Effects of higher spin mesons in elastic electron-nucleon scattering

Nikolay Volchanskiy[1]
Research Institute of Physics, Southern Federal University,
prospekt Stachki 194, 344090 Rostov-na-Donu, Russia

We study multiphoton exchange effects in elastic electron-nucleon scattering. An effective field theory of eN scattering mediated by exchange of mesons with arbitrary high spin is constructed in the Born approximation. In the particular case of spin-2 mesons, we construct a general vertex for interactions of spin-2 mesons with two photons or vector mesons and calculate corresponding one-loop contributions to eN scattering.

Elastic lepton-nucleon scattering is of fundamental importance as a source of information on the properties and internal structure of the nucleons. These properties are parametrized as the elastic form factors (FFs) of the nucleons. A large bulk of quite precise experimental data on the elastic FFs is available. Some features of these data are intriguing and still not completely understood.

Historically, the nucleon FFs were obtained mostly from the data on unpolarized elastic electron-nucleon scattering. The method of extraction of the FFs from unpolarized data is the Rosenbluth separation [1]. It gives both electric and magnetic nucleon FF. In recent decades progress in the experimental field made it possible to use polarized beams and targets intensively. The polarization experiments allow us to determine the ratio of the elastic FFs [2, 3]. The ratios extracted from unpolarized and polarization experiments are in significant disagreement (see review [4]). To understand the origin of this discrepancy is important not only because of general importance to understand the nucleon structure, but also because of a number of reasons, of which I want to mention at least one.

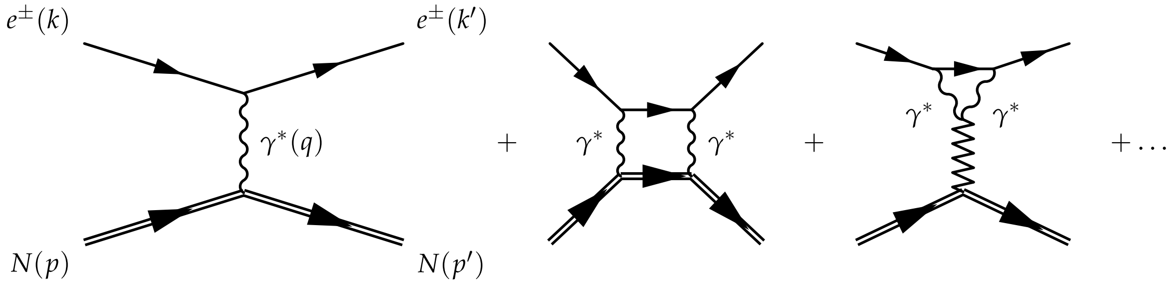
The recent proton charge radius measurements from the Lamb shift in muonic hydrogen disagree sharply with the values obtained from atomic hydrogen and electron-proton scattering (see review [5]). In the literature, this so called proton radius puzzle is often considered as a possible clue to physics beyond the Standard Model, in particular to a violation of lepton flavor universality [6]. In order to draw conclusions, however, we have to be sure that we have taken into account all relevant contributions and consequently all lepton mass effects. This report is about a step in this direction.

The mainstream in explaining the discrepancy between polarization and unpolarized measurements is two-photon exchange (TPE) effects (see reviews [4, 7]). Indeed, TPE [second diagram in (1)] can account for at least a part of the discrepancy. The recent polarization data show, however, that the situation is not perfect—many models have difficulties describing the data [8].

Instead of considering photons that couple directly to the nucleon, we can study a different parametrization of two-photon and multiphoton exchange effects with photons hadronizing into a meson that consequently interacts with the nucleons directly [an example is shown as a triangle loop in (1)]. Such parametrization was considered in Refs. [9, 10] and more recent paper [11]. There is not much experimental information on multiphoton hadronization amplitudes and couplings of higher-spin mesons to nucleons at the low momentum transfers, which leaves some space for varying the parameters of the triangle loop in (1).

[1]nikolay.volchanskiy@gmail.com

The contributions of scalar and pseudoscalar mesons are suppressed by a lepton mass due to lepton helicity conservation and can be safely neglected in the case of the electron nucleon scattering. Therefore, the spin of the exchanged meson should be not less then two.



(1)

Dealing with higher-spin hadrons as effective fields can be tricky. In the simplest case of spin-2 mesons, the corresponding field is a symmetric rank-2 tensor $\varphi_{\mu\nu}$. The field contains 10 degrees of freedom (DOF), half of which are redundant. The Fierz-Pauli Lagrangian of the free symmetric tensor field is written here in terms of the linearized Einstein tensor $G_{\mu\nu}$:

$$\mathscr{L} = -\varphi_{\mu\nu}G^{\mu\nu} - \frac{m^2}{2}\left[\varphi_{\mu\nu}\varphi^{\mu\nu} - \left(\varphi^{\lambda}_{\lambda}\right)^2\right], \tag{2}$$

$$G_{\mu\nu} = \varphi_{\mu\lambda\nu}{}^{\lambda} - \frac{1}{2}g_{\mu\nu}\varphi_{\lambda\sigma}{}^{\lambda\sigma}, \tag{3}$$

$$\varphi_{\mu\nu\lambda\sigma} = \frac{1}{2}\left(\partial_\mu\partial_\lambda\varphi_{\nu\sigma} - \partial_\nu\partial_\lambda\varphi_{\mu\sigma} - \partial_\mu\partial_\sigma\varphi_{\nu\lambda} + \partial_\nu\partial_\sigma\varphi_{\mu\lambda}\right). \tag{4}$$

The kinetic term of the free Lagrangian (2) possesses an internal gauge symmetry, $\delta\varphi_{\mu\nu} = \partial_\mu\theta_\nu + \partial_\nu\theta_\mu$. The gauge symmetry is broken by the mass term in such a way that the massive field is subject to supplementary conditions $\varphi^{\mu}_{\mu} = 0 = \partial^\nu\varphi_{\nu\mu}$, which leaves only 5 physical DOF.

Introducing interactions to the theory might change the number of constraints, which results in different pathologies including superluminal propagation and problems with quantization. Similar problems plague the field theories of all higher spins $J \geq 3/2$ and do not have a known general solution in a closed form [12–14]. Constructing consistent interactions of higher-spin fields is not an easy task and usually makes it necessary to work with nonminimal or even nonlocal interactions. The interactions we consider here are much simpler (linear in the higher-spin field) and can be made consistent by assuming their invariance under the gauge transformation of the free field, which means that vertices should be transversal:

$$q^{\mu_1}\Gamma^{\text{lepton}}_{\mu_1\mu_2\cdots\mu_J} = 0 = q^{\mu_1}\Gamma^{\text{hadron}}_{\mu_1\mu_2\cdots\mu_J}. \tag{5}$$

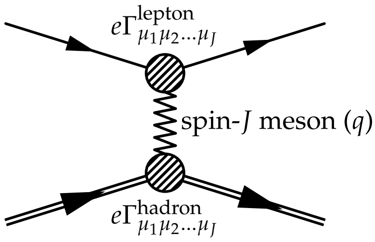
We can consider general decomposition of the vertices of spin-J meson interactions with leptons and nucleons. These vertices depend on two invariant amplitudes. In the lepton vertex we can omit one amplitude due to the helicity conservation and negligible electron mass:

$$\Gamma^{\text{lepton}}_{\mu_1\mu_2\cdots\mu_J} = \frac{h_{(J)}}{JM^{J-1}}\left(\gamma_{\mu_1}K_{\mu_2}\cdots K_{\mu_J} + \text{permutations}\right), \tag{6}$$

$$\Gamma^{\text{hadron}}_{\mu_1\mu_2\cdots\mu_J} = \frac{g_{(J)M}}{JM^{J-1}}\left(\gamma_{\mu_1}P_{\mu_2}\cdots P_{\mu_J} + \text{permutations}\right) - \frac{g_{(J)2}}{M^J}P_{\mu_1}\cdots P_{\mu_J}, \tag{7}$$

$$K_\mu = \frac{1}{2}(k_\mu + k'_\mu), \qquad P_\mu = \frac{1}{2}(p_\mu + p'_\mu).$$

For spin-1 Eqs. (6) and (7) simplify to the QED and Rosenbluth nucleon vertecies.



(8)

Now we can calculate the differential cross section of elastic eN scattering with the transfer of polarization from the electron beam to the recoil target. In the laboratory frame, we obtain a modified Rosenbluth formula including the contributions of one-meson-exchange for arbitrarily high spin J of the meson:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(e^{\pm}N \to e^{\pm}N) = \sigma_{(0)a}\sigma_a^{\pm}(\tau,\epsilon)\left[1 + h(\boldsymbol{\zeta}\hat{\mathbf{z}})P_l^a(\tau,\epsilon) + h(\boldsymbol{\zeta}\hat{\mathbf{x}})P_t^a(\tau,\epsilon)\right], \tag{9}$$

$$\sigma_a^{\pm} = \tau|\mathscr{G}_{Ma}^{\pm}|^2 + \epsilon|\mathscr{G}_{Ea}^{\pm}|^2 + \tau|\mathscr{G}_{Ta}^{\pm}|^2, \tag{10}$$

$$P_l^a = \tau\sqrt{1-\epsilon^2}\,\frac{1}{\sigma_p^-}\Re\left[\left(\mathscr{G}_{Ma}^- + \sqrt{\frac{1-\epsilon}{1+\epsilon}}\mathscr{G}_{Ta}^-\right)\left(\mathscr{G}_{Ma}^- - \sqrt{\frac{1+\epsilon}{1-\epsilon}}\mathscr{G}_{Ta}^-\right)^*\right], \tag{11}$$

$$P_t^a = -\sqrt{2\tau\epsilon(1-\epsilon)}\,\frac{1}{\sigma_a^-}\Re\left[\mathscr{G}_{Ea}^-\left(\mathscr{G}_{Ma}^- - \sqrt{\frac{1+\epsilon}{1-\epsilon}}\mathscr{G}_{Ta}^-\right)^*\right], \tag{12}$$

Here $a = p$, or n, $\tau = Q^2/(4M_N^2)$, $\sigma_{(0)a}$ is the Mott cross section, h is the beam helicity, $\boldsymbol{\zeta}$ is the polarization vector of the recoiled nucleon, $\epsilon = [1 + 2(1+\tau)\tan^2\frac{\theta}{2}]^{-1}$ is the polarization parameter of the virtual photon, and the kinematics is conventional [15]. The unpolarized part of the cross section, reduced cross section $\sigma_a^{\pm}(\tau,\epsilon)$, and the longitudinal P_l^a and transverse P_t^a polarization coefficients depend on the generalized FFs:

$$\mathscr{G}_{Ma}^{\pm}(\tau,\epsilon) = G_M^a(Q^2) + \sum_{J\geqslant 2}\left(\mp\frac{\alpha}{\pi}\tilde{\nu}\right)^{J-1} R_M^{(J)}(\epsilon)G_{Ma}^{(J)}(Q^2), \tag{13}$$

$$\mathscr{G}_{Ea}^{\pm}(\tau,\epsilon) = G_E^a(Q^2) + \sum_{J\geqslant 2}\left(\mp\frac{\alpha}{\pi}\tilde{\nu}\right)^{J-1} R_E^{(J)}(\epsilon)G_{Ea}^{(J)}(Q^2), \tag{14}$$

$$\mathscr{G}_{Ta}^{\pm}(\tau,\epsilon) = \epsilon\sqrt{\frac{1-\epsilon}{1+\epsilon}}\sum_{J\geqslant 2}\left(\mp\frac{\alpha}{\pi}\tilde{\nu}\right)^{J-1} R_T^{(J)}(\epsilon)G_{Ma}^{(J)}(Q^2), \tag{15}$$

where

$$\tilde{\nu} = \sqrt{\tau(1+\tau)}\sqrt{\frac{1+\epsilon}{1-\epsilon}} \tag{16}$$

and $R_i^{(J)}(\epsilon)$, $i = E,\ M,\ T$ are positive regular rational functions of $\epsilon \in [0,\,1]$. First terms of the electric and magnetic generalized FFs are the Sachs FFs, while all other terms come from exchanges of higher spin mesons, which makes the generalized FFs be functions of all two independent kinematic variables, ϵ and Q^2.

However, it is also important to consider Q^2-dependencies of the lepton coupling $h_{(J)}(Q^2)$ of higher spin mesons (6). It means that we have to calculate triangle loop in (1). To do this in the simplest case of spin $J = 2$, we need a Lagrangian of the tensor meson interactions with two vector fields. Besides, it is preferable that the effective interaction Lagrangian does not lead to ultraviolet (UV) divergences. To this end, it is assumed that the interaction Lagrangian is invariant under "point" transformations of the tensor field

$$\varphi_{\mu\nu} \to \varphi_{\mu\nu} + \eta g_{\mu\nu} \phi^{\lambda}_{\lambda}, \tag{17}$$

i.e. the current is traceless. Note that the free field Fierz-Pauli Lagrangian (2) is not invariant under the point transformation, but the free field equations and constraints do not change under the transformation (17), that is the point transformation forms a non-unitary symmetry group of the equivalent class of free Lagrangians. The invariants of the lowest differential orders can be written as follows:

$$\begin{gathered} I_1 = \frac{1}{M^3} C_{\mu\nu\lambda\sigma} V_1^{\mu\nu} V_2^{\lambda\sigma}, \qquad I_2 = \frac{1}{M^5} C_{\mu\nu\lambda\sigma} \partial^\mu \partial^\lambda \left(V_1^{\nu\rho} V_{2\,\rho}^{\sigma} \right), \qquad I_3 = \frac{1}{M^5} C_{\mu\nu\lambda\sigma} \partial^\lambda V_1^{\mu\rho} \partial^\sigma V_{2\,\rho}^{\nu}, \\ I_4 = \frac{1}{M^5} C_{\mu\nu\lambda\sigma} \left(\partial_\rho V_1^{\nu\rho} \partial^\mu V_2^{\lambda\sigma} - \partial_\rho V_2^{\nu\rho} \partial^\mu V_1^{\lambda\sigma} \right), \qquad I_5 = \frac{1}{M^7} C_{\mu\nu\lambda\sigma} \partial^\mu \partial^\lambda \left(\partial^\nu V_1^{\rho\omega} \partial^\sigma V_{2\rho\omega} \right). \end{gathered} \tag{18}$$

Here

$$C_{\mu\nu\lambda\sigma} = \varphi_{\mu\nu\lambda\sigma} - \frac{1}{2} \left(\varphi_{\mu\rho\lambda}{}^{\rho} g_{\nu\sigma} - \varphi_{\nu\rho\lambda}{}^{\rho} g_{\mu\sigma} - \varphi_{\mu\rho\sigma}{}^{\rho} g_{\nu\lambda} + \varphi_{\nu\rho\sigma}{}^{\rho} g_{\mu\lambda} \right) + \frac{1}{6} \varphi_{\rho\omega}{}^{\rho\omega} \left(g_{\mu\lambda} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\lambda} \right). \tag{19}$$

is the linearized Weyl tensor. Isospinor indices are omitted for simplicity. All other Lagrangian terms permitted by the symmetry are reduced (up to a total derivative) to the invariants (18) or nonlocal corrections obtained by inserting pairs of derivatives with contracted indices. For estimates of tensor exchange in eN scattering it suffices to take into account just the lowest in derivatives invariant I_1. It is the only one in its differential order, i.e. it is selected by the gauge and point symmetry unambiguously, and it gives UV finite contribution to the loop despite the large number of derivatives involved. It can be checked that the invariant I_1 is the only one gauge invariant Lagrangian with four derivatives of field operators that leads to the cancellation of UV divergences. It justifies the requirement of the point invariance of the interaction Lagrangian.

Let us now summarize the results. Effective gauge invariant vertices for the interactions of higher-spin mesons with electrons and nucleons have been written. In the simplest case of spin-2 mesons, the general Lagrangian for the interactions with two vector fields has been constructed. The symmetry of the Lagrangian ensures its physical and mathematical consistency. Further, the symmetry results in automatic cancellation of UV divergences in the lepton vertex of spin-2 mesons (at least in the case of the Lagrangian term with the minimal number of the derivatives of field operators). We have computed the cross section of eN-scattering mediated by exchange of arbitrarily-high–spin meson. In the case of spin-2 mesons, we have taken into account Q^2-dependence of the lepton-meson vertex using the spin-2–spin-1–spin-1 Lagrangian constructed.

To describe experimental data on the elastic eN-scattering, we could employ a model of dominance of vector and tensor mesons constrained by high-Q^2 pQCD predictions similarly to Ref. [16]. However, we have to extend our model to include both box and triangle two-photon effects as well as other relevant corrections. Besides, higher-spin exchange results in singularity at $\epsilon = 1$, if the standard propagators for higher-spin mesons are used in calculations [see Eqs. (13)–(15)]. The problem could be resolved by using Regge propagators for higher-spin mesons as in Ref. [11] or, possibly, considering a coherent exchange of an infinite tower of mesons with arbitrarily high spins $J \geq 2$.

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