Internal structure of Maxwell-Gauss-Bonnet black hole

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Subject

The talk is based on the paper

S.O. Alexeyev, A. Barrau, K.A. Rannu

"Internal structure of a Maxwell-Gauss-Bonnet black hole"

Phys. Rev. **D 79** 067503 (2009)

S.O. Alexeyev, M.V. Pomazanov

Phys. Rev. D **55** 2110 (1997)

S.O. Alexeyev, M.V. Sazhin

Gen. Relativ. Grav **30** 1187 (1998)

S.O. Alexeyev, M.V. Sazhin, M.V. Pomazanov

Int. J. Mod. Phys **D 10** 225 (2001)

Action and metric

Low-energy effective string action with second order curvature corrections:

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(-R + 2\partial_\mu\phi\partial^\mu\phi - e^{-2\phi}F_{\mu\nu}F^{\mu\nu} + \lambda e^{-2\phi}S_{GB}\right),$$

$$S_{GB} = R_{ijkl}R^{ijkl} - 4R_{ij}R^{ij} + R^2$$

Spherically-symmetric metric in GHS coordinates:

$$ds^2 = \Delta dt^2 - \frac{1}{\Delta}dr^2 - f^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

Anzats for Maxwell tensor: $F = q\sin\theta\; d\theta\wedge d\varphi$

Variable change: $E = e^{-2\phi}, \quad E_0 = e^{-2\phi_0}$

Lagrange-Euler (Einstein) equations

$$\begin{pmatrix} 0 & f-4\lambda f'E' & -2\lambda(\Delta f'^2-1) \\ f-4\lambda\Delta f'E' & 2\Delta(1-2\lambda\Delta'E') & -4\lambda\Delta\Delta'f' \\ 4\lambda E(\Delta f'^2-1) & 8\lambda\Delta\Delta'f'E & \dfrac{\Delta f^2}{E} \end{pmatrix} \begin{pmatrix} \partial\Delta'/\partial r \\ \partial f'/\partial r \\ \partial E'/\partial r \end{pmatrix} =$$

$$= \begin{pmatrix} -\dfrac{f^2}{4}\dfrac{E'^2}{E^2} \\ 2E\dfrac{q^2}{f^3} - 2\Delta'f' - \dfrac{\Delta f}{2}\dfrac{E'^2}{E^2} + 4\lambda\Delta'^2 f'E' \\ 2E\dfrac{q^2}{f^2} - f(\Delta'f + 2\Delta f')\dfrac{E'}{E} + \Delta f^2\dfrac{E'^2}{E^2} - 4\lambda\Delta'^2 f'^2 E \end{pmatrix}$$

Constraint:

$$(1+\Delta f^2(\phi')^2 - \Delta' f f' - \Delta(f')^2) + 4E\lambda\Delta'\phi'(1-3\Delta(f')^2) - Eq^2 f^{-2} = 0$$

Asymptotics

- Asymptotics near the horizon $\left((r-r_h)\ll 1\right)$:

$$\Delta = d_1(r-r_h)+d_2(r-r_h)^2+O\left((r-r_h)^2\right)$$
$$f = f_0+f_1(r-r_h)+f_2(r-r_h)^2+O\left((r-r_h)^2\right)$$
$$E = E_0+\phi_1(r-r_h)+\phi_2(r-r_h)^2+O\left((r-r_h)^2\right)$$

- Asymptotics at the infinity — GM-GHS solution:

$$ds^2=\left(1-\frac{2M}{r}\right)dt^2-\left(1-\frac{2M}{r}\right)^{-1}dr^2-r\left(r-\frac{q^2E_0}{M}\right)d\Omega$$
$$E=E_0-\frac{q^2}{Mr}$$

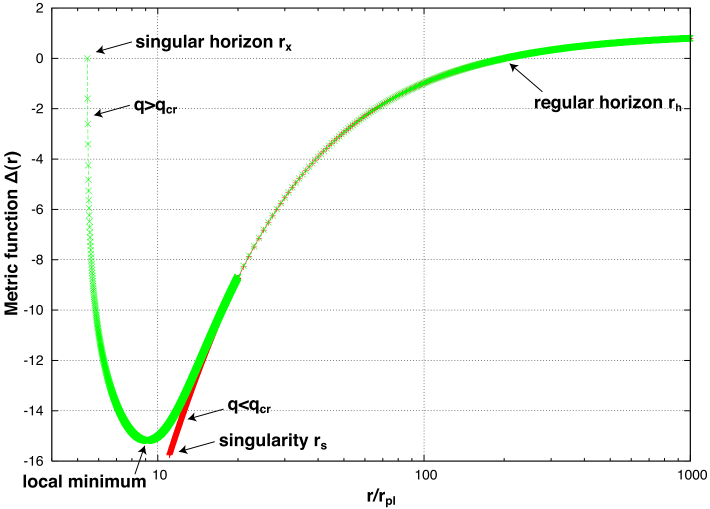
Curvature invariant in GHS coordinates

$$R_{ijkl}R^{ijkl} = \Delta''^2 + 4\Delta'^2\frac{f'^2}{f^2} + 8\Delta^2\frac{f''^2}{f^2} + 8\Delta\Delta'\frac{f'f''}{f^2} + \frac{4}{f^4} - 8\Delta\frac{f'^2}{f^4} + 4\Delta^2\frac{f'^4}{f^4}$$

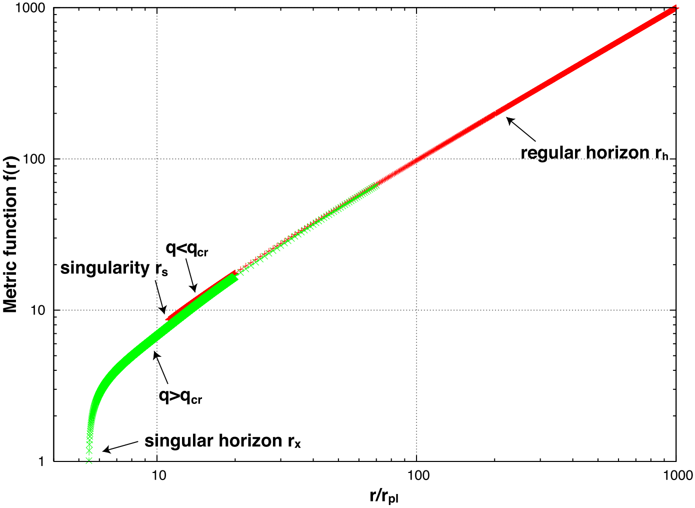
Determining the singularity type:

- curvature invariant diverges $\Rightarrow$ real scalar singularity (Clarke, 1993)
- curvature invariant is finite $\Rightarrow$ coordinate singularity, that can be neglected by appropriate coordinate transformation

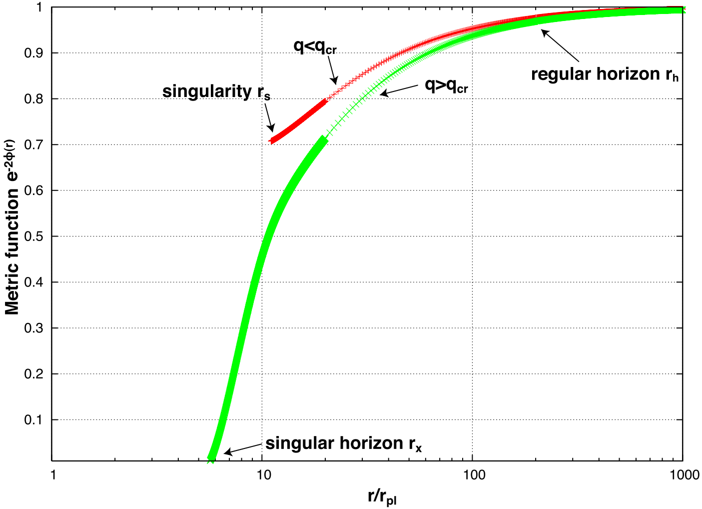
Metric function $\Delta(r)$



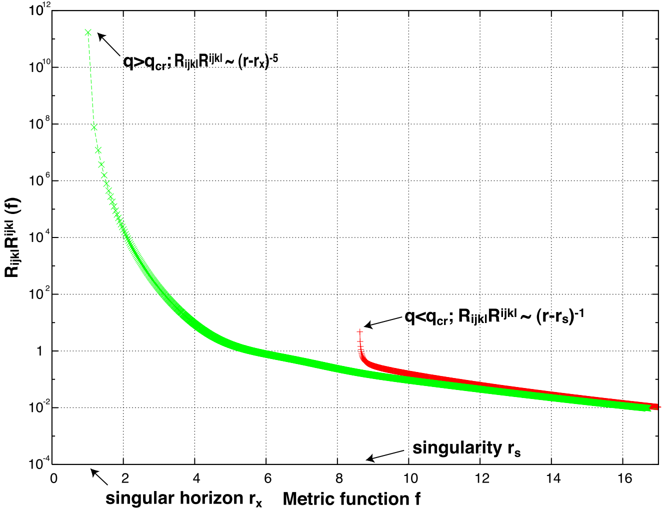
Metric function $f(r)$



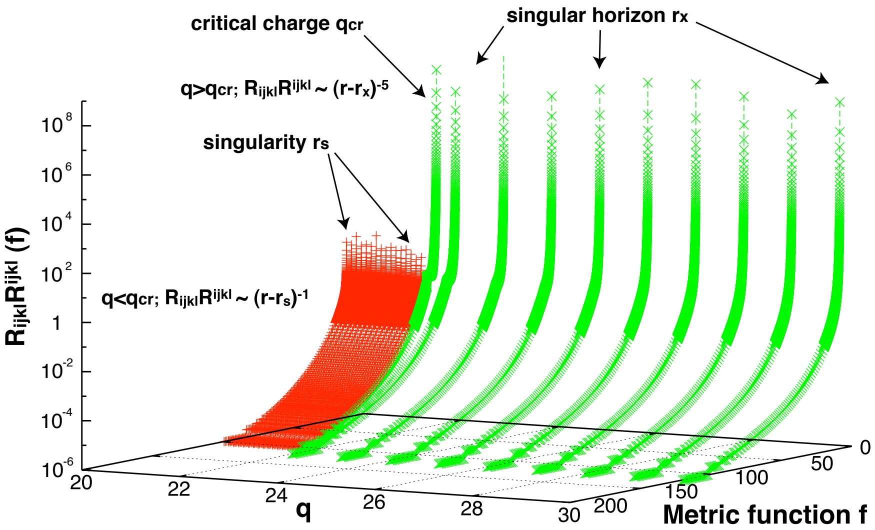
Dilatonic exponent $exp(-2\phi(r))$



Curvature invariant $R_{ijkl}R^{ijkl}(f)$



Curvature invariant $R_{ijkl}R^{ijkl}(q,f)$



Asymptotics near the particular points

Metric function $f(r)$ should be regarded as a radial coordinate.

$$f(r \to r_s) = f_s + f_{s2}(\sqrt{r-r_s})^2 + f_{s3}(\sqrt{r-r_s})^3 + \dots$$
$$f(r \to r_x) = f_x + f_{x1}\sqrt{r-r_x} + f_{x2}(\sqrt{r-r_s})^2 + \dots$$

$$\text{for} \quad f \to f_s \quad R_{ijkl}R^{ijkl} \sim \text{const}_1 \times (f - f_s)^{-1}$$
$$\text{for} \quad f \to f_x \quad R_{ijkl}R^{ijkl} \sim \text{const}_2 \times (f - f_x)^{-5}$$

Results

- When the black hole charge becomes larger than the critical value the singularity r_s is replaced by a local minimum of the fuction $\Delta(r)$ and the solution exists till the singular horizon r_x.
- Function $f(r)$ is the radius of S^2, so it plays the role of the radial coordinate. If $q < q_{cr}$ it decreases monotonously till $r = r_s$ like in GHS. When r_s disappears the function $f(r)$ reaches its zero in the new point r_x.
- Curvature invariant increases much more rapidly (as $(r - r_x)^{-5}$) near the singular horizon r_x than near the singularity r_s (as $(r - r_s)^{-1}$), so the singularity in r_x is much stronger than the one in r_s.
- New kind of singularity inside black hole was found. Unfortunately Maxwell-Gauss-Bonnet black hole cannot help wormholes' or multiverse theories because this singularity is very strong.

Thank you
for attention

Critical charge

Critical charge value — a meaning at which the inner singularity disappears being replaced by a local minimum for $\Delta(r)$