Theoretical aspects of EW symmetry breaking in SUSY models

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Outline

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- Supersymmetry
- EW symmetry breaking in the MSSM
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  - Little hierarchy problem
- Higgs sector of the NMSSM
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Symmetries in the SM

- The Lagrangian of the SM is invariant under Poincare group and $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge symmetry transformations.
- The Poincare group is an extension to Lorentz group that includes time and space translations

$$|\Psi > \longrightarrow \exp\{-i\hat{H}t\}|\Psi >, \qquad |\Psi > \longrightarrow \exp\{i\hat{\mathbf{P}} \cdot \mathbf{x}\}|\Psi >,$$

  - The transformations of Lorentz group involve rotations about three axises and Lorentz boosts along them. Lorentz transformations of spin J particle are given by

$$|J > \rightarrow \exp\{i\omega^{\mu\nu} M_{\mu\nu}\}|J >, \qquad M_{\mu\nu} = -M_{\nu\mu}.$$

  - The translation operators $\hat{P}_\mu = (\hat{H}, \hat{P}_1, \hat{P}_2, \hat{P}_3)$ and the angular momentum operators $M_{\mu\nu}$ form a complete set of generators of Poincare group.

- The commutation relations between the generators of Poincare group can be presented in the following form

$$[M_{\mu\nu}, M_{\rho\sigma}] = i\left(g_{\nu\rho}M_{\mu\sigma} - g_{\mu\rho}M_{\nu\sigma} - g_{\nu\sigma}M_{\mu\rho} + g_{\mu\sigma}M_{\nu\rho}\right) ,$$

$$\left[\hat{P}_\mu, \, \hat{P}_\nu\right] = 0 , \qquad \left[M_{\mu\nu}, \hat{P}_\lambda\right] = i\left(g_{\nu\lambda}\hat{P}_\mu - g_{\mu\lambda}\hat{P}_\nu\right) .$$

- The elements of $SU(N)$ groups can be written as

$$UU^\dagger = 1 , \qquad \det U = 1 , \qquad \Longrightarrow \qquad U = \exp\left\{i\omega^a T^a\right\},$$

$$T^\alpha = T^{a\dagger} , \qquad Tr\,(T^a) = 0 ,$$

where generators T^a obey commutation relations

$$\left[T^a, T^b\right] = if_{abc}T^c , \qquad \left[T^a, \hat{P}_\mu\right] = 0 , \qquad [T^a, M_{\mu\nu}] = 0 .$$

- There are 3 generators of $SU(2)$ and 8 generators of $SU(3)$.

- $SU(2)_W \times U(1)_Y$ symmetry is broken down to $U(1)_{em}$.
  - $W^{\pm}$ and Z bosons that are associated with the weak interactions have been observed.
- Quarks and gluons that participate in the strong interactions are confined inside mesons and baryons.
  - Theory of strong interactions based on $SU(3)_C$ provides a good description for the spectrum of mesons and baryons, e^+e^- annihilation data, deep inelastic scattering and etc.
- Higgs boson plays a key role in the SM.
  - Higgs field acquires vacuum expectation value (VEV) breaking electroweak (EW) symmetry and generating masses of all bosons and fermions.

Supersymmetry

- In order to achieve the unification of gauge interactions with gravity we need to combine Poincare and internal symmetries.
- But according to the Coleman-Mandula theorem the most general symmetry which quantum field theory can have is a tensor product of the Poincare group and an internal group, i.e. $G \otimes$ Poincare symmetry.
- However Coleman and Mandula restricted themselves to Lie algebras.
- Graded Lie algebras have general structure

$$\left[\hat{B},\hat{B}\right] = \hat{B}, \qquad \left[\hat{B},\hat{F}\right] = \hat{F}, \qquad \left\{\hat{F},\hat{F}\right\} = \hat{B},$$

where $\hat{B}$ and $\hat{F}$ are bosonic and fermionic generators.

- Graded Lie algebras that contain the Poincare algebra are called supersymmetries.
- The simplest $N = 1$ supersymmetry (SUSY) involves Weyl spinor operators Q_α and $Q^*_\alpha = \overline{Q}_{\dot\alpha}$

$$Q_\alpha|fermion >= |boson >, \qquad \overline{Q}_{\dot\alpha}|boson >= |fermion > .$$

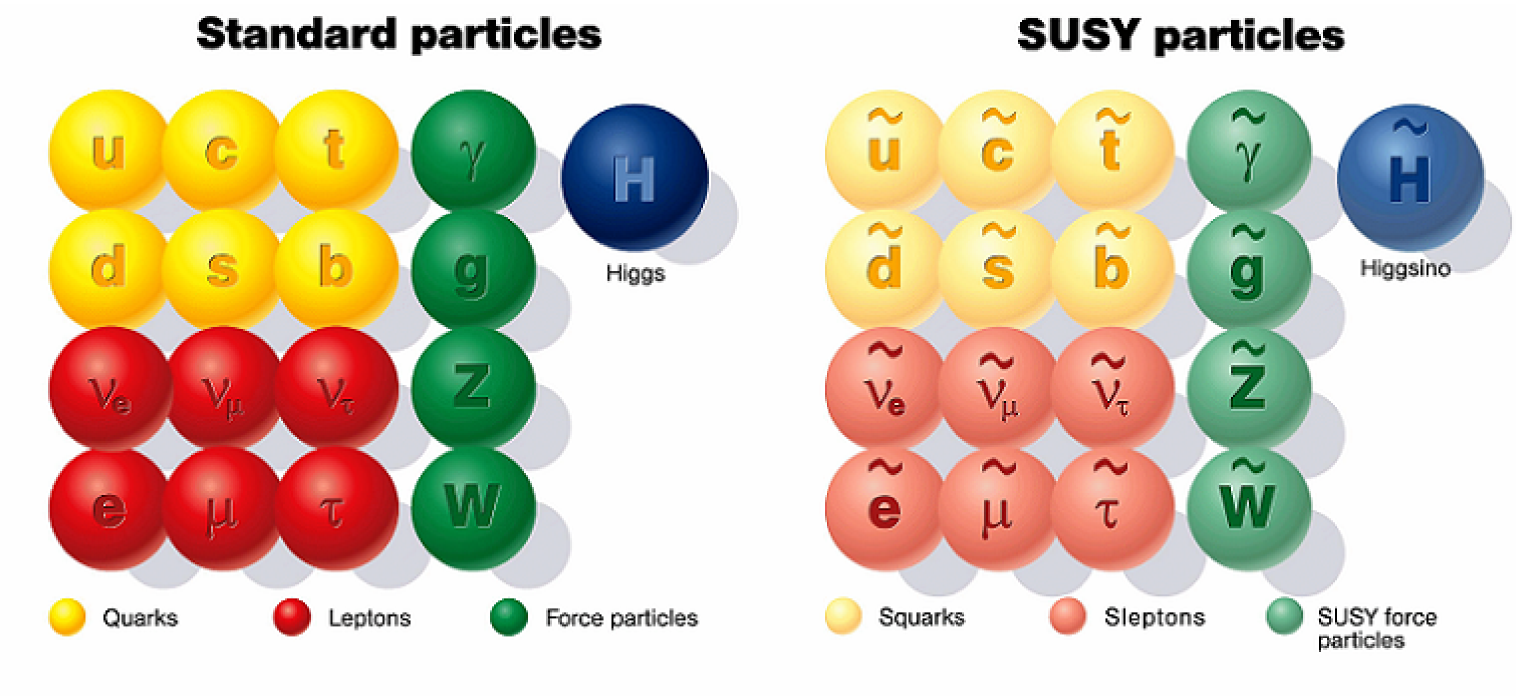
- The $N = 1$ SUSY algebra is Poincare algebra plus

$$\{Q_\alpha, \overline{Q}_{\dot\alpha}\} = 2\,\sigma^\mu_{\alpha\dot\alpha}\hat{P}_\mu, \qquad \{Q_\alpha, Q_\beta\} = \{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\alpha}\} = 0,$$
$$\left[\hat{P}_\mu, Q_\alpha\right] = 0, \qquad [M^{\mu\nu}, Q_\alpha] = -i\,(\sigma^{\mu\nu})_\alpha^{\ \beta}\, Q_\beta,$$
$$\left[\hat{P}^\mu, \overline{Q}_{\dot\alpha}\right] = 0, \qquad \left[M^{\mu\nu}, \overline{Q}^{\dot\alpha}\right] = -i\,(\sigma^{\mu\nu})^{\dot\alpha}_{\ \dot\beta}\, \overline{Q}^{\dot\beta},$$

where $\sigma^{\mu\nu} = \frac{1}{4}\left(\sigma^\mu\overline{\sigma}^\nu - \sigma^\nu\overline{\sigma}^\mu\right)$, $\overline{\sigma}^{\mu\nu} = \frac{1}{4}\left(\overline{\sigma}^\mu\sigma^\nu - \overline{\sigma}^\nu\sigma^\mu\right)$, $\sigma^\mu = (1, \sigma^i)$, $\overline{\sigma}^\mu = (1, -\sigma^i)$, σ^i are Pauli matrices.

- The local version of SUSY (supergravity) leads to a partial unification of gauge interactions with gravity.

- There are several consequences of the SUSY algebra:
  - SUSY multiplets have equal number of bosonic and fermionic degrees of freedom;
  - Members of SUSY multiplet have the same mass.
- The particle content of the minimal supersymmetric standard model (MSSM) involves:



- SUSY models are defined by the field content, structure of gauge interactions and superpotential.
- The most general renormalizable gauge invariant superpotential of the MSSM is given by

$$W = W_{MSSM} + W_{NR},$$

$$W_{MSSM} = \epsilon_{ij}(y^U_{ab}Q^j_a u^c_b H^i_2 + y^D_{ab}Q^j_a d^c_b H^i_1 + y^L_{ab}L^j_a e^c_b H^i_1 + \mu H^i_1 H^j_2),$$

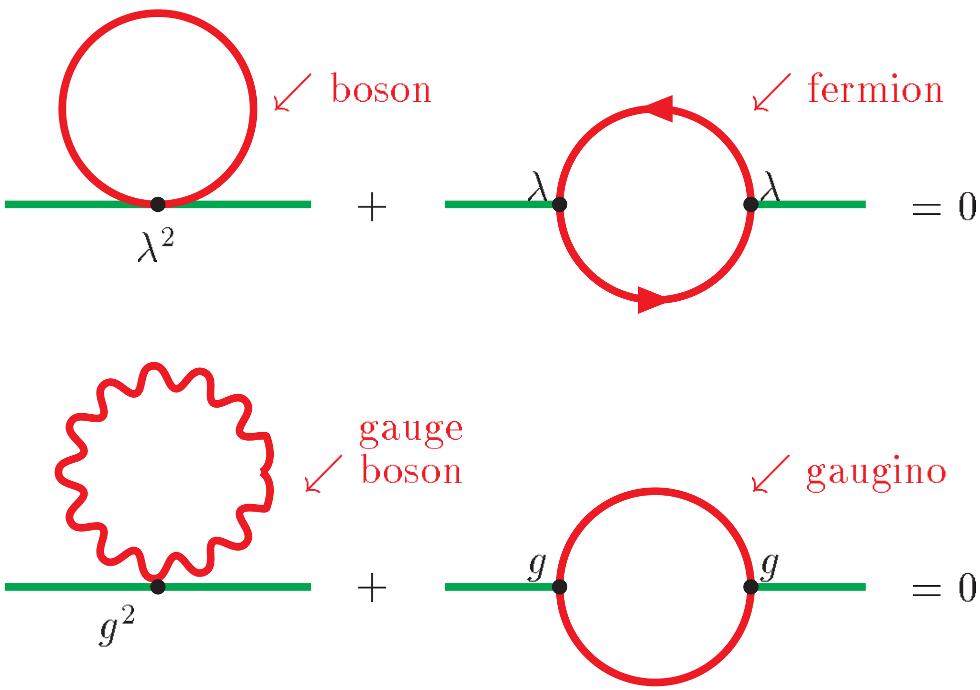
$$W_{NR} = \epsilon_{ij}(\lambda^L_{abd}L^i_a L^j_b e^c_d + \lambda^{L\prime}_{abd}L^i_a Q^j_b d^c_d + \mu'_a L^i_a H^j_2) + \lambda^B_{abd}u^c_a d^c_b d^c_d.$$

- Terms in W_{NR} violate either lepton or baryon number.
- To prevent rapid proton decay R-parity is normally imposed

$$R = (-1)^{3(B-L)+2S}\,.$$

- R-parity forbids all terms in W_{NR}.

- If R–parity is conserved the lightest SUSY particle is absolutely stable and can play the role of dark matter.
- Softly broken supersymmetry ensures the cancellation of quadratic divergences stabilising mass hierarchy.
- In order to avoid the degeneracy between bosons and fermions SUSY must be broken at low energies.



- The supersymmetry breaking couplings should not spoil the the cancellation of quadratic divergences (soft breakdown of SUSY).
- The Lagrangian of SUSY models based on the softly broken supersymmetry can be written as

$$\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{soft}\,.$$

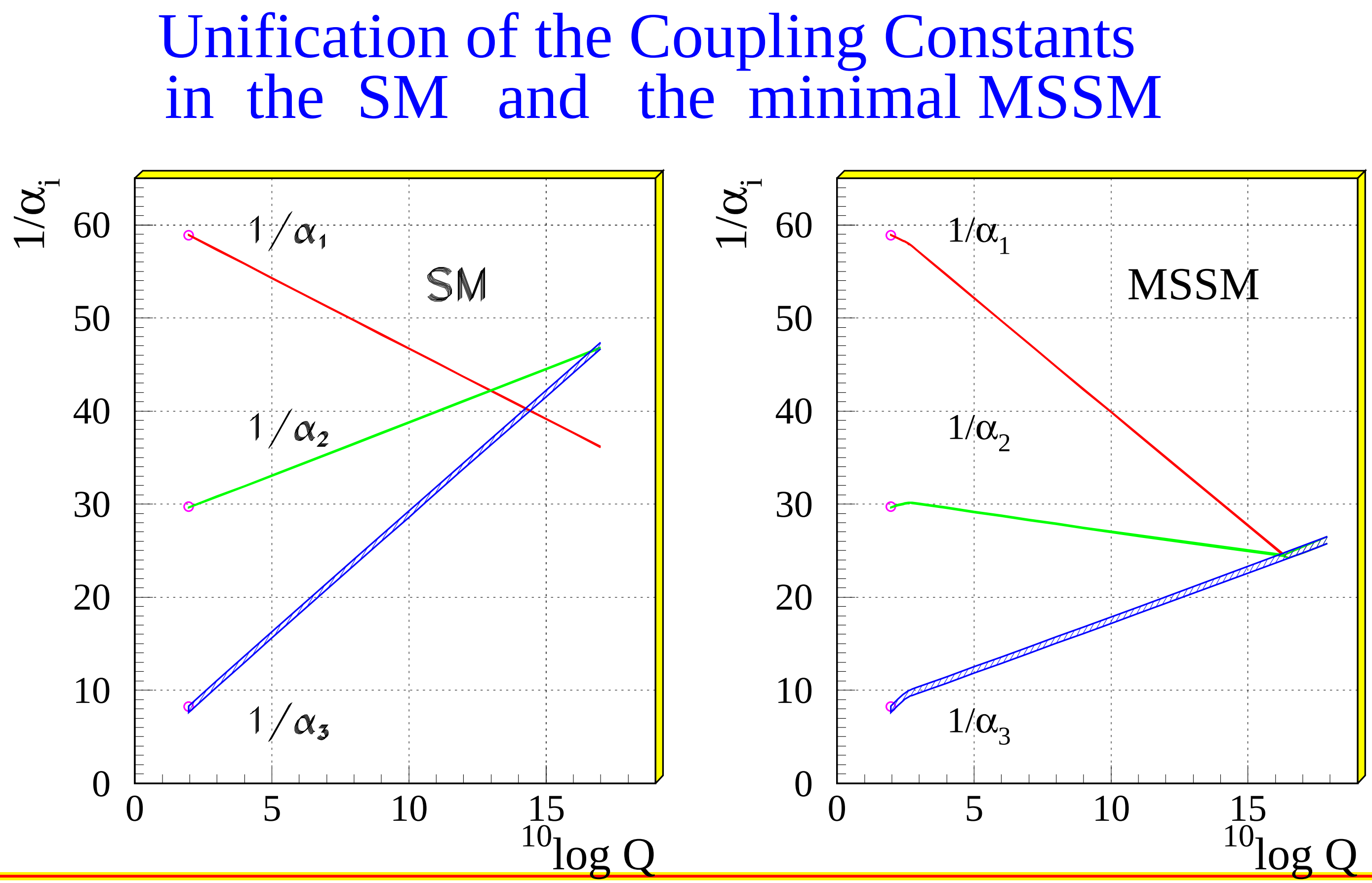
- In the MSSM the set of the soft SUSY breaking terms includes

$$-\mathcal{L}_{soft} = \sum_i m_i^2|\varphi_i|^2 + (\tfrac{1}{2}\sum_\alpha M_\alpha\tilde{\lambda}_\alpha\tilde{\lambda}_\alpha + \sum_{a,b}[A^U_{ab}y^U_{ab}\tilde{Q}_a\tilde{u}^c_b H_2$$

$$+A^D_{ab}y^D_{ab}\tilde{Q}_a\tilde{d}^c_b H_1 + A^L_{ab}y^L_{ab}\tilde{L}_a\tilde{e}^c_b H_1] + B\mu H_1 H_2 + h.c.)\,,$$

  where φ_i are scalar fields, $\tilde{\lambda}_\alpha$ are gaugino fields.
- To avoid fine–tuning SUSY breaking mass parameters should be in the TeV range.

- Gauge couplings in the MSSM converge to a common value at $M_X \simeq 10^{16}\,\mathrm{GeV}$ that allows to embed the SM gauge group into Grand Unified Theories (GUTs) based on $SU(5)$, $SO(10)$ and etc.



- The fundamental representation of $SU(5)$ can be chosen so that

$$\bar{5}_i = \begin{pmatrix} d_i^c \\ L_\alpha \end{pmatrix} = (\bar{3},\, 1,\, \tfrac{1}{3}) \oplus (1,\, \bar{2},\, -\tfrac{1}{2})\,, \qquad 5^i = (3,\, 1,\, -\tfrac{1}{3}) \oplus (1,\, 2,\, \tfrac{1}{2})\,.$$

- Then other quarks and leptons fill in antisymmetric tensor representation of rank 2

$$5^i \otimes 5^j = V^{ij} = \tfrac{1}{2} V^{\{ij\}} + \tfrac{1}{2} V^{[ij]} = 15 \oplus 10,$$

$$5 \otimes 5 = \left((3,\, 1,\, -\tfrac{1}{3}) \oplus (1,\, 2,\, \tfrac{1}{2}) \right) \otimes \left((3,\, 1,\, -\tfrac{1}{3}) \oplus (1,\, 2,\, \tfrac{1}{2}) \right),$$

$$(3,\, 1,\, -\tfrac{1}{3}) \otimes (3,\, 1,\, -\tfrac{1}{3}) = (\bar{3},\, 1,\, -\tfrac{2}{3}) \oplus (6,\, 1,\, -\tfrac{2}{3}),$$

$$(1,\, 2,\, \tfrac{1}{2}) \otimes (3,\, 1,\, -\tfrac{1}{3}) = (3,\, 1,\, -\tfrac{1}{3}) \otimes (1,\, 2,\, \tfrac{1}{2}) = (3,\, 2,\, \tfrac{1}{6}),$$

$$(1,\, 2,\, \tfrac{1}{2}) \otimes (1,\, 2,\, \tfrac{1}{2}) = (1,\, 3,\, 1) \oplus (1,\, 1,\, 1),$$

$$10 = (\bar{3},\, 1,\, -\tfrac{2}{3}) \oplus (3,\, 2,\, \tfrac{1}{6}) \oplus (1,\, 1,\, 1) = u_i^c + Q^i + e^c,$$

- Thus GUTs provide a simple explanation of electric charge quantisation in the SM.
- Each family of quarks and leptons fits into one spinor representation of $SO(10)$.
  - 16 dimensional spinor representation of $SO(10)$ decomposes under the $SU(5)$ subgroup as follows

$$16 \to 1 \oplus 10 \oplus \bar{5}.$$

  - $SO(10)$ predicts the existence of right–handed neutrinos.
- If the breakdown of $SO(10)$ to the SM group takes place at very high energies the right–handed neutrino may be superheavy.
- Then the three known left–handed neutrinos acquire small Majorana masses via the seesaw mechanism.

EWSB in the MSSM

- The Higgs potential in the MSSM can be written as

$$V = m_1^2|H_1|^2 + m_2^2|H_2|^2 - m_3^2(H_1H_2 + h.c.) + \frac{g^2}{8}\left(H_1^+\sigma_a H_1 + H_2^+\sigma_a H_2\right)^2 + \frac{g'^2}{8}\left(|H_1|^2 - |H_2|^2\right)^2 + \Delta V\,,$$

  where $m_1^2 = m_{H_1}^2 + \mu^2$, $m_2^2 = m_{H_2}^2 + \mu^2$, $m_3^2 = -B\mu$ and ΔV is a contribution of loop corrections.

- In the leading one–loop approximation

$$\Delta V = \frac{3}{32\pi^2}\left[m_{\tilde{t}_1}^4\left(\ln\frac{m_{\tilde{t}_1}^2}{Q^2} - \frac{3}{2}\right) + m_{\tilde{t}_2}^4\left(\ln\frac{m_{\tilde{t}_2}^2}{Q^2} - \frac{3}{2}\right) - 2m_t^4\left(\ln\frac{m_t^2}{Q^2} - \frac{3}{2}\right)\right],$$

  where $m_{\tilde{t}_{1,2}} = \frac{1}{2}\left(m_Q^2 + m_U^2 + 2m_t^2 \pm \sqrt{(m_Q^2 - m_U^2)^2 + 4m_t^2X_t^2}\right)$

  and $X_t = A_t - \mu/\tan\beta\,.$

- At the tree level the MSSM Higgs potential contains three independent parameters: $m_1^2,\, m_2^2,\, m_3^2$.

- The stable vacuum of the MSSM Higgs potential exists only if

$$m_1^2 + m_2^2 > 2|m_3|^2 \,.$$

- Higgs doublets acquire non-zero VEVs when

$$m_1^2 m_2^2 < |m_3|^4 \,.$$

- At the physical vacuum of the potential

$$< H_1 >= \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \qquad < H_2 >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}.$$

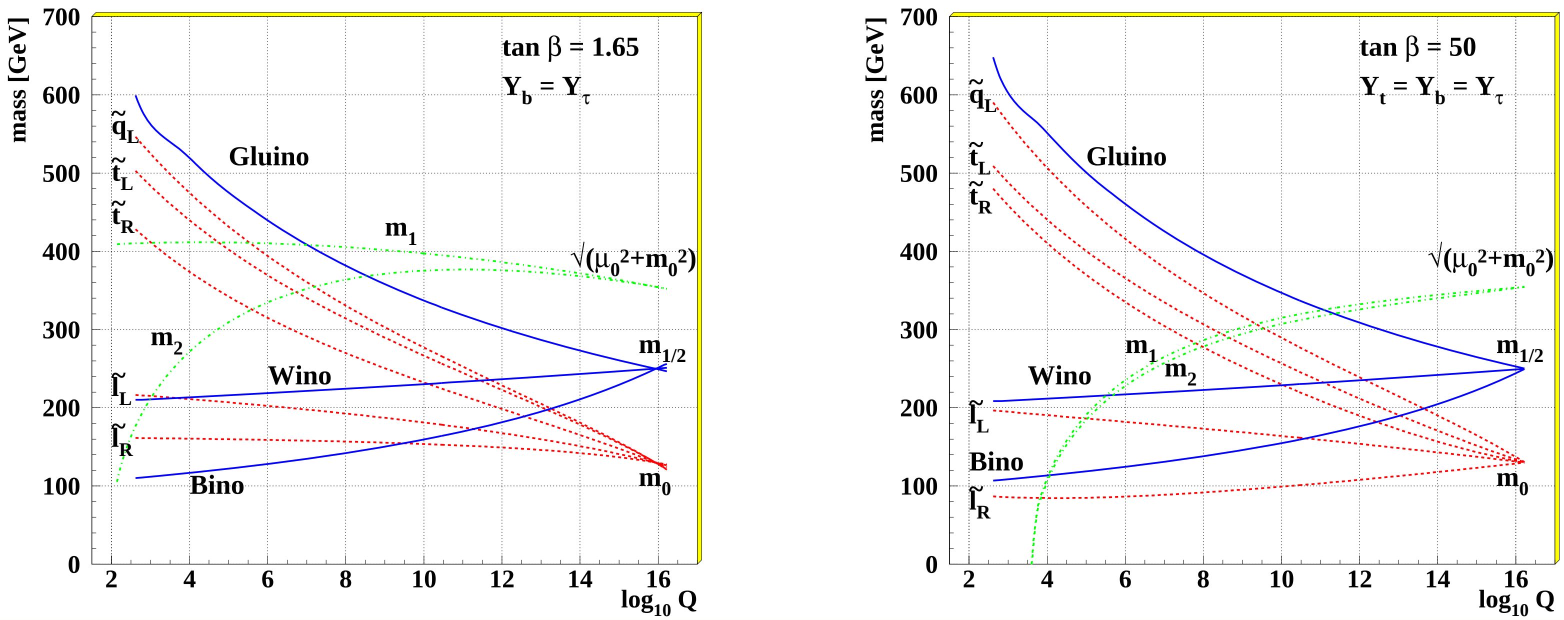
  - One can define: $\tan\beta = v_2/v_1$, $\quad v = \sqrt{v_1^2 + v_2^2} = 246$ GeV.

- Higgs VEVs obey the minimization conditions

$$\left(m_1^2 + \frac{\bar{g}^2}{8}(v_1^2 - v_2^2)\right) v_1 = m_3^2 v_2 \,, \quad \left(m_2^2 + \frac{\bar{g}^2}{8}(v_2^2 - v_1^2)\right) v_2 = m_3^2 v_1 \,.$$

- The breakdown of EW symmetry in the MSSM can be caused by the renormalization group (RG) flow (radiative electroweak symmetry breaking).
- In the cMSSM ($m_i^2(M_X) = m_0^2,\ A_{ab}^k(M_X) = A,\ M_\alpha(M_X) = M_{1/2}$) $m_2^2(M_Z)$ can become either small or negative.

Evolution of sparticle mass parameters



- At the tree level CP in the MSSM Higgs sector is conserved.
- The Higgs sector of the MSSM includes:
  - one CP-odd state $m_A^2 = m_1^2 + m_2^2 + \Delta_A$,
  - two charged states $M_{H^\pm}^2 = m_A^2 + M_W^2 + \Delta_\pm$,
  - two CP-even states.
- In the field basis

$$Re\, H_1^0 = (h\cos\beta - H\sin\beta + v_1)/\sqrt{2}\,, \quad Re\, H_2^0 = (h\sin\beta + H\cos\beta + v_2)/\sqrt{2}\,,$$

the mass matrix of the CP-even Higgs sector takes a form

$$M^2 = \begin{pmatrix} M_{11}^2 & M_{12}^2 \\ M_{21}^2 & M_{22}^2 \end{pmatrix} = \begin{pmatrix} \frac{1}{v^2}\frac{\partial^2 V}{\partial^2 \beta} & \frac{1}{v}\frac{\partial^2 V}{\partial v \partial \beta} \\ \frac{1}{v}\frac{\partial^2 V}{\partial v \partial \beta} & \frac{\partial^2 V}{\partial^2 v} \end{pmatrix}.$$

- The corresponding matrix elements are given by

$$M_{11}^2 = m_A^2 + M_Z^2 \sin^2 2\beta + \Delta_{11} ,$$
$$M_{12}^2 = M_{21}^2 = -\frac{1}{2} M_Z^2 \sin 4\beta + \Delta_{12} ,$$
$$M_{22}^2 = M_Z^2 \cos^2 2\beta + \Delta_{22} .$$

- Since the minimal eigenvalue of a matrix does not exceed its smallest diagonal element

$$m_{h_1}^2 \leq M_Z^2 \cos^2 2\beta + \Delta_{22} .$$

- The masses of the CP-even Higgs states are

$$m_{h_1,h_2}^2 = \frac{1}{2}\left(M_{11}^2 + M_{22}^2 \mp \sqrt{(M_{11}^2 - M_{22}^2)^2 + 4M_{12}^4}\right) .$$

- At the tree level the masses of the CP-even Higgs states obey mass relation

$$m_{h_1}^2 + m_{h_2}^2 = m_A^2 + M_Z^2 .$$

- The Lagrangian, which determines the interactions of the neutral Higgs states with the Z–boson, is given by

$$L_{AZH} = \frac{\bar{g}}{2} M_Z Z_\mu Z_\mu h + \frac{\bar{g}}{2} Z_\mu \Big[H(\partial_\mu A) - (\partial_\mu H) A \Big] .$$

- The normalised R–couplings of the neutral Higgs states to vector bosons can be defined as follows:

$$g_{VVh_i} = R_{VVh_i} \times \text{ SM coupling,} \qquad g_{ZAh_i} = \frac{\bar{g}}{2} R_{ZAh_i} .$$

- The R–couplings are given by

$$R_{VVh_1} = -R_{ZAh_2} = \sin(\beta - \alpha)\,, \qquad R_{VVh_2} = R_{ZAh_1} = \cos(\beta - \alpha)\,,$$

where

$$h_1 = -(Re\, H_1^0 - v_1) \sin\alpha + (Re\, H_2^0 - v_2) \cos\alpha\,,$$

$$h_2 = (Re\, H_1^0 - v_1) \cos\alpha + (Re\, H_2^0 - v_2) \sin\alpha\,, \qquad \tan\alpha = \frac{M_{12}^2}{M_{11}^2 - m_{h_1}^2} .$$

- In the MSSM the couplings of the lightest Higgs boson to the Z pair can be substantially smaller than in the SM.
- As a result the experimental lower bound on the lightest Higgs mass in the MSSM is weaker than in the SM.
- The inclusion of loop effects can give rise to CP-violation in the MSSM Higgs sector.
  - In this case all three neutral states get mixed.
  - This makes the experimental constraints on the lightest Higgs boson mass even weaker.
  - The CP-violation in the Higgs sector might shed light on the origin of the baryon asymmetry.

- The experimental lower bound on the SM–like Higgs boson mass is rather stringent (114 GeV).
- At the tree level the lightest Higgs boson mass in the MSSM does not exceed $M_Z \simeq 91$ GeV
- In order to satisfy the experimental constraints large contribution of loop corrections to the SM-like Higgs mass is needed.
- In the leading one-loop approximation we have

$$m_{h_1}^2 \simeq M_Z^2 \cos^2 2\beta + \Delta_{22} \,,$$

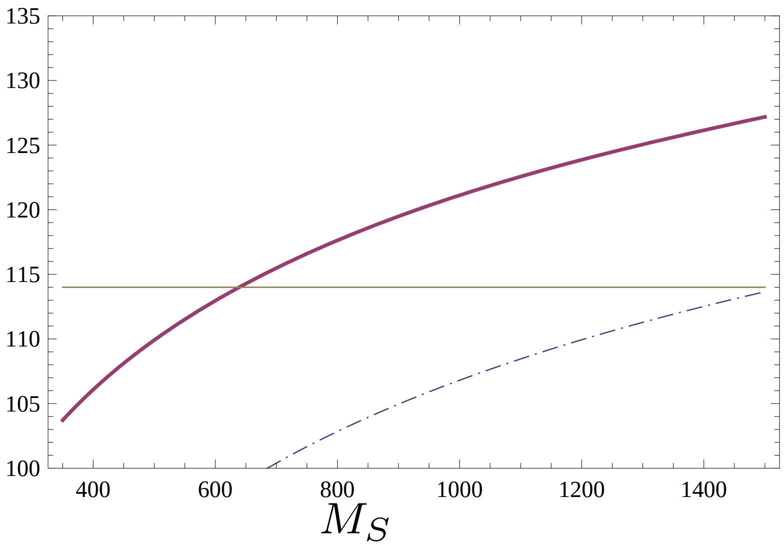
$$\Delta_{22}^{(1)} = \frac{3M_t^4}{2\pi^2 v^2}\left[\frac{X_t^2}{M_S^2}\left(1 - \frac{1}{12}\frac{X_t^2}{M_S^2}\right) + \ln\left(\frac{M_S^2}{m_t^2}\right)\right] \,.$$

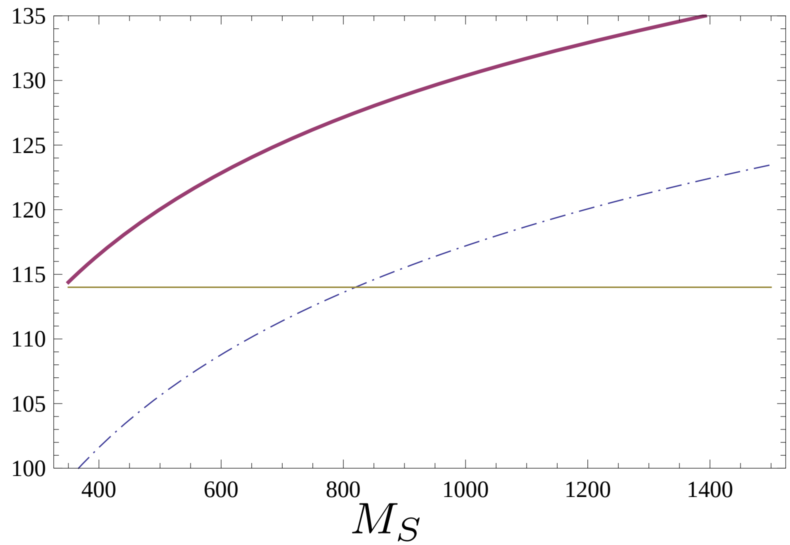
- Large $\Delta_{22}^{(1)}$ can be obtained for large M_S and large mixing parameter X_t.

- Due to the RG flow large stop scalar masses m_Q^2 and m_U^2 induce large mass parameters in the Higgs potential.
- This leads to the fine tuning because m_1^2 and m_2^2 determine the EW scale.

SM-like Higgs mass in the MSSM for $\tan\beta = 2$ and $\tan\beta = 3$

$[m_Q = m_U = M_S,\ X_t = M_S(2M_S)]$





- Indeed, combining the minimization conditions we get

$$M_Z^2 = 2\left(\frac{m_1^2 - m_2^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2\right).$$

- This implies $\sim 1\%$ tuning generically.
- At the same time the fine tuning in QCD is much higher.
- The θ-term in the QCD Lagrangian

$$\mathcal{L}_\theta = \theta_{\text{eff}} \frac{\alpha_s}{8\pi} F^{\mu\nu\, a} \tilde{F}^a_{\mu\nu}, \qquad \theta_{\text{eff}} = \theta + \arg \det M_q \,,$$

  could lead to a neutron electric dipole moment of order $|d_n| \approx |\theta_{\text{eff}}| \times 10^{-16} e \text{ cm}$.
- Current experimental bound on d_n ($d_n < 3 - 6 \times 10^{-26} e \text{ cm}$) implies that $|\theta_{\text{eff}}| \lesssim 10^{-9}$.

- One possible solution of the little hierarchy problem is to increase the the mass of the lightest Higgs boson.
- This can be achieved by adding new gauge and Yukawa interactions.
- One can also try to avoid stringent LEP bound by allowing exotic Higgs decays that implies the inclusion of new particles and interactions as well.
- New interactions can be used to solve the so-called μ problem.
- The superpotential of the MSSM contain only one bilinear term $\mu(\hat{H}_d\hat{H}_u)$ and parameter μ is expected to be of order of $M_X - M_{Pl}$.
- At the same time the correct pattern of EW symmetry breaking requires $\mu \sim 100 - 1000$ GeV.

Higgs sector of the NMSSM

- In the NMSSM the superpotential is invariant under Z_3 discrete symmetry, i.e.

$$\mu(\hat{H}_d\hat{H}_u) \to \lambda\hat{S}(\hat{H}_d\hat{H}_u) + \frac{\varkappa}{3}\hat{S}^3 .$$

- The NMSSM Higgs potential is given by

$$V = V_F + V_D + V_{soft} + \Delta V ,$$

$$V_F = \lambda^2|S|^2(|H_1|^2 + |H_2|^2) + \lambda^2|(H_1H_2)|^2 + \lambda\varkappa\left[S^{*2}(H_1H_2) + h.c.\right] + \varkappa^2|S|^4 ,$$

$$V_D = \frac{g^2}{8}\left(H_1^+\sigma_a H_1 + H_2^+\sigma_a H_2\right)^2 + \frac{g'^2}{8}\left(|H_1|^2 - |H_2|^2\right)^2 ,$$

$$V_{soft} = m_1^2|H_1|^2 + m_2^2|H_2|^2 + m_s^2|S|^2 + \left[\lambda A_\lambda S(H_1H_2) + \frac{\varkappa}{3}A_\varkappa S^3 + h.c.\right] .$$

- It contains seven fundamental parameters:

$$\lambda, \qquad \varkappa, \qquad m_1^2, \qquad m_2^2, \qquad m_s^2, \qquad A_\lambda, \qquad A_\varkappa .$$

- At the physical vacuum of the potential

$$< H_1 >= \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \qquad < H_2 >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \qquad < S >= \frac{s}{\sqrt{2}},$$

and effective μ term is generated, i.e. $\mu = \lambda \frac{s}{\sqrt{2}}$.

  - One can express soft masses m_1^2, m_2^2, m_s^2 via $\tan\beta = v_2/v_1$, s and $v = \sqrt{v_1^2 + v_2^2} = 246$ GeV using the conditions for the extrema

$$\frac{\partial V}{\partial v_1} = \frac{\partial V}{\partial v_2} = \frac{\partial V}{\partial s} = 0\,.$$

- Then Higgs boson spectrum can be parametrized in terms of six variables:

$$\lambda, \qquad \varkappa, \qquad \tan\beta, \qquad \mu = \frac{\lambda s}{\sqrt{2}}, \qquad A_\varkappa, \qquad A_\lambda \Longleftrightarrow x\,(\text{or}\, m_A^2),$$

$$x = \frac{1}{2\mu}\left(A_\lambda + 2\,\frac{\varkappa}{\lambda}\,\mu\right)\sin 2\beta, \qquad m_A^2 = \frac{4\mu^2}{\sin^2 2\beta}\left(x - \frac{\varkappa}{2\lambda}\sin 2\beta\right).$$

- When λ and $\varkappa$ are small, i.e. λv, $\varkappa v \ll M_Z$, new singlet Higgs states decouple (MSSM limit of the NMSSM).
- In the considered limit $\frac{\partial V}{\partial v_1} = 0$ and $\frac{\partial V}{\partial v_2}$=0 are approximately the same as in the MSSM while

$$\frac{\partial V}{\partial s} \approx s \left(m_S^2 + \frac{\varkappa A_\varkappa}{\sqrt{2}} s + \varkappa^2 s^2 \right) \simeq 0 \,.$$

- In this case $m_S^2 > 0$ and Higgs potential has always a minimum in which $s \simeq v_1 \simeq 0$.
- However the corresponding vacuum becomes unstable for large values of $A_\varkappa$.
- When $A_\varkappa^2 > 9m_S^2$ the global minimum is attained at $s = s_1(s_2)$ for negative (positive) $A_\varkappa$ and $v_1, v_2 \neq 0$, where

$$s_{1,2} = \frac{-A_\varkappa \pm \sqrt{A_\varkappa^2 - 8m_S^2}}{2\sqrt{2}\varkappa} \,.$$

- After the EW symmetry breaking three goldstone modes are absorbed by $W^{\pm}$ and Z.
- When CP is preserved the Higgs sector of the NMSSM involves:
  – two charged states $\quad m_{H^{\pm}}^2 \simeq m_A^2 + m_W^2$,
  – two pseudoscalars $\quad m_{A_2}^2 \simeq m_A^2, \qquad m_{A_1}^2 \simeq -3\dfrac{\varkappa}{\lambda} A_{\varkappa}\mu$,
  – three scalars

$$m_{h_3}^2 \simeq m_A^2 + M_Z^2 \sin^2 2\beta + O\left(\frac{M_Z^4}{m_A^2}\right),$$

$$m_{h_2}^2 \simeq M_Z^2 \cos^2 2\beta + O\left(\frac{M_Z^4}{m_A^2}\right),$$

$$m_{h_1}^2 \simeq 4\frac{\varkappa^2}{\lambda^2}\mu^2 + \frac{\varkappa}{\lambda}A_{\varkappa}\mu + \frac{\lambda^2 v^2}{2} x \sin^2 2\beta - \frac{2\lambda^2 v^2 \mu^2 (1-x)^2}{M_Z^2 \cos^2 2\beta}\,.$$

- In the considered limit the MSSM sum rules for the Higgs masses and couplings are reproduced.

- The masses of new singlet states m_{A_1} and m_{h_1} are set by $\frac{\varkappa}{\lambda}\mu$ and grow with increasing $\varkappa$ and s.
- $A_\varkappa$ is responsible for the splitting of m_{A_1} and m_{h_1}.
- Vacuum stability requirement constrains $A_\varkappa$

$$-3\left(\frac{\varkappa}{\lambda}\mu\right)^2 \lesssim A_\varkappa \left(\frac{\varkappa}{\lambda}\mu\right) \lesssim 0\,.$$

- When $\dfrac{\varkappa}{\lambda} << 1$ extra singlet bosons can be the lightest particles in the Higgs spectrum.
- In the approximate Peccei–Quinn symmetry limit ($\frac{\varkappa}{\lambda}\mu \sim \lambda v$) $m_{h_1}^2$ tends to be negative resulting in theoretical restriction on x

$$1-\Delta < x < 1+\Delta\,, \qquad \Delta = \left|\frac{\sqrt{2}\varkappa M_Z \cos 2\beta}{\lambda^2 v}\right|\,.$$

- If $\varkappa$ is small variable x is localized nearby unity.
- Stringent bound on x and LEP constraints lead to a hierarchical structure of the Higgs spectrum.
  - In order to avoid conflict with chargino searches $\mu \gtrsim 100\,\text{GeV}$.
  - The non–observation of the SM-like Higgs particle excludes low values of $\tan\beta$ ($\tan\beta \geq 2.4$).
- The masses of the charged, heaviest CP-odd and heaviest CP-even states are almost degenerate and proportional $m_A \sim \mu\, \tan\beta$.
- The SM-like Higgs boson and extra singlet states are much lighter than the heaviest ones, i.e.

$$m_{A_1},\ m_{h_2},\ m_{h_1} \ll m_A\,.$$

- In general the mass terms in the Higgs boson potential can be written as

$$V_{mass} = M^2_{H^\pm} H^+ H^- + \frac{1}{2}(P\ P_S)\tilde{M}^2 \begin{pmatrix} P \\ P_S \end{pmatrix} + \frac{1}{2}(H\ h\ S) M^2 \begin{pmatrix} H \\ h \\ S \end{pmatrix},$$

where

$$H_1^- = G^- \cos\beta + H^- \sin\beta\,, \qquad H_2^+ = H^+ \cos\beta - G^+ \sin\beta\,,$$

$$Im\,H_1^0 = (P\sin\beta + G^0\cos\beta)/\sqrt{2}\,, \qquad Re\,H_1^0 = (h\cos\beta - H\sin\beta + v_1)/\sqrt{2}\,,$$

$$Im\,H_2^0 = (P\cos\beta - G^0\sin\beta)/\sqrt{2}\,, \qquad Re\,H_2^0 = (h\sin\beta + H\cos\beta + v_2)/\sqrt{2}\,,$$

$$Im\,S = P_S/\sqrt{2}\,, \qquad Re\,S = (s+N)/\sqrt{2}\,.$$

- The Lagrangian that describes the interations of the neutral Higgs particles with the Z-boson is given by

$$L_{AZH} = \frac{\bar{g}}{2} M_Z Z_\mu Z_\mu h + \frac{\bar{g}}{2} Z_\mu \Big[H(\partial_\mu P) - (\partial_\mu H) P \Big]\,.$$

- Charged Higgs boson mass is

$$m_{H^\pm}^2 = m_A^2 - \frac{\lambda^2 v^2}{2} + M_W^2 + \Delta_\pm .$$

- The mass matrix of the CP-odd Higgs states is

$$\tilde{M}^2 = \begin{pmatrix} \tilde{M}_{11}^2 & \tilde{M}_{12}^2 \\ \tilde{M}_{21}^2 & \tilde{M}_{22}^2 \end{pmatrix} ,$$

$$\tilde{M}_{11}^2 = m_A^2 , \qquad \tilde{M}_{12}^2 = \tilde{M}_{21}^2 = \sqrt{2}\lambda v \mu \left(-2\frac{\varkappa}{\lambda} + \frac{x}{\sin 2\beta} \right) + \tilde{\Delta}_{12} ,$$

$$\tilde{M}_{22}^2 = -3\frac{\varkappa}{\lambda} A_\varkappa \mu + \frac{\lambda\varkappa}{2} v^2 \sin 2\beta + \frac{\lambda^2 v^2 x}{2} + \tilde{\Delta}_{22} .$$

- The masses of Higgs pseudoscalars are

$$m_{A_1,A_2}^2 = \frac{1}{2}\left(\tilde{M}_{11}^2 + \tilde{M}_{22}^2 \mp \sqrt{(\tilde{M}_{11}^2 - \tilde{M}_{22}^2)^2 + 4\tilde{M}_{12}^4} \right) .$$

- The mass matrix of CP-even Higgs sector has a form

$$M^2 = \begin{pmatrix} M_{11}^2 & M_{12}^2 & M_{13}^2 \\ M_{21}^2 & M_{22}^2 & M_{23}^2 \\ M_{31}^2 & M_{32}^2 & M_{33}^2 \end{pmatrix} = \begin{pmatrix} \frac{1}{v^2}\frac{\partial^2 V}{\partial^2 \beta} & \frac{1}{v}\frac{\partial^2 V}{\partial v \partial \beta} & \frac{1}{v}\frac{\partial^2 V}{\partial s \partial \beta} \\ \frac{1}{v}\frac{\partial^2 V}{\partial v \partial \beta} & \frac{\partial^2 V}{\partial^2 v} & \frac{\partial^2 V}{\partial v \partial s} \\ \frac{1}{v}\frac{\partial^2 V}{\partial s \partial \beta} & \frac{\partial^2 V}{\partial v \partial s} & \frac{\partial^2 V}{\partial^2 s} \end{pmatrix},$$

$$M_{11}^2 = m_A^2 + \left(\frac{\bar{g}^2}{4} - \frac{\lambda^2}{2}\right) v^2 \sin^2 2\beta + \Delta_{11} \,,$$

$$M_{12}^2 = M_{21}^2 = \left(\frac{\lambda^2}{4} - \frac{\bar{g}^2}{8}\right) v^2 \sin 4\beta + \Delta_{12} \,,$$

$$M_{13}^2 = M_{31}^2 = -\frac{\sqrt{2}\lambda v \mu x}{\tan 2\beta} + \Delta_{13} \,,$$

$$M_{22}^2 = M_Z^2 \cos^2 2\beta + \frac{\lambda^2}{2} v^2 \sin^2 2\beta + \Delta_{22} \,,$$

$$M_{23}^2 = M_{32}^2 = \sqrt{2}\lambda v \mu (1 - x) + \Delta_{23} \,,$$

$$M_{33}^2 = 4\frac{\varkappa^2}{\lambda^2}\mu^2 + \frac{\varkappa}{\lambda} A_\varkappa \mu + \frac{\lambda^2 v^2 x}{2} - \frac{\lambda\varkappa}{2} v^2 \sin 2\beta + \Delta_{33} \,.$$

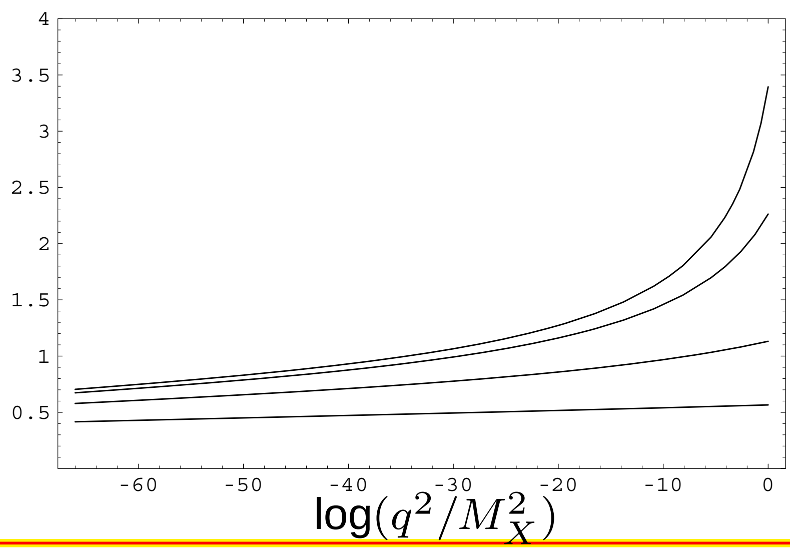
- In the considered field basis the mass of the SM-like Higgs is always less than

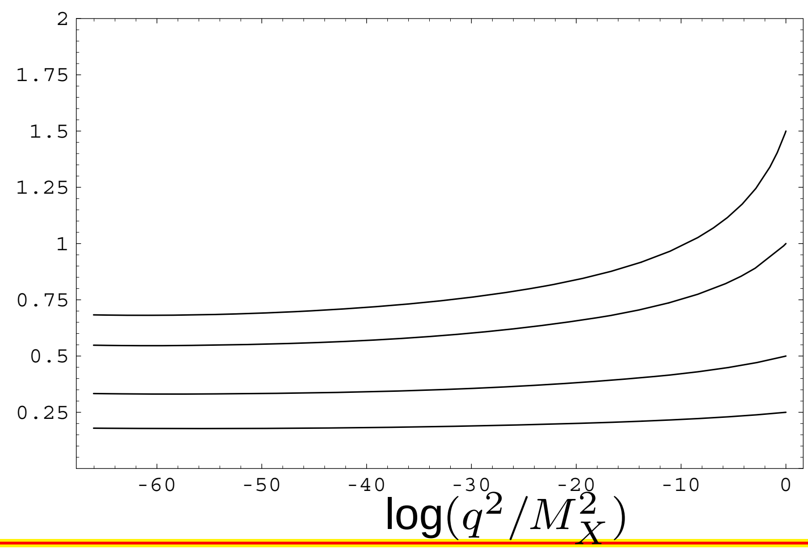
$$m_{h_2}^2 \lesssim M_{22}^2 = \frac{\partial^2 V}{\partial^2 v} = M_Z^2 \cos^2 2\beta + \frac{\lambda^2}{2} v^2 \sin^2 2\beta + \Delta_{22}\,.$$

- The requirement of validity of perturbation theory up to the GUT scale constrains the allowed range of λ and $\varkappa$

$$\lambda^2(M_Z) + \varkappa^2(M_Z) \lesssim 0.5\,.$$

Renormalization group flow of $\sqrt{\kappa^2+\lambda^2}$ and κ/λ.

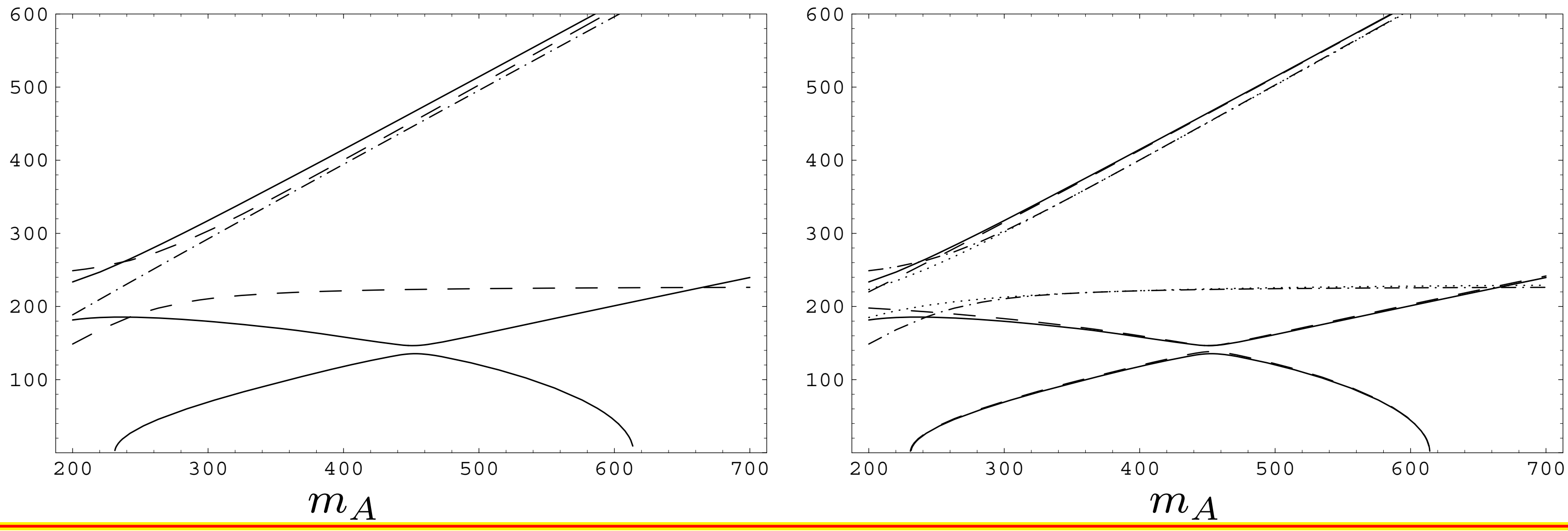




- As a result in the leading two–loop approximation the mass of the lightest Higgs boson in the NMSSM does not exceed 135-140 GeV.
- The analysis of RG flow indicates that $\varkappa^2(M_Z)$ tends to be considerably smaller than $\lambda(M_Z)$ resulting in the hierarchical structure of the Higgs spectrum.

Exact and approximate solutions for the Higgs masses (GeV):

$\lambda = 0.6$, $\kappa = 0.36$, $\mu = 150$ GeV, $\tan\beta = 3$ and $A_\kappa = 135$ GeV



- The hierarchical structure of the Higgs mass matrices allows to obtain an approximate solution.

$$m_{h_3}^2 \simeq M_{11}^2 + \frac{M_{13}^4}{M_{11}^2}\,, \qquad m_{A_1}^2 \simeq \tilde{M}_{22}^2 + \frac{\tilde{M}_{12}^4}{\tilde{M}_{11}^2}\,, \qquad m_{A_2}^2 \simeq \tilde{M}_{11}^2 - \frac{\tilde{M}_{12}^4}{\tilde{M}_{11}^2}\,,$$

$$m_{h_2,h_1}^2 = \frac{1}{2}\left(M_{22}^2 + M_{33}^2 - \frac{M_{13}^4}{M_{11}^2} \pm\sqrt{\left(M_{22}^2 - M_{33}^2 + \frac{M_{13}^4}{M_{11}^2}\right)^2 + 4\left(M_{23}^2 - \frac{M_{13}^2 M_{12}^2}{M_{11}^2}\right)^2}\right).$$

- It is convenient to define the relative R–couplings of the Higgs bosons to Z

$$g_{ZZh_i} = \frac{\bar{g}}{2} M_Z R_{ZZh_i}\,, \qquad g_{ZA_jh_i} = \frac{\bar{g}}{2} R_{ZA_jh_i}\,.$$

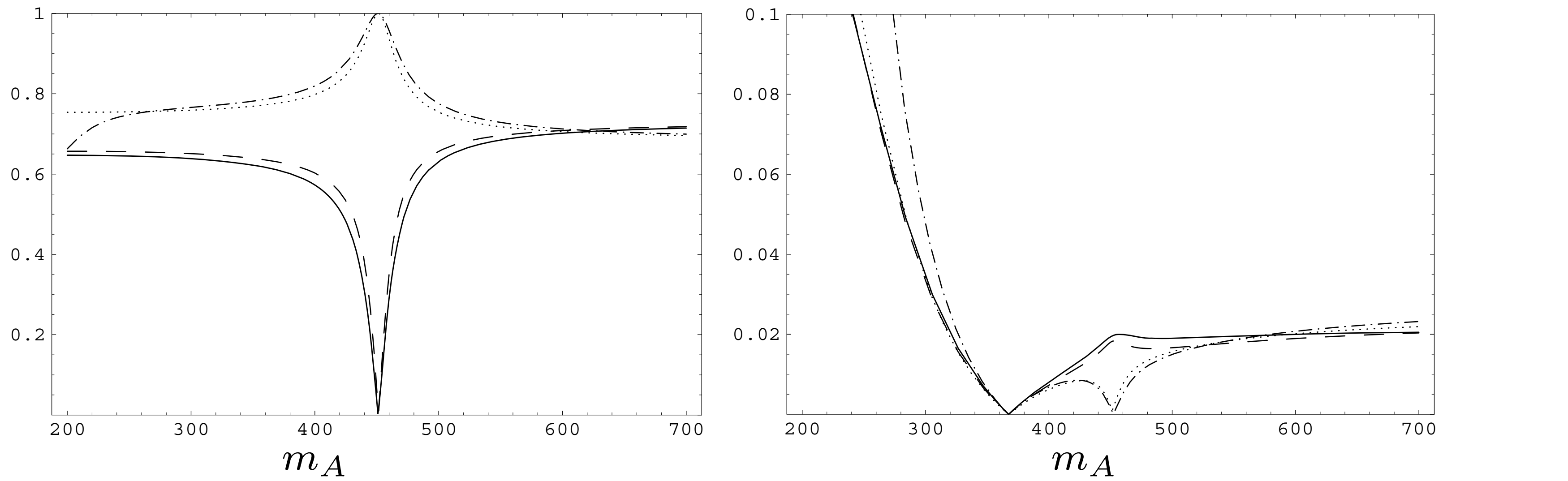
- In general these couplings obey sum rules:

$$\sum_i R_{ZZi}^2 = 1\,, \qquad \sum_{ij} R_{ZA_ji}^2 = 1\,, \qquad \sum_i R_{ZZi} R_{ZA_ji} = 0\,.$$

- In the approximate Peccei–Quinn symmetry limit the R–couplings of the lightest Higgs scalars and pseudoscalar satisfy

$$R^2_{ZZh_2} + R^2_{ZZh_3} \simeq 1\,, \qquad R^2_{ZA_2h_2} + R^2_{ZA_2h_3} \simeq O\left(\frac{\lambda^4 v^4}{4m_A^4}\right) \ll 1.$$

The couplings $|R_{ZZi}|$ and $|R_{ZA_1i}|$ of the lightest Higgs bosons:
$\lambda = 0.6$, $\kappa = 0.36$, $\mu = 150\,$GeV, $\tan\beta = 3$ and $A_\kappa = 135\,$GeV



- Normally in the MSSM and NMSSM the lightest Higgs boson decays predominantly into $b\bar{b}$.
- However the couplings of the SM-like Higgs boson to the bottom quark is rather small $\sim (m_b/v) < 1/50$.
- In principle new light particles which could escape detection at LEP can have relatively large couplings to the lightest Higgs boson leading to the drastic change in the strategy of Higgs boson searches.
- In the NMSSM such scenario can be realized if new singlet pseudoscalar state A_1 is light [Dermisek, Gunion].
- In this case A_1 can decay into $\tau\bar{\tau}$ (if A_1 is relatively light $m_{A_1} \lesssim 10$ GeV) or into $\tau\bar{\tau}$ and into $b\bar{b}$ resulting in four fermion decay of the SM-like Higgs boson, i.e. $h \to A_1 A_1 \to f\bar{f}f'\bar{f}'$.

Conclusions

- Supersymmetry is a very promissing extension of the SM. It provides dark matter candidate, lead to the unification of gauge couplings and stabilize mass scale hierarchy.
- SUSY models predict relatively light SM-like Higgs boson that can be discovered in the near future.
- The stringent experimental lower bound on the lightest Higgs boson mass lead to the little hierarchy problem.
- In the extensions of the MSSM the heaviest CP–odd, heaviest CP–even and charged Higgs states become very heavy ($m_{H^\pm} \simeq m_A \simeq m_H \gtrsim 1$ TeV) if λ is large.
- In the nonminimal SUSY models new particles and interactions may give rise to new channels of Higgs decays resulting in the drastic change in the strategy of Higgs searches.