Multi-Higgs models. Perspectives for identification of wide set of models in future experiments in the SM-like scenario

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Higgs mechanism of EWSB can be realized with both the well known minimal model and more complex non-minimal Higgs models.
These non-minimal models contain new Higgs bosons – neutral h_a and charged $H_b^\pm$. Necessary step in the discovery of such model is observation of these additional Higgses. We discuss the potential of such researches at modern and future colliders in the light of recent LHC results, for wide set of models, including 2HDM as a simplest example.

Our conclusion is rather pessimistic. The discovery of new neutral Higgs bosons at LHC is in general a very difficult task. (Nevertheless, some favorable values of parameters of the theory can exist, allowing such observation.) We propose the regular way for the discovery of models, which consists in the study of processes with production of charged Higgs bosons (better, at Linear Collider).

1 Introduction

This report continues the series of papers [1]- [5].

We base on the following interpretation of modern data [6].
1. Higgs boson h with mass 125 GeV is discovered at LHC.
2. Its properties are close to those in minimal Standard Model (SM) [7], i.e. *SM-like scenario* (sect. 1.3) is realized.

This situation does not exclude an extended Higgs sector (Beyond Standard Model – BSM). Higgs mechanism of EWSB can be realized with both well known minimal model and more complex non-minimal Higgs sector, containing new neutral Higgs bosons h_a and charged Higgs bosons $H_b^\pm$.

Here we consider from common point of view a large group of BSM models with more or less standard description of Higgs phenomenon but with richer set of fundamental scalar fields. Those are models with n fundamental weak isodoublets, p_2 complex weak isosinglets S_c and p_1 real weak isosinglets S_r: $nHDM + p_2(HS_cM) + p_1(HS_rM)$. The models of this group are under wide discussion now.

Examples:

$1HDM$ – model with single Higgs doublet don't allow CP violation and FCNC. This model is commonly referred to as SM (Standard Model).

$2HDM$ with huge literature – see e.g. [8], [9]. At some values of parameters, 2HDM can explain CP violation, FCNC, etc. At other values of parameters, 2HDM gives Dark Matter (Inert doublet Model) (without CP violation in Higgs sector and FCNC) (see e.g. [10]). One more set of parameters realizes Higgs sector of MSSM.

$2HDM + 1(HS_cM)$ describes Higgs sector of nMSSM (see also [12]).

$3HDM$ at suitable set of parameters describes models with Dark Matter and possible CP violation and FCNC [13,14].

$nHDM$ at $n \geqslant 3$, in particular $n = 6$ [15], can describe Dark asymmetric Matter (see e.g. [16]).

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These models give $n-1$ pairs of charged Higgs bosons $H_b^{\pm}$ with masses $M_b^{\pm}$ and $2n-1+2p_2+p_1$ neutral scalar Higgs particles h_a with masses M_a, having either definite or indefinite CP parity (in the latter case we have CP violation). Variants with suitable Yukawa sector allow to have flavour changing neutral currents (FCNC), etc.

♢ We don't discuss models with alternative explanations of situation or (and) with additional mechanisms, supplementing or changing the standard Higgs mechanism – little Higgs, orbifold, radion, models with Higgs triplets,... We have not found a general description for this group of models.

- To identify the model, it is necessary to measure many parameters. They can be subdivided for three group. The first group consists from masses M_a, $M_b^{\pm}$ of all particles h_a, $H_b^{\pm}$. The second group contains couplings of neutral Higgses with gauge bosons, χ_a^V (see (1)). The third group contains Higgs self-couplings (for 2HDM minimal set of such observables can consist from 3 triple couplings $H^+H^-h_a$ and one quartic coupling $H^+H^-H^+H^-$ [4]).

Main subject of this report is the observation of additional neutral Higgses h_a. Such discovery is a challenge for the next stage of LHC and e^+e^- LC. A number of papers, devoted this task, study different non-minimal models with various benchmark parameters of new particles and (or) parameters of Lagrangian [17]. They usually find that many "natural" approaches in these problems turn out either non-realistic or very difficult (for example, demand extremely high luminosity integral).

In this report we show that such conclusion is not caused by an unfortunate choice variant of non-minimal model or its parameters but is general feature for all models of the considered class with almost arbitrary parameters, compatible with modern data (sect. 1.3). To come to this conclusion we use sets of Sum Rules (sect. 2) which form either extensions of known in 2HDM Sum Rules for more wide class of models with arbitrary set of parameters or new Sum Rules founded recently.

1.1 Relative couplings

We use the relative couplings for each neutral Higgs boson h_a:

$$\chi_a^P = \frac{g_a^P}{g_{\text{SM}}^P}, \quad \chi_a^{\pm b} = \frac{g(H_b^+ H_b^- h_a)}{2M_{b\pm}^2/v}, \qquad \chi_a^{H_b^+ W^-} = \frac{g(H_b^+ W^- h_a)}{M_W/v}. \tag{1}$$

The quantities χ_a^P are the ratios of the couplings of h_a with the fundamental particles $P = V\,(W,Z)$, $q = t, b, ...,$ $\ell = \tau, ...$ to the corresponding couplings for the would be SM Higgs boson with $M_h = M_a$. The other relative couplings describe interaction of h_a with charged Higgs boson $H_b^{\pm}$. The quantity $\chi_a^{\pm b}$ describes interaction $H_b^+ H_b^- h_a$, the quantity $\chi_a^{H_b^+ W^-}$ describes off-diagonal interaction $H_b^{\pm} W^{\mp} h_a$. (Below we omit the adjective "relative" and subscript b for the models with single charged Higgs boson $H^{\pm}$.)

The neutrals h_a generally have no definite CP parity. Couplings χ_a^V and $\chi_a^{\pm b}$ are real due to Hermiticity of Lagrangian, while other couplings are generally complex. The $Re(\chi_a^f)$ and $Im(\chi_a^f)$ are responsible for the interaction of fermion f with CP-even and CP-odd components of h_a respectively.

1.2 Conditions for CP-conservation

In the CP conserving case some of h_a are scalars, others are pseudoscalars. In this case we have

$$(a)\ \boxed{\prod_a \chi_a^V = 0}, \quad (b)\ \boxed{\prod_a \chi_a^{\pm b} = 0}, \quad (c)\ \boxed{\left|\prod_a \chi_a^f\right| = \prod_a \left|\chi_a^f\right| \quad \text{for each fermion } f}. \tag{2}$$

(In the 2HDM with CP conservation we have $h_3 = A$ (pseudoscalar) and $\chi_3^V = 0$, $\chi_3^{\pm} = 0$, $Im(\chi_{2,1}^f) = 0$, $Re(\chi_3^f) = 0$. In this model the relationship (2c) follows from the (2a).)

1.3 Modern status. SM-like scenario

The intensive study of recently discovered Higgs boson makes very probable that the *SM-like scenario* [18], (or *SM alignment limit [19]*) is realized in the nature:

1) Single observed Higgs boson h has mass $M_h \approx 125$ GeV, we denote it as h_1.
2) Its couplings to fundamental particles P (gauge bosons V and fermions f) are close to the SM expectations within experimental accuracy (see e.g. [7]):

$$\varepsilon_P = \left|1 - |\chi_1^P|^2\right| \ll 1 \qquad (P = V(W, Z), \quad f = (t, b, \tau, ...))\,. \tag{3}$$

This statement remains only a plausible hypothesis before we are able to measure the coupling with a sufficient accuracy.

The existence of SM-like scenario don't close the doors for realization of non-minimal Higgs models. No doubts that the SM-like scenario in the non-minimal model can occur if additional Higgs bosons are very heavy and are coupled only weakly with usual matter (decoupling limit) (see e.g. [8]). 15 years ago it was found that, at finite inaccuracy of future experiments (at both the LHC and the planned high energy e^+e^- collider), even the simplest non-minimal model 2HDM with the special choice of the Yukawa interaction 2HDM-II allows several possible windows significantly differing from the decoupling limit and admitting the SM-like scenario [18]. Naturally, such windows exist in other models as well.

The future experiments reduce ε_P and, consequently, the region of the allowed parameters of each non-minimal model.

2 Sum Rules

The freedom in parameters of discussed models is limited by Sum Rules (SR), which are going to be the key point in our discussion.

- **The coupling of the EW gauge boson V to each neutral Higgs scalar h_a (χ_a^V)** is real due to Hermiticity of Lagrangian. In models like nHDM (with $p_i = 0$) $\chi_a^Z = \chi_a^W \equiv \chi_a^V$.

These χ_a^V coincide with elements of rotation matrix, describing transition from neutral components of Higgs fields Φ_j to the physical neutral Higgs bosons h_a in the Higgs basis. The unitarity of this transformation matrix results in the Sum Rules, which are valid for all discussed models both with and without CP violation:

$$\sum_a (\chi_a^W)^2 = 1\,, \;\; \sum_a (\chi_a^Z)^2 = 1 : \qquad \boxed{\text{all Higgs models with doublets and singlets}}\,. \tag{4}$$

This argumentation spreads the approach of [4] developed for the most general 2HDM. (For particular case of 2HDM such SR's were obtained in [20], [1]). One can say that these SR's mean that masses of gauge bosons are given by the single v.e.v. v.

- **The couplings χ_a^f** of each definite fermion f (quark or lepton) to all neutral Higgs scalars h_a are generally complex. The Sum Rules for these couplings naturally appear in models $nHDM + p_2HSn_2M + p_1HSn_1M$ at arbitrary n and p_i, where weak isosinglets are not coupled with fermions. To prove this, we start with Sum Rule proved for 2HDM with definite Yukawa interaction (Model I or Model II) in [20], [18], [1]. Let us write general Yukawa term for interaction of down-type fermion f to neutral components[1] $\phi_{j,0}$ of ϕ_j as $\Delta L_Y = \sum_j g_{jf}\bar{\psi}^\dagger\phi_{j,0}\psi_f$. Simple reparameterization $\phi'_{1,0} = N\sum_j g_{jf}\phi_{j,0}$ (where N – the normalization factor) transforms this term to the form $\Delta L_Y = g'_{1f}\bar{\psi}^\dagger\phi'_{1,0}\psi_f$, which coincides with that for Model I (or II) in 2HDM (we call that *f-selective reparameterization basis* [4]). For the latter case Sum Rules have been proved in [20, 21] in the form[2]

$$\sum_a (\chi_a^f)^2 = 1 : \quad \boxed{\text{models without Yukawa interaction with isoscalars } S_i}\,. \tag{5}$$

Our argumentation shows that these SR's are valid for much more general class of models than those discussed in [20, 21].

- **The non-diagonal couplings to EW gauge bosons $H^\pm W^\mp h_a$** are generally complex. The Higgs potential is naturally invariant under *rephasing transformation* $\phi_i \to \phi'_i e^{i\alpha_i}$, compensated by the corresponding phase rotation of some coefficients. This rephasing freedom results in the phase freedom of couplings

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[1] Similar argumentation is valid for up-type fermion with the change $\phi_{j,0} \to \phi^*_{j,0}$.

[2] Another proof of these SR's is similar to that developed in [4] for 2HDM. Couplings χ_a^f can be expressed via couplings χ_a^V, $\chi_a^{H^+W^-}$ and parameters of transformation of Higgs basis to the f-selective basis. The orthogonality of this transformation leads to SR's (5).

$\chi_a^{H^\pm W^\mp} \to \chi_a^{H^\pm W^\mp} e^{i\beta}$, keeping phase differences between $\chi_a^{H^\pm W^\mp}$ for different a. The SR's for these quantities were obtained firstly in [4] for the most general 2HDM. The method of derivation of these SR's allows to extend result for all models with single charged Higgs boson and arbitrary number of Higgs singlets, $2HDM + p_2(HS_cM) + p_1(HS_rM)$,

$$|\chi_a^V|^2 + |\chi_a^{H^\pm W^\mp}|^2 = 1 : \boxed{2HDM + isoscalars, \text{ with arbitrary Yukawa sector}}\,. \tag{6}$$

◊ **Some relations for different Yukawa sectors in 2HDM**. The Yukawa couplings for different fermions are generally independent on each other. In some widely discussed models of Yukawa sector these couplings are correlated.

For the 2HDM-I, the f-selective bases coincide for all fermions, $\beta_t = \beta_b$, $\xi_t = \xi_b$, and

$$\chi_a^u = \chi_a^d = \chi_a^\ell\,. \tag{7a}$$

For the 2HDM-II, the u-selective bases coincide for all up-quarks, and the d-selective bases coincide for all down quarks, $\beta_b = \pi/2 - \beta_t \equiv \beta$, $\xi_b = \xi$, $\xi_t = 0$. In this case the pattern relations among Yukawa couplings for different fermions take place [1]

$$(\chi_a^u + \chi_a^d)\chi_a^V = 1 + \chi_a^u\chi_a^d\,. \tag{7b}$$

3 Consequences from SR's in the SM-like scenario

3.1 Scalar sector

1. Because of (4), couplings of neutrals h_a to gauge bosons χ_a^V are small (these Higgses are *gaugefobic*),

$$|\chi_a^V|^2 < \varepsilon_V \ll 1\,. \tag{8}$$

2. Because of (6), (4), the absolute values of non-diagonal couplings to EW gauge bosons $\chi_a^{W^\pm H^\mp}$ are close to their maximal values while similar coupling for the observed Higgs $\chi_1^{W^\pm H^\mp}$ is small:

$$a)\ |\chi_a^{W^\pm H^\mp}|^2 \approx 1\,; \quad b)\ |\chi_1^{W^\pm H^\mp}|^2 \sim \varepsilon_V \ll 1\,. \tag{9}$$

(The calculations of $H^- \to W^- h_1$ decay at LHC in [22] are made in the specific case of CP-conserving 2HDM and, in fact, in the case when strong SM-like scenario is violated.)

3.2 Yukawa interactions

The SR's for couplings to the given fermion f (5) can be written as $\sum\limits_{a\geqslant 2}(\chi_a^f)^2 \approx 0$. We will write here about the most important case $f = t$. Since couplings χ_a^t are generally complex, this SR can be saturated by different ways, we discuss the simplest limiting cases

$$\begin{array}{lc} a) & |\chi_a^t| < 1 \quad \text{for all} \quad h_a\,, \end{array} \tag{10a}$$

$$\begin{array}{lc} b) & (bI)\quad |\chi_a^t| \approx |\chi_a^b| \gg 1; \qquad (bII)\quad |\chi_a^t| \approx |1/\chi_a^b| \gg 1\,, \end{array} \tag{10b}$$

$$\begin{array}{lc} c) & |\chi_{a_2}^t| \approx |\chi_{a_1}^t| > 1\,, \quad \chi_{a_2}^t \approx i\chi_{a_1}^t \quad \text{for some } h_{a_1} \text{ and } h_{a_2}\,. \end{array} \tag{10c}$$

The case (10a) provides no new interesting opportunities in the discovery of h_a.

In the opposite case, if some couplings χ_a^t are large in their absolute value, new interesting opportunities appear[3], depending on variant of organization of Yukawa sector.

The eq. (10b) describes two limiting options in the organization of Yukawa sector. The case (bI) corresponds to the Yukawa sector similar to that in 2HDM-I. The case (bII) corresponds to the Yukawa sector similar to that in 2HDM-II.

One particular opportunity in saturation of (5) is described by eq. (10c). In this case the absolute values of couplings $\chi_{a_i}^t$ are large only for two neutrals h_a. For one of these neutrals $Re(\chi_{a_i}^t) > Im(\chi_{a_i}^t)$, for another neutral the imaginary part dominates. (This variant can be realized in CP conserving 2HDM with $h_{a_1} = H$, $h_{a_2} = A$.)

The opportunities (10b) and (10c) can coexist or not coexist.

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[3] The standard perturbative estimates, used here and in other papers, become invalid if this coupling is enormously large, at $|\chi_{a_1}^t| > 2\pi$ we come into the region of the strong interaction in Higgs sector, transferred by t-quarks.

4 Properties and production of neutral Higgses h_a

In all considered models the masses $M_{a>1}$ and $M_b^\pm$ as well as some couplings of neutrals to vector bosons can be treated as independent free parameters, limited only by SR's written above. (The theoretical limitations for the masses can result from some additional hypotheses, implemented in model.) Some triple and quartic couplings are also independent parameters of theory (see 2HDM [4] where this complete set contains masses of Higgs bosons M_a, $M_\pm$, two of three couplings χ_a^V, and couplings $g(H^+H^-h_a)$, $g(H^+H^-H^+H^-)$). The Yukawa couplings form additional set of input parameters.

For definiteness, we assume[4] $M_a > 150$ GeV and $|\chi_a^f| < 40$ for $f \neq t$. To make some statements more transparent, we will compare discussed quantities with those for the would-be SM Higgs boson having the same mass $H_{SM}^{(wb)}(M_a)$, for example total width $\Gamma_{SM}^{(wb)}(M_a)$ and some cross sections, like $\sigma_{SM}^{(wb)}(gg \to h|M_a)$.

Some conclusions below about total width and observability can be changed by effects of moderately strong interaction in Higgs sector with large triple Higgs vertices like $h_a h_1 h_1$, etc. They should be considered separately. Here we neglect this opportunity.

4.1 Effects from coupling h_a to gauge bosons

For the would be Higgs boson with mass $M_a > 150$ GeV the main contribution to the width comes from the decays $h \to W^+W^-$ and $h \to ZZ$. These decays and processes like W fusion provide the main signal for detection of the Higgs boson. The production of this $H_{SM}^{(wb)}(M_a)$ through a gauge vertex provides the best signal/background ratio and the least inaccuracy in the measurement of its parameters both at the LHC and at the LC. We present, for example, some properties of the would-be SM Higgs boson with mass $M = 300$ GeV.

$$\Gamma_{SM,tot}^{(wb)} = 8.4\, GeV, \quad BR_{SM}^{(wb)}(h \to b\bar{b}) \approx 0.0008, \quad \Gamma_{SM}^{(wb)}(h \to gg) \approx 3.4\, MeV. \tag{11}$$

For the actual Higgs bosons with mass $M_a > 150$ **GeV**, according to eq. (8),
(i) $\Gamma_a \ll \Gamma_{SM}^{(wb)}(M_a)$.
(ii) The decay $h_a \to W^+W^-/ZZ$ is suppressed, observation of h_a via this decay is hardly probable.
(iii) The search for new neutral Higgs bosons in the W fusion at the LHC, $e^+e^- \to Zh_a$ and $e^+e^- \to \nu\bar{\nu}h_a$ at the ILC, and $e\gamma \to \nu W^- h_a$ at the PLC (photon collider) cannot be successful [2], their cross sections are roughly by one order of value lower then those calculated for $H_{SM}^{(wb)}(M_a)$.

4.2 If $h_a\bar{t}t$ interaction is enhanced

The decreasing of partial widths and production cross sections, caused by small coupling to gauge bosons can in principle be compensated by the interaction $h_a\bar{t}t$ (cases (10b), (10c)) mainly via the increasing of the cross section of gluon fusion.

The cross section of gluon fusion $\sigma(gg \to h_a)$ is saturated by contribution of t-quark loop (just as two gluon width of Higgs boson Γ_a^{gg}). Therefore, this cross section is enhanced comparing with the would-be SM case by a factor $|\chi_a^t|^2$. In the one-loop approximation

$$\sigma(gg \to h_a) = \sigma_{SM}^{(wb)}(gg \to h|M_a)\left[|Re(\chi_a^t)|^2 + |Im(\chi_a^t)|^2\Phi^{(O/E)}(4M_t^2/M_a^2)\right]. \tag{12}$$

Here $\Phi^{(O/E)}(r)$ is the ratio of two well known loop integrals, defined for CP-odd and CP-even Higgs bosons respectively (see e.g. [23]), at $M_a = 300$ GeV we have $\Phi^{(O/E)}(r) \approx 2.7$.

The case $M_a < 350 \div 400$ GeV.

In this case Higgs boson h_a is very narrow. However it can be obtained in some cases due to enhancement of production cross section.

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[4] The opportunity that some neutrals are lighter than 125 GeV cannot be excluded, but this opportunity is strongly constrained by modern data.

∇ If Yukawa sector is similar to Model I (variant (bI)), we have $\chi_a^b \approx \chi_a^t$. For the $H_{SM}^{(wb)}(M_a)$ we have the $BR_{SM}^{(wb)}(h \to b\bar{b}|M_a > 200\ GeV) < 0.003$, therefore the $b\bar{b}$ channel for the hunting for such Higgs is practically closed. Oppositely, at $|\chi_a^b| \approx |\chi_a^t| \gg 1$ one can hope to observe h_a as the narrow peak in the production of $\bar{b}b$ pairs at LHC.

Benchmark example for CP-even h_2 with $M_2 = 300$ GeV – compare (11).
Let $\boxed{|\chi_2^t| = |\chi_2^b| = 6,\ |\chi_2^V| = 0.2}$. In this case
$\Gamma(h_2 \to b\bar{b}) = |\chi_2^b|^2 \cdot 7\text{ MeV} \approx 250\text{ MeV}$,
$\Gamma(h_2 \to W^+W^-(ZZ)) = |\chi_2^V|^2 \cdot 8.4\text{ GeV} \approx 340\text{ MeV}$,
$\Gamma(h_2 \to gg) = |\chi_2^t|^2 \cdot 3.4\text{ MeV} \approx 120\text{ MeV}$.
It gives $\Gamma_2 \approx 0.7$ GeV with $BR(h_2 \to b\bar{b}) \approx 0.36$.
The cross section $\sigma(gg \to h_a \to \bar{b}b) \approx |\chi_a^t|^2 \sigma_{SM}^{(wb)}(gg \to h|M_a) BR(h_2 \to b\bar{b})$.
The account of CP odd admixture in h_2 increases result – see (12).

∇ **If $M_2 > 250$ GeV and $h_2h_1h_1$ coupling is high enough**, the process $gg \to h_2 \to h_1h_1$ can be seen as resonant production of h_1h_1 pair. It allow in principle to discover h_2 at LHC (see examples in [24], [25], [26]). Similar opportunity is absent at e^+e^- colliders.

If both neutrals mentioned in (10c) are not very heavy, $M_{a_1}, M_{a_2} < 350$ GeV, one can hope to observe two narrow peaks in $b\bar{b}$ production, well separated from each other in general.

∇ If Yukawa sector is similar to Model II (variant (bII)), the eq. (7b) results in $\chi_a^b \approx 1/\chi_a^t$. In this case the gg partial width can be even larger than the $\bar{b}b$ one. The cross section $\sigma(gg \to h_a \to \bar{b}b) \approx \sigma_{SM}^{(wb)}(gg \to h_a \to \bar{b}b)$, and it is difficult to hope for observation of signal of this process in comparison with background signal $gg \to \bar{b}b$.

The case $M_a > 350$ GeV.

In this case the contribution of $h_a \to t\bar{t}$ decay is enlarged so that one can hope to see h_a in $t\bar{t}$ mode (see interesting analysis in [27] for particular case of 2HDM-II).

If both neutrals, mentioned in (10c) are heavy, $M_{a_1} > 350$ GeV, $M_{a_2} > 350$ GeV, one can hope to observe either two separated enhancements in $t\bar{t}$ production or even one enhancement (at $|M_{a_1} - M_{a_2}| \leqslant \Gamma_{a_1} + \Gamma_{a_2}$).

• **Two photon width**. The widths $h_a \to \gamma\gamma$, $h_a \to Z\gamma$ are described by loop integrals with W-loop (contribution is $\propto \chi_a^V$), t-loop (contribution is $\propto \chi_a^t$), and H^+ loop (contribution is $\propto \chi_a^{\pm}$). The knowledge of all masses and couplings χ_a^V don't limit values of $\Gamma(h_a \to \gamma\gamma)$ even in 2HDM [4].

As it was found for the simple SM-like scenario ($\chi_a^t \approx 1$) for 2HDM in [18], at $\chi_1^{\pm} \approx 1$ the contribution of the charged Higgs loop into $\Gamma(h_1 \to \gamma\gamma)$ reduces the mentioned width by about 10% , that is within modern inaccuracy of data. One can also realized SM-like scenario with $\chi_a^t \approx -1$. In this case two photon width can be enhanced by factor $2 \div 2.5$ vs SM value [18], which contradicts to modern data. However variation of $\chi_1^{\pm}$ can reduce this enhancement (see modern studies in [28]).

• Many results obtained in recent studies can be treated as examples of discussed general picture for separate sets of parameters and particular models. Our discussion shows that almost negative results of many such studies have the common origin.

Some authors, estimating opportunities of future experiments, in fact either don't take into account realization SM-like scenario or assume only its weak version (see e.g. [29]). Some of their results can be treated as too optimistic if it appears that the strong SM-like scenario is realized indeed.

5 Using of charged Higgses, etc.

Here we limit ourself by the group of models with single charged Higgs boson $H^{\pm}$ (models $2HDM + p_2(HS_cM) + p_1(HS_rM)$). The discovery of this charged Higgs boson and the study of its decays should be discussed separately. Below we assume that the mass $M^{\pm}$ is not extremely large and these particles have good signature.

In the SM-like scenario (3) the Sum Rules (6) shows that the coupling $H^{\pm}W^{\mp}h_1$ is weak while the couplings $H^{\pm}W^{\mp}h_a$ with $a \geqslant 2$ are close to possible maximal value (9). It gives following results:

• The partial width $\Gamma(H^+ \to W^+ h_1)$ is small, while at $M^\pm > M_W + M_a$ the partial width $\Gamma(H^+ \to W^+ h_a)$ is relatively large ($a \geqslant 2$) – see for example [30].

• The production of Higgs boson h_1 in association with $H^\pm W^\mp$ or in decay $H^+ \to W^+ h_1$ is hardly observable.

• The search for Higgs bosons h_a can be successful in the following channels:

$q_1 \bar{q}_2 \to H^+ h_a$, $q\bar{q} \to W^\mp H^\pm h_a$ at LHC,

$e\gamma \to \nu H^- h_a$, $e^+e^- \to H^\pm W^\mp h_a$ at e^+e^- collider,

$\gamma\gamma \to H^\pm W^\mp h_a$ at PLC (Photon Collider).

Some calculations of this type for special variant of 2HDM can be found in [31]. Certainly, e^+e^- collider and PLC have advantages due to much better background conditions.

6 Summary

The big class of Higgs models with arbitrary Yukawa sector obeys simple Sum Rules (4)-(6), helpful for analysis of future experiments with searching for phenomena beyond SM.

Assuming that the SM-like scenario is realized, for wide class of Higgs models many now discussed ways to observe additional neutral scalars (new Higgses) are difficult (or inaccessible).

The production of h_a together with charged Higgses looks as the most perspective approach.

One can consider the opportunity to find scalars h_a with mass $M_a < 350$ GeV as a relatively narrow peaks in $b\bar{b}$ and (or) $h_1 h_1$ channels at LHC.

We hope that similar picture is realized in many other models.

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References

[1] I.F. Ginzburg, M. Krawczyk, *Phys. Rev.* **D 72** (2005) 115013; hep-ph/0408011

[2] I.F. Ginzburg, M. Krawczyk, arXiv: 1310.5881 [hep-ph]

[3] I.F. Ginzburg, M. Krawczyk, in preparation.

[4] I.F. Ginzburg, K.A. Kanishev, *Phys. Rev.* **D 92** (2015) 015024; arXiv:1502.06346 [hep-ph]

[5] I.F. Ginzburg, *JETP Lett.* **99** (2014) 742-751; ArXiv: 1410.0873 [hep-ph]

[6] ATLAS Collaboration, Phys. Lett. B **716**, 1 (2012) [arXiv:1207.7214 [hep-ex]], ATLAS-CONF-2014-010, 2014; CMS Collaboration, Phys. Lett. B **716**, 30 (2012) [arXiv:1207.7235 [hep-ex]]; CMS-PAS-HIG-14-009, 2014.

[7] G. Belanger, B. Dumont, U. Ellwanger, J.F. Gunion, S. Kraml, JHEP **1302** (2013) 053 [arXiv:1212.5244 [hep-ph]]; B. Dumont, arXiv:1305.4635 [hep-ph]

[8] T.D. Lee, *Phys. Rev.* **D8** (1973) 1226; J.F. Gunion, H.E. Haber, G. Kane, S. Dawson, *The Higgs Hunter's Guide* (Addison-Wesley, Reading, 1990); G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher and J. P. Silva, Phys. Rept. **516**, 1 (2012) [arXiv:1106.0034 [hep-ph]].

[9] J.F. Gunion, H.E. Haber, *Phys. Rev.* **D72** (2005) 095002, hep-ph/0506227; S. Davidson, H.E. Haber, *Phys. Rev.* **D72** (2005) 035004, hep-ph/0504050.

[10] I.F. Ginzburg, K.A. Kanishev, M. Krawczyk, D. Sokolowska, *Phys. Rev.* **D 82** (2010) 123533, 1-9, Arxiv. hep-ph/1009.4593; G. C. Dorsch, S. J. Huber and J. M. No, JHEP **1310**, 029 (2013) [arXiv:1305.6610 [hep-ph]].

[11] I.F. Ginzburg, arXiv:1505.01984; 1510.08270 [hep-ph]

[12] C.-Y. Chen, M. Freid, M. Sher, arXiv:1312.3949 [hep-ph]

[13] B. Grzadkowski, O.M. Ogreid, P. Osland, *Phys. Rev.* D **D 80** (2009) 055013 [arXiv:0904.2173 [hep-ph]].

[14] See e.g. A.Arando, C. Bonilla, A.D. Rojas, *Phys. Lett.* **B717** (2012) 248

[15] M. Lisanti, J G. Wacker, arXiv:0704.2816 [hep-ph]

[16] I. P. Ivanov1, V. Keus, arXiv:1203.3426 [hep-ph]

[17] B. Holdom, M. Ratzlaff, arXiv:1412.1513 [hep-ph]; S. Kanemura, K. Tsumura, H. Yokoya, arXiv:1305.5424 [hep-ph]; N. Craig, J. Galloway, S. Thomas, arXiv:1305.2424 [hep-ph]; B. Dumont, J.F. Gunion, Y. Jiang, S. Kraml, arXiv:1405.3584 [hep-ph]; C. Lange, on behalf of the ATLAS and CMS, arXiv:1411.7279 [hep-ex]; D. Cline, X. Ding, J. Lederman, arXiv:1204.6700 [hep-ph]; J. Baglio, O. Eberhardt, U. Nierste, M. Wiebusch, arXiv:1403.1264 [hep-ph]

[18] I.F. Ginzburg, M. Krawczyk, P. Osland, 2nd ECFA/DESY Study 1998-2001, 997-1001; hep-ph/9909455;// Nucl. Instrum. Meth. **A 472** (2001) 149-154; hep-ph/0101229;
AIP Conf. Proc. Series 578 (2001) 304, IFT-2001-2, 304; hep-ph/0101331;// Proc. SUSY02 Conference 703-706; hep-ph/0211371.

[19] P.S.B. Dev, A. Pilaftsis. arXiv:1503.09140 [hep-ph]

[20] J.F. Gunion, H.E. Haber, J. Wudka, *Phys. Rev.* **D 43**, 904 (1991).

[21] A. Celis, V.Jlsie, A. Pich, JHEP **1307** (2013) 053 [arXiv:1302.4022 [hep-ph]]

[22] R. Enberg, W. Klemm, S. Moretti, S. Munir, G. Wouda, *Nucl. Phys.* **B893** (2015) 420-442, arXiv:1412.5814[hep-ph]; arXiv:1502.02931 [hep-ph]

[23] I.F. Ginzburg, arXiv:1505.01984 [hep-ph]

[24] J. Baglio, O. Eberhardt, U. Nierste, M. Wiebusch, *Phys. Rev.* **D 90**, 015008 (2014) arXiv:1403.1264 [hep-ph].

[25] V. Barger, L.L. Everett, C.B. Jackson, A.D. Peterson, G. Shaughnessy, *Phys. Rev.* **D 90**, 095006 (2014) arXiv:1408.2525 [hep-ph].

[26] S. Kanemura, H. Yokoya, Y.-J. Zheng, arXiv:1505.01089 [hep-ph].

[27] P.S.B. Dev, A. Pilaftsis, arXiv:1408.3405 [hep-ph]

[28] P.M. Ferreira, R. Guedes, J.F. Gunion, H.E. Haber, M.O.P. Sampaio, R. Santos, arXiv:1410.1926 [hep-ph]

[29] J. Hajer, Y.-Y. Li, T. Liu, J.F.H. Shiua, arXiv:1504.07617 [hep-ph]; N. Craig, F. D'Eramo, P. Draper, S. Thomas, H. Zhang, arXiv:1504.04630 [hep-ph]

[30] F. Kling, A. Pyarelal, S. Su, arXiv:1504.06624 [hep-ph]

[31] M. Hashemi1, I. Ahmed, arXiv:1401.4841 [hep-ph]; arXiv:1502.06445 [hep-ph]; M. Hashemi, S.M. Zebarjad, H. Bakhshalizadeh, arXiv:1407.3436 [hep-ph]