$$

direct form of Baikov, Chetyrkin, Kuhn (2010) results

Results of advanced QCD analytical calculation 2

QCD β-function – a measure of the conformal symmetry breaking effects, $\boldsymbol{\beta(a_s)/a_s}$ – fix renormalization of trace of energy momentum tenzor

$$\beta(a_s) = \mu^2 \frac{d}{d\mu^2} a_s = -a_s^2 \left(\beta_0 + \beta_1 a_s + \beta_2 a_s^2 + \beta_3 a_s^3\right)$$

$$\beta_0 = \frac{11}{12}\mathrm{C_A} - \frac{1}{3}\mathrm{T_F N_F} \quad , \quad \beta_1 = \frac{17}{24}\mathrm{C_A^2} - \frac{5}{12}\mathrm{C_A T_F N_F} - \frac{1}{4}\mathrm{C_F T_F N_F}$$

$$\beta_2 = \frac{2857}{3456}\mathrm{C_A^3} - \frac{1451}{1728}\mathrm{C_A^2 T_F N_F} - \frac{205}{576}\mathrm{C_A C_F T_F N_F} + \frac{79}{864}\mathrm{C_A T_F^2 N_F^2} + \frac{1}{32}\mathrm{C_F^2 T_F N_F} + \frac{11}{144}\mathrm{C_F T_F^2 N_F^2}$$

Tarasov,Vladimirov,Zharkov (1980);

$$\begin{aligned}\beta_3 &= \left(\frac{150653}{124416} - \frac{11}{576}\zeta_3\right)\mathrm{C_A^4} + \left(-\frac{39143}{20735} + \frac{17}{96}\zeta_3\right)\mathrm{C_A^3 T_F N_F} + \frac{23}{128}\mathrm{C_F^3 T_F N_F} \\ &+ \left(\frac{7073}{62208} - \frac{41}{144}\zeta_3\right)\mathrm{C_F C_A^2 T_F N_F} - \left(\frac{1051}{1728} - \frac{11}{72}\right)\mathrm{C_A C_F^2 T_F N_F} + \frac{53}{7776}\mathrm{C_A T_F^3 N_F^3} \\ &+ \left(\frac{3965}{10368} + \frac{7}{72}\zeta_3\right)\mathrm{C_F^2 T_F^2 N_F^2} + \left(\frac{219}{7776} + \frac{7}{36}\zeta_3\right)\mathrm{C_F C_A T_F^2 N_F^2} \\ &+ \frac{77}{15552}\mathrm{C_F T_F^3 N_F^3} - \left(\frac{5}{144} - \frac{11}{12}\zeta_3\right)\frac{\mathrm{d_A^{abcd} d_A^{abcd}}}{\mathrm{d_R}} + \left(\frac{2}{9} - \frac{13}{6}\zeta_3\right)\frac{\mathrm{d_F^{abcd} d_A^{abcd}}}{\mathrm{d_R}}\mathrm{N_F} \\ &- \left(\frac{11}{36} - \frac{2}{3}\zeta_3\right)\frac{\mathrm{d_F^{abcd} d_F^{abcd}}}{\mathrm{d_R}}\mathrm{N_F^2}\end{aligned}$$

van Ritbergen,Vermaseren,Larin (1997)

Notice appearance of 3 new $SU(N_c)$-group structures

Perturbative QCD violation of the conformal symmetry of massless quark-parton model - source: the procedure of renormalization leads to non-zero QCD β-function

Conformal symmetry is the generalization of Poincaré symmetry

Symmetry under following transformations

- Space-time translations $x'^{\mu} = x^{\mu} + \alpha^{\mu}$
- Lorentz transformations $x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu}$
- Special conformal transformations $x'^{\mu} = \frac{x^{\mu} + \beta^{\mu} x^2}{1 + 2\beta x + \beta^2 x^2}$
- Scale transformations $x'^{\mu} = \rho x^{\mu}$

Generalized CBK relation in the $\overline{MS}$-scheme

- Conformal symmetry relates e^+e^--annihilation Adler D-function with DIS sum rules (Bjorken/Gross-Llewellyn Smith) in the massless quark-parton model - **[Crewther (1972)]**
- Explicit role of conformal symmetry breaking effects – factorization of QCD β-function in the analogous massless QCD relation, discovered at a_s^3–level **[Broadhurst, Kataev (93)]**.
- Shown to be true in all orders by applying operator product expansion to the 3-point AVV triangle diagram in momentum p – space **[Gabadadze, Kataev (95), Kataev (96)- INR-09296]**;
- Proved in coordinate x-space by **[Crewther (97)]** and **[D. Mueller (97)]**, this proof is in the review **[V.Braun, G. Korchemsky, D. Mueller (2003)]**

$$\begin{aligned} D_A(a_s) \times C^{NS}(a_s) &= \mathbb{1} + \Delta_{\mathrm{CSB}}(a_s) \\ \Delta_{\mathrm{CSB}}(a_s) &= \frac{\beta(a_s)}{a_s}\,\mathcal{P}(a_s),\ \text{polynomial } \mathcal{P}(a_s) = \sum_{m\geq 1} K_m a_s^m \end{aligned}$$

$K_1 = K_1[1,0,0]\mathrm{C_F}$,
$K_2 = K_2[2,0,0]\mathrm{C_F^2} + K_2[1,1,0]\mathrm{C_F C_A} + K_2[1,0,1]\mathrm{C_F T_F N_F}$ - **notice $\mathrm{T_F N_F}$-dependence (!)**

Explicit form of CBK relation at a_s^3 and a_s^4

Validity of Generalized **CBK** at a_s^3-strong check of different a_s^3 analytical calculations of the Adler function and DIS sum rules **[Broadhurst,Kataev (93)]**:

$K_1[1,0,0] = -\frac{21}{8} + 3\zeta_3$; $K_2[2,0,0] = \frac{397}{96} + \frac{17}{2}\zeta_3 - 15\zeta_5$;

$K_2[1,1,0] = -\frac{629}{32} + \frac{221}{12}\zeta_3$; $K_2[1,0,1] = \frac{163}{24} - \frac{19}{3}\zeta_3$. Explicit demonstration of β-function factorization at a_s^4-level

5 New explicit terms+ 1 known [Baikov, Chetyrkin, Kuhn (2010)]: $K_3 = K_3[3,0,0]\mathrm{C_F^3} + K_3[2,1,0]\mathrm{C_F^2 C_A} + K_3[1,2,0]\mathrm{C_F C_A^2} + K_3[2,0,1]\mathrm{C_F^2 T_F N_F} + K_3[1,1,1]\mathrm{C_F C_A T_F N_F} + K_3[1,0,2]\mathrm{C_F T_F^2 N_F^2}$

(Contain additional $\mathrm{T_F N_F}$-terms)

$K_3[3,0,0] = \frac{2471}{768} + \frac{61}{8}\zeta_3 - \frac{715}{8}\zeta_5 + \frac{315}{4}\zeta_7$;

$K_3[2,1,0] = \frac{99757}{2304} + \frac{8285}{96}\zeta_3 - \frac{1555}{12}\zeta_5 - \frac{105}{8}\zeta_7$

$K_3[1,2,0] = -\frac{406043}{2304} + \frac{18007}{144}\zeta_3 + \frac{2975}{48}\zeta_5 - \frac{77}{4}\zeta_3^2$

$K_3[2,0,1] = -\frac{7729}{1152} - \frac{917}{16}\zeta_3 + \frac{125}{2}\zeta_5 + 9\zeta_3^2$

$K_3[1,1,1] = \frac{1055}{9} - \frac{2521}{36}\zeta_3 - \frac{125}{3}\zeta_5 - 2\zeta_3^2$

$K_3[1,0,2] = -\frac{307}{18} + \frac{203}{18}\zeta_3 + 5\zeta_5$ **agrees with BK(93)**

Validity at a_s^4- strong check of advanced a_s^4 analytical calculations!

New 3 gauge group contributions in β_3 will not spoil factorization of the QCD β-function in a_s^5-they should be multiplied by $K_1[1,0,0]$-coefficient.

New form of the relation between e^+e^--characteristics and DIS sum rules in Euclidean region – **power series in $\beta(a_s)/a_s$–term** -more detailed understanding of structure of QCD generalization of [Crewther] relation [Kataev, Mikhailov (09-10)]- CKM relation

Q: Is it possible to unravel structure of $\Delta_{\rm CSB}(a_s)$–term? **Guess** : **Yes!** **[Kataev, Mikhailov (09-10)]** CERN-PH-TH/2009-203; PoS (RADCOR2009) 036, 2010 (prior learning **[BChK2010]** a_s^4 results, arXiv:1001.0728)

$$\Delta_{\rm CSB}(a_s) = \sum_{n\geq 1} \left(\frac{\beta(a_s)}{a_s}\right)^n \mathcal{P}_n(a_s),$$

$$\mathcal{P}_n(a_s) = \sum_{k\geq 1} P_n^{(k)} a_s^k$$

$$\mathcal{P}_1(a_s) = a_s \mathrm{C_F} \Bigg[\left(-\tfrac{21}{8} + \zeta_3\right) \text{ (}= K_1[1,0,0] \text{ - BK-expansion coeff.) } +$$

$$\left[\left(-\tfrac{47}{48} + \zeta_3\right) \mathrm{C_A} + \left(\tfrac{397}{96} + \tfrac{17}{2}\zeta_3 - 15\zeta_5\right) \mathrm{C_F}\right] a_s \Bigg] + O(a_s^3)$$

$$\mathcal{P}_2(a_s) = a_s \mathrm{C_F} \left(\frac{163}{8} - \frac{19}{\zeta}_3\right) \text{ (}\mathcal{P}_3(a_s) = O(a_s)\text{- was unfixed.}$$

Relation obtained by:

a) requiring $\mathrm{T_F N_F}$-independence of $\mathcal{P}_n(a_s)$ and absorption them into $\beta(a_s)$- coeff. (leads to unique system of equations, which relates P_n^k to β_i and K_i);

b) using expansions of $\boldsymbol{D_A}(a_s)$ and $\boldsymbol{C^{NS}}(a_s)$–coeff. $\boldsymbol{d_n}$ and $\boldsymbol{c_n}$ $(1 \leq n \leq 3)$ through β_0, β_1 (**[S.V. Mikhailov Quarks-2004 and JHEP(2007)]**).

More general structure : $\Delta_{\rm CSB} = \sum_{n\geq 1} \left(\frac{\beta(a_s)}{a_s}\right)^n \mathcal{P}_n(a_s)$ with

$\mathcal{P}_n(a_s) = \sum_{k\geq 1} P_n^{(k)} a_s^k = \sum_{r\geq 1} P_n^{(r)}[\rm k, m] C_F^{\rm k} C_A^{\rm m}\ a_s^r$ where $r = \rm k + m$

After learning the results of **[Baikov, Chetyrkin, Kuhn (09-10)]** the guess was confirmed at a_s^4-level. We get additional 3 contributions:

$$\mathcal{P}_1^{(3)}(a_s) = \Big[\mathrm{C_F^3}\left(\frac{2471}{768} + \frac{61}{8}\zeta_3 - \frac{715}{8}\zeta_5 + \frac{315}{4}\zeta_7\right)$$
$$+\mathrm{C_F^2 C_A}\left(\frac{16649}{1536} - \frac{11183}{192}\zeta_3 + \frac{1015}{24}\zeta_5 - \frac{105}{8}\zeta_7 + \frac{99}{4}\zeta_3^2\right)$$
$$+\mathrm{C_F C_A^2}\left(\frac{2107}{192} + \frac{2503}{72}\zeta_3 - \frac{355}{18}\zeta_5 - 33\zeta_3^2\right)\Big] a_s^3$$
$$\mathcal{P}_2^{(2)}(a_s) = \Big[\mathrm{C_F^2}\left(-\frac{13597}{384} - \frac{2523}{16}\zeta_3 + \frac{375}{2}\zeta_5 + 27\zeta_3^2\right) +$$
$$\mathrm{C_F C_A}\left(\frac{1433}{32} - \frac{1}{4}\zeta_3 - \frac{170}{4}\zeta_5 - 6\zeta_3^2\right)\Big] a_s^2$$
$$\mathcal{P}_3^{(1)}(a_s) = \mathrm{C_F}\left(-\frac{307}{2} + \frac{203}{2}\zeta_3 + 45\zeta_5\right) a_s$$

Higher order parts: the terms, leading in large powers of N_F

$$
\begin{aligned}
&\sum_{n<10} S_n x^n = \\
&\left[-\frac{21}{2} + 12\zeta_3\right]x + \left[\frac{326}{3} - \frac{304}{3}\zeta_3\right]x^2 + \left[-\frac{9824}{9} + \frac{6496}{9}\zeta_3 + 320\zeta_5\right]x^3 + \\
&\left[\frac{2760448}{243} - \frac{1268480}{243}\zeta_3 - \frac{48640}{9}\zeta_5\right]x^4 + \left[-\frac{280736320}{2187} + \frac{89300480}{2187}\zeta_3 + \frac{5196800}{81}\zeta_5 + \right. \\
&\left. 17920\zeta_7\right]x^5 + \left[\frac{10320047360}{6561} - \frac{2327111680}{6561}\zeta_3 - \frac{507392000}{729}\zeta_5 - \frac{1361920}{3}\zeta_7\right]x^6 + \\
&\left[-\frac{3723517199360}{177147} + \frac{611395563520}{177147}\zeta_3 + \frac{50008268800}{6561}\zeta_5 + \frac{203714560}{27}\zeta_7 + \right. \\
&\left. \frac{48742400}{27}\zeta_9\right]x^7 + \left[\frac{485484017500160}{1594323} - \frac{59933178265600}{1594323}\zeta_3 - \frac{5212730163200}{59049}\zeta_5 - \right. \\
&\left. \frac{79559065600}{729}\zeta_7 - \frac{14817689600}{243}\zeta_9\right]x^8 + \left[-\frac{7616109282344960}{1594323} + \frac{726735764193280}{1594323}\zeta_3 + \right. \\
&\left. \frac{195646580326400}{177147}\zeta_5 + \frac{1120185221120}{729}\zeta_7 + \frac{316630630400}{243}\zeta_9 + \frac{7821721600}{27}\zeta_{11}\right]x^9
\end{aligned}
$$

Large $\mathrm{N_F}$-dependence of polynom, multiplied by $\beta(a_s)/a_s$ factor - BK (93); Here $x = \mathrm{T_F N_F} a_s/4$.
This gives us first coefficients of the expansion:

´$_{\mathrm{CSB}} = \sum_{n\geq 1} \left(\frac{\beta(a_s)}{a_s}\right)^n \mathbf{P}_n^{(1)} \mathbf{a_s}$, where $\mathbf{P}_n^{(1)} = \frac{S_n}{4^n} \mathbf{3}^{(n-1)} \mathrm{C_F}$.

Question:

Is it possible to apply this NEW EXPRESSION for the conformal symmetry breaking term in practical QCD applications ?

Answer:

give NEW constraints between 5-loop results of advanced complicated computer calculations by **[BChK (09-10)]**

Consider Mikhailov (04/07) representations for the $\boldsymbol{D_A}(a_s)$ coefficients

$$\boldsymbol{d_2} = \beta_0\, d_2[1] + \boldsymbol{d_2[0]}$$
$$\boldsymbol{d_3} = \beta_0^2\, d_3[2,0] + \beta_1\, d_3[0,1] + \beta_0\, d_3[1,0] + \boldsymbol{d_3[0]}$$
$$\boldsymbol{d_4} = \beta_0^3\, d_4[3] + \beta_1\,\beta_0\, d_4[1,1] + \beta_2\, d_4[0,0,1] + \beta_0^2\, d_4[2] + \beta_1\, d_4[0,1] + \beta_0\, d_4[1] + \boldsymbol{d_4[0]}$$

and the similar representations for the $\boldsymbol{C^{NS}}(a_s)$ coefficients $\boldsymbol{c_n}$. At the order a_s^3 it is possible to define all coefficients for $\boldsymbol{d_2, d_3}$ (**[Mikhalov (04/07)]**

Using similar representations for $\boldsymbol{c_n}$ and the original Crewther relation, valid in the conformal-invariant limit of $\beta(a_s) = 0$ we get all coefficients for $\boldsymbol{c_2, c_3}$. **[Kataev-Mikhailov (this work)]**

Concrete **new** relations:

From the original **Crewther** relation
$D_A(a_s) \times C^{NS}|_{ci} = \mathbb{1}$ we get constraint
for 3,4 and 5 -loop results from the lower loops ones: $c_2[0] + d_2[0] = d_1^2$
$c_3[0] + d_3[0] = 2d_1 d_2[0] - d_1^4$
$c_4[0] + d_4[0] = 2d_1 d_3[0] - 3d_1^2 d_2[0] + (d_2[0])^2 + d_1^4$

$$
\begin{aligned}
\mathcal{P}_1(a_s) &= -a_s\Big\{P_1^{(1)} + a_s P_1^{(2)} + a_s^2 P_1^{(3)}\Big\} \\
&= -a_s\Big\{c_2[1] + d_2[1] + a_s\left(c_3[1] + d_3[1] + d_1(c_2[1] - d_2[1])\right) \\
&+ a_s^2\left(c_4[1] + d_4[1] + d_1(c_3[1] - d_3[1]) + d_2[0]c_2[1] + d_2[1]c_2[0]\right)\Big\} \\
\mathcal{P}_2(a_s) &= a_s\{P_2^{(1)} + a_s P_2^{(2)}\} = a_s\Big(c_3[2] + d_3[2]) + a_s^2\left(c_4[2] + d_4[2] + d_1(k_3[2] - c_3[2])\right)\Big) \\
\mathcal{P}_3(a_s) &= a_s\{P_3^{(1)}\} = -a_s\left(c_4[3] + d_4[3]\right)); \mathcal{P}_n(a_s) = (-1)^n a_s\Big(c_n[n-1] + d_n[n-1]\Big)
\end{aligned}
$$

Last two equations result from the **chain of "renormalon"graphs**
These equations allow to get by another method the N_F-independent coefficients $P_1^{(1)}$, $P_1^{(2)}$, $P_2^{(1)}$. However to check these constraints for $P_1^{(3)}$, $P_2^{(2)}$ some additional 5-loop calculations are needed (contribution from gluino - element of SUSY QCD); $P_n^{(1)}$ can extracted from $c_n[n-1]$ and $d_n[n-1]$-results of [Broadhurst, Kataev (93)]

Application: New cross-checks for **[BChK(2010)]** results from **Banks-Zaks (1982)** condition:

$\beta_0(T_F N_F^*) = 0$ gives relation $T_F N_F^* = \frac{11}{4} C_A$

Our new representation for $\Delta_{\rm CSB}$, which is **POLYNOMIAL** in $(\beta(a_s)/a_s)$, leads to the identity:

$$\boldsymbol{d_4} + \boldsymbol{c_4}|_{BZ} = \boldsymbol{d_4}[\mathbf{0}] + \boldsymbol{c_4}[\mathbf{0}](\mathbf{CI})$$
$$-\beta_1(BZ)\left[\boldsymbol{d_4}[\mathbf{0,1}] + \boldsymbol{c_4}[\mathbf{0,1}]\right] - \beta_2(BZ)\left[\boldsymbol{d_4}[\mathbf{0,0,1}] + \boldsymbol{c_4}[\mathbf{0,0,1}]\right]$$

where $\beta_1(BZ) = -\frac{1}{16}\left[7C_A^2 + 11 C_F C_A\right]$

$$\beta_2(BZ) = -C_A^3 \frac{1127}{1536} - C_F C_A^2 \frac{77}{192} + \frac{11}{128} C_F^2 C_A$$

Thus $\boldsymbol{d_4}[\mathbf{0}] + \boldsymbol{c_4}[\mathbf{0}](\mathbf{CI}) = -\frac{333}{1024} C_F^4 + C_F^2 C_A^2 \left[\frac{525}{512} - \frac{81}{16}\zeta_3\right] + C_F^3 C_A \left[\frac{99}{64}\right]$

Next: $\boldsymbol{d_4}[\mathbf{0,1}] + \boldsymbol{c_4}[\mathbf{0,1}] = P_1^{(2)} - (\boldsymbol{c_3}[\mathbf{0,1}] - \boldsymbol{d_3}[\mathbf{0,1}])\, d_1$

$$= \left[C_F C_A\left(-\frac{47}{48} + \zeta_3\right) + C_F^2\left(\frac{1117}{96} + \frac{7}{4}\zeta_3 - 15\zeta_5\right)\right]$$

$$\boldsymbol{d_4}[\mathbf{0,0,1}] + \boldsymbol{c_4}[\mathbf{0,0,1}] = -P_1^{(1)} = -C_F\left(\frac{21}{8} - 3\zeta_3\right)$$

$$-\beta_1(BZ)\,[\boldsymbol{d_4}[0,1]+\boldsymbol{c_4}[0,1]] = C_F C_A^3\left[\frac{7}{16}\zeta_3 - \frac{329}{768}\right] +$$

(no ζ_5 – but they exist in d_4 and c_4 !)

$$C_F^2 C_A^2\left[-\frac{3295}{1536}+\frac{471}{64}\zeta_3-\frac{105}{16}\zeta_5\right] + C_F^3 C_A\left[-\frac{3553}{1536}+\frac{671}{64}\zeta_3-\frac{165}{16}\zeta_5\right]$$

$$-\beta_2(BZ)\,[\boldsymbol{d_4}[0,0,1]+\boldsymbol{c_4}[0,0,1]] =$$

$$C_F C_A^3\left[-\frac{7889}{4069}+\frac{1127}{512}\zeta_3\right] + C_F^2 C_A^2\left[-\frac{539}{512}+\frac{77}{64}\zeta_3\right] + C_F^3 C_A\left[\frac{231}{1024}-\frac{33}{128}\zeta_3\right]$$

Finally:

$$\boldsymbol{d_4}(BZ)+\boldsymbol{c_4}(BZ) = -\frac{333}{1024}C_F^4 + C_A C_F^3\left(-\frac{1661}{3072}+\frac{1309}{128}\zeta_3-\frac{165}{16}\zeta_5\right) +$$
$$C_A^2 C_F^2\left(-\frac{3337}{1536}+\frac{7}{2}\zeta_3-\frac{105}{16}\zeta_5\right) + C_A^3 C_F\left(-\frac{28931}{12288}+\frac{1351}{512}\zeta_3\right)$$

These results are satisfied for **Baikov-Chetyrkin-Kuhn (2010)** 5-loop symbolic analytical results - additional confirmation of the correctness of the results of advanced analytic calculations

Conclusions:

1. We present new QFT (QCD and QED) expression for the conformal symmetry breaking term in the relation between e^+e^- annihilation and DIS sum rules - addition to **Crewther** unity:
   $\Delta_{\rm CSB} = \sum_{n\geq 1} \left(\frac{\beta(a_s)}{a_s}\right)^n \sum_{r\geq 1} P_n^{(r)}[\mathrm{k},\mathrm{m}] C_F^{\mathrm{k}} C_A^{\mathrm{m}}$
   where $r = \mathrm{k} + \mathrm{m}$ – **CKM**-relation–
   and fix there coefficients at a_s^4- level and in the large $\mathrm{N_F}$ expansion up to a_s^9.
2. Applications: new constraints between $d_4 + c_4$ **[BChK(2010)]** new results. for $\beta_0 = 0$ – **Banks-Zaks** condition.
3. Odd ζ-function studies- confirm that at **BZ** condition ζ_7 and ζ_3^2 disappear – reason: proportional to β_0-term– confirmation of observation of Baikov, Chetyrkin, Kuhn (10)
   ζ_{2n+1}-studies - possible link to $N = 4$ SUSY YM oriented theoretical multiloop studies ?
4. In QED diagrammatic representations of new generalizations is straightforward.
5. QCD applications- form-factors ? Summations of power series with expansion parameter being RG β-function, other possible applications ?

Analytical "experiments" detect detailed structures of QFT models