Vacuum polarization and quadrupole corrections to the hyperfine splitting of P-states in muonic deuterium

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On the basis of quasipotential approach in quantum electrodynamics we calculate vacuum polarization and quadrupole corrections in first and second orders of perturbation theory in hyperfine structure of P-states in muonic deuterium. All corrections are presented in integral form and evaluated analytically and numerically. The obtained results can be used for the improvement of the transition frequencies between levels 2P and 2S.

In last years a significant theoretical interest in the investigation of fine and hyperfine energy structure of simple atoms is related with light muonic atoms: muonic hydrogen, muonic deuterium and ions of muonic helium. This is conditioned due to essential progress achieved by experimental collaboration CREMA (Charge Radius Experiment with Muonic Atoms) in studies of such simple atoms [1,2]. The measurement of the transition frequency $2S^{f=1}_{1/2}-2P^{f=2}_{3/2}$ leads to a new more precise value of the proton charge radius. For the first time the hyperfine splitting (HFS) of 2S state in muonic hydrogen was measured. Analogous measurements in muonic deuterium are also carried out and planned for the publication. It is important to point out that the CREMA experiments set a task to improve by an order of the magnitude numerical values of charge radii of simplest nuclei (proton, deuteron, helion and α-particle) [3]. Successful realization of such program is based on precise theoretical calculations of different corrections to the energy intervals of fine and hyperfine structure of muonic atoms.

Let us consider the HFS of P-states in muonic deuterium. Our approach is based on quasipotential method in QED [4–7], in which two-particle bound state is described by the Schroedinger equation. Main contribution to hyperfine splitting in muonic deuterium is given by hyperfine part of the Breit Hamiltoian [8,9]:

$$\Delta V_B^{hfs}(r)=\frac{Z\alpha(1+\kappa_d)}{2m_1m_2r^3}\left[1+\frac{m_1\kappa_d}{m_2(1+\kappa_d)}\right](\boldsymbol{L}\cdot\boldsymbol{s}_2)-\frac{Z\alpha(1+\kappa_d)(1+a_\mu)}{2m_1m_2r^3}\left[(\boldsymbol{s}_1\cdot\boldsymbol{s}_2)-3(\boldsymbol{s}_1\cdot\boldsymbol{n})(\boldsymbol{s}_2\cdot\boldsymbol{n})\right], \quad (1)$$

where m_1, m_2 are muon and deuteron masses respectively, κ_d, a_μ are anomalous magnetic moments of deuteron and muon, $\boldsymbol{L}$, $\boldsymbol{s}_1$ are orbital momentum and spin of muon, $\boldsymbol{s}_2$ is the deuteron spin, $\boldsymbol{n}=\boldsymbol{r}/r$. This operator doesn't commute with the operator of total angular momentum of muon $\boldsymbol{J}=\boldsymbol{L}+\boldsymbol{s}_1$, which leads to non-zero off-diagonal matrix elements. The Coulomb wave function for 2P-state has the following form:

$$\Psi_{2P}(\boldsymbol{r})=\frac{1}{2\sqrt{6}}W^{\frac{5}{2}}re^{-\frac{Wr}{2}}Y_{1m}(\theta,\phi),\ W=\mu Z\alpha. \quad (2)$$

Averaging (1) over wave function (2) we obtain the contribution of order α^4 to HFS of P-states:

$$E_B^{hfs}=\frac{\alpha^4\mu^3(1+\kappa_d)}{48m_1m_2}\left[\overline{T_1}+\frac{m_1\kappa_d}{m_2(1+\kappa_d)}\overline{T_1}-(1+a_\mu)\overline{T_2}\right], \quad (3)$$

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where we introduce the following designations:

$$T_1 = (\boldsymbol{L}\cdot\boldsymbol{s}_2),\; T_2 = \Big[(\boldsymbol{s}_1\cdot\boldsymbol{s}_2) - 3(\boldsymbol{s}_1\cdot\boldsymbol{n})(\boldsymbol{s}_2\cdot\boldsymbol{n})\Big],\; T_3 = \Big[(\boldsymbol{s}_1\cdot\boldsymbol{s}_2) - (\boldsymbol{s}_1\cdot\boldsymbol{n})(\boldsymbol{s}_2\cdot\boldsymbol{n})\Big]. \tag{4}$$

The angle averaging of (4) in the case of diagonal matrix elements can be performed with the help of the following relations [10]:

$$\boldsymbol{s}_1 \to \boldsymbol{J}\frac{\overline{(\boldsymbol{s}_1\cdot\boldsymbol{J})}}{J^2},\; \boldsymbol{L} \to \boldsymbol{J}\frac{\overline{(\boldsymbol{L}\cdot\boldsymbol{J})}}{J^2},\; \overline{(\boldsymbol{s}_1\cdot\boldsymbol{J})} = \frac{1}{2}\Big[j(j+1) - l(l+1) + \frac{3}{4}\Big],\; \overline{(\boldsymbol{L}\cdot\boldsymbol{J})} = \frac{1}{2}\Big[j(j+1) + l(l+1) - \frac{3}{4}\Big],$$
$$\langle \delta_i j - 3n_i n_j\rangle = -\frac{1}{5}(4\delta_{ij} - 3L_iL_j - 3L_jL_i). \tag{5}$$

In the case of off-diagonal matrix elements an averaging of (4) can be expressed in terms of 6j-symbols:

$$\overline{T}_1 = 2\overline{T}_2 = -2\overline{T}_3 = (-1)^{-J-F-I+L+3/2+J'}\sqrt{(2J'+1)(2J+1)}\sqrt{(2I+1)(I+1)I(2L+1)(L+1)L}\times \tag{6}$$
$$\times\begin{Bmatrix} j & I & F \\ I & j' & 1 \end{Bmatrix}\begin{Bmatrix} l & j' & \frac{1}{2} \\ j & l & 1 \end{Bmatrix} = \begin{cases} -\frac{\sqrt{2}}{3},\; F = 1/2, \\ -\frac{\sqrt{5}}{3},\; F = 3/2. \end{cases}$$

Relativistic correction to the hyperfine structure of 2P-state is also known in analytical form:

$$E^{hfs}_{rel}(2P_{1/2}) = \frac{\alpha^6(1+\kappa_d)\mu^3}{48m_1m_2}\frac{m_1^3}{\mu^3}\frac{47}{9}\times\frac{1}{2}[F(F+1) - J(J+1) - I(I+1)], \tag{7}$$

$$E^{hfs}_{rel}(2P_{3/2}) = \frac{\alpha^6(1+\kappa_d)\mu^3}{48m_1m_2}\frac{m_1^3}{\mu^3}\frac{7}{45}\times\frac{1}{2}[F(F+1) - J(J+1) - I(I+1)], \tag{8}$$

$$E^{hfs,off-diag}_{rel,F=1/2} = -\frac{\alpha^6(1+\kappa_d)\mu^3}{48m_1m_2}\frac{m_1^3}{\mu^3}\frac{3\sqrt{2}}{32},\; E^{hfs,off-diag}_{rel,F=3/2} = -\frac{\alpha^6(1+\kappa_d)\mu^3}{48m_1m_2}\frac{m_1^3}{\mu^3}\frac{3\sqrt{5}}{32}. \tag{9}$$

For one-loop vacuum polarization (VP) correction to (1) we obtain the following potential [10]:

$$\Delta V^{hfs}_{1\gamma,VP}(r) = \frac{Z\alpha(1+\kappa_d)}{2m_1m_2r^3}\frac{\alpha}{3\pi}\int_1^\infty \rho(\xi)d\xi e^{-2m_e\xi r}\Bigg\{\Bigg(1+\frac{m_1\kappa_d}{m_2(1+\kappa_d)}\Bigg)(\boldsymbol{L}\cdot\boldsymbol{s}_2)(1+2m_e\xi r) -$$
$$-(1+a_\mu)\Bigg(4m_e^2\xi^2r^2[(\boldsymbol{s}_1\cdot\boldsymbol{s}_2) - (\boldsymbol{s}_1\cdot\boldsymbol{n})(\boldsymbol{s}_2\cdot\boldsymbol{n})] + (1+2m_e\xi r)[(\boldsymbol{s}_1\cdot\boldsymbol{s}_2) - 3(\boldsymbol{s}_1\cdot\boldsymbol{n})(\boldsymbol{s}_2\cdot\boldsymbol{n})]\Bigg)\Bigg\}. \tag{10}$$

The contribution of (10) to hyperfine splitting is given by integral expression:

$$E^{hfs}_{1\gamma,VP}(r) = \frac{\alpha^4\mu^3(1+\kappa_d)}{24m_1m_2r^3}\frac{\alpha}{6\pi}\int_1^\infty \rho(\xi)d\xi\int_0^\infty xdxe^{-x[1+\frac{2m_e\xi}{W}]}\Bigg[\Bigg(1+\frac{m_1\kappa_d}{m_2(1+\kappa_d)}\Bigg)\times$$
$$\times\overline{T_1}(1+\frac{2m_e\xi}{W}x) - (1+a_\mu)\Bigg(\frac{4m_e^2\xi^2x^2}{W^2}\overline{T_3} + (1+\frac{2m_e\xi}{W}x)\overline{T_2}\Bigg)\Bigg]. \tag{11}$$

The integration in (11) is performed analytically over x and numerically over ξ. Numerical results are presented in Table 1. For two-loop VP contribution with two sequential loops we obtain the following

Table 1: Numerical values of corrections to $2P$-state hyperfine structure

Correction	$2P^2_{1/2}$, μeV	$2P^4_{1/2}$, μeV	$2P^2_{3/2}$, μeV	$2P^4_{3/2}$, μeV	$2P^6_{3/2}$, μeV	$2P^{F=1/2}_{1/2\to 3/2}$, μeV	$2P^{F=3/2}_{1/2\to 3/2}$, μeV
α^4	-1380.3359	690.1679	8162.2889	8583.2316	9284.8027	-126.0372	-199.2824
rel α^6	-0.1676	0.0838	-0.0125	-0.0050	0.0075	-0.0043	-0.0067
VP α^5	-1.0706	0.5353	-0.2802	-0.1121	0.1681	-0.1437	-0.2271
VP α^6	-0.0011	0.0005	-0.0014	-0.0006	0.0008	0.00005	0.0001
quad α^4	0	0	434.2329	-347.3863	86.8466	614.0980	-194.1948
quad VP α^5	0	0	0.3561	-0.2849	0.0713	0.1589	-0.0502
$\sum$	-1381.5710	690.7855	8596.5838	8235.4421	9371.88950	488.0717	-393.7611

potential [8,10]:

$$\Delta V^{hfs}_{1\gamma,VPVP}(r)=\frac{Z\alpha(1+\kappa_d)}{2m_1m_2r^3}\left(\frac{\alpha}{3\pi}\right)^2\int_1^\infty\rho(\xi)d\xi\int_1^\infty\rho(\eta)d\eta\frac{1}{\xi^2-\eta^2}\left[\left(1+\frac{m_1\kappa_d}{m_2(1+\kappa_d)}\right)(\boldsymbol{L}\cdot\boldsymbol{s}_2)[\xi^2(1+2m_e\xi r)\times\right.$$
$$\times e^{-2m_e\xi r}-\eta^2(1+2m_e\eta r)e^{-2m_e\eta r}]-(1+a_\mu)\Bigl(4m_e^2r^2[\xi^4e^{-2m_e\xi r}-\eta^4e^{-2m_e\eta r}][(\boldsymbol{s}_1\cdot\boldsymbol{s}_2)- \quad (12)$$
$$\left.-(\boldsymbol{s}_1\cdot\boldsymbol{n})(\boldsymbol{s}_2\cdot\boldsymbol{n})]+[\xi^2(1+2m_e\xi r)e^{-2m_e\xi r}-\eta^2(1+2m_e\eta r)e^{-2m_e\eta r}]\times[(\boldsymbol{s}_1\cdot\boldsymbol{s}_2)-3(\boldsymbol{s}_1\cdot\boldsymbol{n})(\boldsymbol{s}_2\cdot\boldsymbol{n})]\Bigr)\right].$$

For two-loop VP contribution with one nested loop in coordinate representation we get the following potential [8,10]:

$$\Delta V^{hfs}_{2-loop}(r)=\frac{Z\alpha(1+\kappa_d)}{2m_1m_2r^3}\frac{2}{3}\left(\frac{\alpha}{\pi}\right)^2\int_0^1\frac{f(v)dv}{1-v^2}e^{-\frac{2m_er}{\sqrt{1-v^2}}}\left[\left(1+\frac{m_1\kappa_d}{m_2(1+\kappa_d)}\right)\left[1+\frac{2m_er}{\sqrt{1-v^2}}\right](\boldsymbol{L}\cdot\boldsymbol{s}_2)-\right. \quad (13)$$
$$\left.-(1+a_\mu)\left(\frac{4m_e^2r^2}{1-v^2}[(\boldsymbol{s}_1\cdot\boldsymbol{s}_2)-(\boldsymbol{s}_1\cdot\boldsymbol{n})(\boldsymbol{s}_2\cdot\boldsymbol{n})]+\left(1+\frac{2m_er}{\sqrt{1-v^2}}\right)[(\boldsymbol{s}_1\cdot\boldsymbol{s}_2)-3(\boldsymbol{s}_1\cdot\boldsymbol{n})(\boldsymbol{s}_2\cdot\boldsymbol{n})]\right)\right].$$

After averaging (12) and (13) we obtain numerical values of corresponding corrections to the HFS that are included in Table 1. Muonic VP correction of order α^6 can de derived by means of simple replacement m_e to m_1 in (11). Main contribution of vacuum polarization to the HFS in second order PT (SOPT) has the following general form [8,10]:

$$\Delta E^{hfs}_{SOPT\ VP\ 1}=2<\psi|\Delta V^C_{VP}\cdot\tilde{G}\cdot\Delta V^{hfs}_B|\psi>, \quad (14)$$

where $\Delta V^C_{VP}(r)$ is the Coulomb potential that was modified by the vacuum polarization. The Coulomb Green's function with two non-zero arguments for 2P-state was obtained in [11] in the form:

$$G_{2P}(\boldsymbol{r},\boldsymbol{r}')=-\frac{\mu^2(Z\alpha)}{36z^2z'^2}\left(\frac{3}{4\pi}\boldsymbol{n}\boldsymbol{n}'\right)e^{-(z+z')/2}g(z,z'), \quad (15)$$

$$g(z,z')=24z^3_<+36z^3_<z_>+36z^3_<z^2_>+24z^3_>+36z_<z^3_>+36z^2_<z^3_>+49z^3_<z^3_>-3z^4_<z^3_>-$$
$$-12e^{z_<}(2+z_<+z^2_<)z^3_>-3z^3_<z^4_>+12z^3_<z^3_>[-2C+Ei(z_<)-lnz_<-lnz_>],$$

where $C = 0.5772...$ is the Euler constant, $z = Wr$, $z_< = min(z,z')$, $z_> = max(z,z')$. After substitution of (1) and (15) into (14) we get analytical expression for the correction:

$$E^{hfs,VP}_{SOPT} = \frac{\alpha^4\mu^3(1+\kappa_d)}{24m_1m_2}\frac{\alpha}{54\pi}\int_1^\infty \rho(\xi)d\xi\int_0^\infty dx\int_0^\infty \frac{e^{-x'}}{x'^2}dx' e^{-x(1+\frac{2m_e\xi}{W})}\left[\overline{T_1} + \frac{m_1\kappa_d}{m_2(1+\kappa_d)}\overline{T_1} - (1+a_\mu)\overline{T_2}\right]. \quad (16)$$

The integration is performed analytically over x, x' and numerically over ξ. For two-loop contributions in second order PT we use the potential (10) and the modifications of the Coulomb potential [8,10]:

$$\Delta V^C_{VP-VP}(r) = \left(\frac{\alpha}{3\pi}\right)\int_1^\infty \rho(\xi)d\xi\int_1^\infty \rho(\eta)d\eta\left(-\frac{Z\alpha}{r}\right)\frac{1}{\xi^2-\eta^2}(\xi^2 e^{-2m_e\xi r} - \eta^2 e^{-2m_e\eta r}), \quad (17)$$

$$\Delta V^C_{2-loopVP}(r) = -\frac{2Z\alpha^3}{3\pi^2 r}\int_0^1 \frac{f(v)dv}{(1-v^2)}e^{-\frac{2m_e r}{\sqrt{1-v^2}}}. \quad (18)$$

The contribution of the VP of order α^6 to the HFS of P-states in muonic deuterium also exists in third order PT. This correction has the following general structure:

$$\Delta E^{hfs}_{TOPT} = \left\langle \psi_n \left| \Delta V^C_{VP}\cdot\tilde G\cdot V^{hfs}\cdot\tilde G\cdot\Delta V^C_{VP}\right|\psi_n\right\rangle + 2\left\langle \psi_n \left|\Delta V^C_{VP}\cdot\tilde G\cdot V^C_{VP}\cdot\tilde G\cdot\Delta V^{hfs}\right|\psi_n\right\rangle - \quad (19)$$

$$-\left\langle\psi_n\left|\Delta V^{hfs}\right|\psi_n\right\rangle\left\langle\psi_n\left|\Delta V^C_{VP}\cdot\tilde G\cdot\tilde G\cdot\Delta V^C_{VP}\right|\psi_n\right\rangle - 2\left\langle\psi_n\left|\Delta V^C_{VP}\right|\psi_n\right\rangle\left\langle\psi_n\left|\Delta V^C_{VP}\cdot\tilde G\cdot\tilde G\cdot\Delta V^{hfs}\right|\psi_n\right\rangle.$$

Numerous matrix elements are calculated by means of (15) similar to previous contributions.

The deuteron has a non-zero quadrupole moment which leads to additional quadrupole interaction correction to hyperfine structure of P-states. The quadrupole interaction can be written in the following form [12]:

$$H^{quad}_{\mu d} = \sum_q (-1)^q T^2_q(d)\cdot T^2_{-q}(\mu), \quad (20)$$

where $T^2(d)$, $T^2(\mu)$ are irreducible tensor operators of rank 2 that describe quadrupole moment of nucleus and muon respectively. The nucleus irreducible tensor operator has the form:

$$T^2_q(d) = \sqrt{\frac{4\pi}{5}}\int \rho(r)r^2 Y^2_q(\theta,\phi)d^3r \quad (21)$$

The quadrupole moment of the nucleus is equal to

$$Q = 2\left\langle II\left|T^2_0(d)\right|II\right\rangle = 2\langle I\parallel T(d)\parallel I\rangle\begin{pmatrix} I & 2 & I\\ -I & 0 & I\end{pmatrix}. \quad (22)$$

The muon irreducible tensor operator is determined by the expression

$$q_{ij} = \frac{\partial^2 V}{\partial x_i\partial x_j},\ T^2_0(\mu) = q_{zz},\ T^2_1(\mu) = -q_{xz} - iq_{yz},\ T^2_2(\mu) = \frac{1}{2}(q_{xx}-q_{yy}) + q_{xy}, \quad (23)$$

$$T^2_{-m}(\mu) = (-1)^m T^2_m(\mu)^*,\ T^2_q(\mu) = -\frac{e}{r^3}Y^2_q(\theta,\phi).$$

To obtain the quadrupole interaction contribution we have to calculate diagonal and off-diagonal matrix elements of (20). In general form for both diagonal and off-diagonal matrix elements of (20) we obtain the following expression [12]:

$$\left\langle j'IF \left| (T^2(d)\cdot T^2(\mu)) \right| jIF \right\rangle = (-1)^{I+J'-F} W(jIj'I;F2) \langle j' \parallel T(\mu) \parallel j \rangle \langle I \parallel T(d) \parallel I \rangle, \tag{24}$$

$$\langle I \parallel T(d) \parallel I \rangle = \frac{Q}{2}\left[\begin{pmatrix} I & 2 & I \\ -I & 0 & I \end{pmatrix}\right]^{-1}, \tag{25}$$

$$\langle j' \parallel T(\mu) \parallel j \rangle = -\sqrt{2J+1}\sqrt{2J'+1}(-1)^{J'+1/2}\begin{pmatrix} J' & 2 & J \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix} < \frac{\alpha}{r^3} >, \tag{26}$$

Using (20), (25) and (26) we finally get:

$$\left\langle j'IF \left| H^{quad}_{\mu d} \right| jIF \right\rangle = (-1)^{J'+1/2-F-J}\begin{Bmatrix} J & I & F \\ I & J' & 2 \end{Bmatrix}\frac{Q}{2}\left[\begin{pmatrix} I & 2 & I \\ -I & 0 & I \end{pmatrix}\right]^{-1}\times \tag{27}$$

$$\times\sqrt{2J+1}\sqrt{2J'+1}\begin{pmatrix} J' & 2 & J \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix} < \frac{\alpha}{r^3} > .$$

For diagonal matrix elements we get following analytical expressions:

$$E^{quad}_{\mu d}(j=1/2) = 0,\ E^{quad}_{\mu d}(j=3/2) = \frac{\alpha^4\mu^3 Q}{48}(\delta_{F,1/2} - 4/5\delta_{F,3/2} + 1/5\delta_{F,5/2}), \tag{28}$$

where the value of deuteron quadrupole moment is equal to $Q = 0.285783(30)\ fm^2$ [13]. Off-diagonal matrix elements have the form:

$$E^{quad}_{\mu d}(j=3/2, j'=1/2) = \frac{\alpha^4\mu^3 Q}{48}(\sqrt{2}\delta_{F,1/2} - 1/\sqrt{5}\delta_{F,3/2}). \tag{29}$$

For the calculation of vacuum polarization correction to quadrupole contribution we use the modification of the muon quadrupole moment tensor:

$$q^{VP}_{ij} = \int_1^\infty d\xi\rho(\xi)\left\{\frac{(1+2m_e\xi r)e^{-2m_e\xi r}(3n_in_j - \delta_{ij})}{r^3} + \frac{4m_e^2\xi^2 e^{-2m_e\xi r}(3n_in_j-\delta_{ij})}{3r} + \frac{4m_e^2\xi^2e^{-2m_e\xi r}}{3r}\delta_{ij}\right\}. \tag{30}$$

The reduced matrix element in (24) can be written as:

$$\langle j' \parallel T(\mu) \parallel j \rangle = (-1)^{J-2-S+L}\sqrt{(2L+1)(2J'+1)(2J+1)}W(llj j';2s)\langle l \parallel T(\mu) \parallel l \rangle,\ \langle l \parallel T(\mu) \parallel l \rangle = \tag{31}$$

$$= \sqrt{2l+1}(C^{l2l}_{0q0})^{-1}\left\langle lm \left| T(\mu)^2_q \right| lm \right\rangle,\ \left\langle lm \left| T(\mu)^2_q \right| lm \right\rangle = \int_0^\infty r^2dr\int d\Omega\Psi^*_{2p}(r)Y^*_{lm}T(\mu)^2_q\Psi(r)_{2p}Y_{lm}R(r).$$

Averaging of third term in (30) which is proportional to δ_{ij} gives zero. Thus we have to calculate the sum of two integrals over r and Ω:

$$\langle l \parallel T(\mu) \parallel l \rangle = \sqrt{2l+1}(C^{l2l}_{000})^{-1}\left[R_1(r)\int Y^*_{lm}\frac{1}{2}(3cos^2\theta - 1)Y_{lm}d\Omega + R_2(r)\int Y^*_{lm}\frac{1}{2}(cos^2\theta - \frac{1}{3})Y_{lm}d\Omega\right], \tag{32}$$

$$R_1 = \int_0^\infty r^2dr\int_1^\infty d\xi\rho(\xi)\frac{(1+2m_e\xi r)e^{-2m_e\xi r}}{r^3}|\Psi(r)_{2p}|^2,\ R_2 = \int_0^\infty r^2dr\int_1^\infty d\xi\rho(\xi)\frac{4m_e^2\xi^2e^{-2m_e\xi r}}{r}|\Psi(r)_{2p}|^2$$

Integration in (32) can be performed analytically. Numerical results are presented in Table 1. To evaluate quadrupole correction in second order PT we use the same approach as for VP correction. We use (14) and the following potentials:

$$V^C_{VP}(r) = \frac{\alpha}{3\pi}\int_1^\infty \rho(\xi)d\xi\left(-\frac{Z\alpha}{r}\right)e^{-2m_e\xi r},\ V^{hfs}_Q(r) = \frac{\alpha Q}{2r^3}\times\Big[(\boldsymbol{s}_2\cdot\boldsymbol{s}_2) - 3(\boldsymbol{s}_2\cdot\boldsymbol{n})(\boldsymbol{s}_2\cdot\boldsymbol{n})\Big]. \quad (33)$$

The quadrupole corrections to diagonal and off-diagonal matrix elements in second order PT (numerical coefficient is in μeV) are equal respectively:

$$E^{hfs}_{SOPT\ VP\ Q}(j=3/2) = 0.112326(\delta_{F,1/2} - 4/5\delta_{F,3/2} + 1/5\delta_{F,5/2}), \quad (34)$$

$$E^{hfs}_{SOPT\ VP\ Q}(j=3/2, j'=1/2) = 0.112326(\sqrt{2}\delta_{F,1/2} - 1/\sqrt{5}\delta_{F,3/2}). \quad (35)$$

We present corrections in integral form and evaluate them numerically. After diagonalization of the results from the matrix in Table 1 we obtain final values of 2P-state hyperfine structure in muonic deuterium: $E^{F=1/2}_{1/2} = -1405.3877\ \mu eV$, $E^{F=3/2}_{1/2} = 670.2905\ \mu eV$, $E^{F=1/2}_{3/2} = 8620.4005\ \mu eV$, $E^{F=3/2}_{3/2} = 8255.9371\ \mu eV$, $E^{F=5/2}_{3/2} = 9371.8895\ \mu eV$. The detailed calculation of 2P-state HFS in muonic deuterium was performed in [14], where first order PT vacuum polarization corrections were included. Vacuum polarization corrections in [14] were evaluated approximately thus they differ from our values (11) by $\sim 30\%$. Other differences are connected with second order PT α^5 and α^6 corrections. Obtained results can be used for improved estimates of transition frequencies between 2P and 2S states regarding to CREMA experiments.

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