Upgraded LHC experiments as a check of a would-be approach to calculations of the SM fundamental parameters

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The problem of a mutual dependence of parameters of the Standard Model is considered in the framework of the compensation approach. Conditions for a spontaneous generation of four electro-weak boson effective interactions are shown to lead to a set of equations for parameters of the interaction. In case of a realization of a non-trivial solution of a set of compensation equations, parameter $\sin^2\theta_W$ is defined. The existence of non-trivial solutions is demonstrated, which provide a satisfactory value for the electromagnetic fine structure constant α at scale M_Z: $\alpha(M_Z) = 0.007756$. Within the range of experimental limitations we demonstrate the existence of two solutions for the problem. There is a solution with high effective cut-off being close to the Planck mass by the order of magnitude. Another solution corresponds to the effective cut-off in 10^2 TeV range and leads to prediction of significant effects in the production of a top pair accompanied by an electro-weak boson, which might be observed in experiments at the upgraded LHC.

1 Introduction

The Standard Model of particles' interactions is fairly considered to be a quite successful theory. It describes phenomena in high energy physics and give us the consecutive interpretation of the totality of data. But till now we cannot take SM as the accomplished theory. First of all, by the reason of that it does not include at least lowest energy gravitational interaction in coupling with other ones and in the same way as others. Just this circumstance forces theorists to invent different SM extensions. However, secondly, even provided we shall leave such problems as scale hierarchy aside, we immediately face another, so to say, more prosaic problem. We cannot admit the Standard Model to be just an accomplished theory simply because there are too many external parameters we have to bring into it to maintain it's expository power. The number of these parameters (such as coupling constants, masses and mixing angles including those of neutrinos) reach up to 29. Of course, one may hope that determination of their values will be supplied with a wouldbe extension of the SM, for example, in the framework of the superstring theory.

However we see striking contrast between existing theory's insularity and it's excellent experimental validation just in this insularity, on the one hand, and the variety of extensions of the SM, none of them acquiring even a hint for a confirmation despite the richest facilities of the LHC, on the other hand. Provided the wouldbe extended theory will be really able to reduce the totality of all data to one fundamental source, we have to make extremely great progress in experiment technology for making this theory as such validated as SM is validated just now.

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This problem prompts us to make efforts in other direction. Particularly, we can attempt to perform necessary evaluations just within the existing theory structure. The minimal extensions which we'll have to build anyway, have rather to be non-structural and to deal not with new notions, fields and particles, but with new types of effective interactions inherent in the Standard Model. The searching region for such interactions may be, of course, indicated by the fact that SM and general quantum field theory conception are of the less successfulness in describing of low energy processes. We can ask ourselves: may be, there is some deep correlation between the fact of failing of perturbation theory in an explanation of such phenomena and the presence of special low energy quantum effects, which we cannot imagine with the same clearness and distinctness as higher energy interactions, but which we are to take into account in some way? This situation patently corresponds with such effects as superconductivity and superfluidity, where classical local theory was powerless and consecutive analysis in terms of fundamental quantum theory equations was inaccessible also, but where the solution was found. We can not say from the beginning, what "force" cause electrons to produce their coupling into a Cooper's pair, but we can describe their behavior in this pairing. The way to consecutive understanding of the phenomenon is fairly provided by the Bogoliubov transformation, which follows from his general compensation approach [1, 2].

The method of a spontaneous generation of effective non-local interactions, which we shall try to apply in this work to the problem being mentioned above, was just grown up from N.N. Bogoliubov's compensation conception [1, 2] developed and successfully applied just in the superconductivity theory. Although in a field theory it acquires some new specialities, the stated above analogy seems to be quite encouraging for us. On the other hand, we have also quite successfully applied this approach to the range of the particle physics low energy processes. The compensation approach was applied [3, 4] to the problem of the spontaneous generation of effective interactions in quantum field theories. Most impressive effectiveness of the method was demonstrated in the light meson physics, where the spontaneously generated Nambu - Iona-Lazinio interaction [5, 6] allowed us [7] to calculate main light mesons' properties with good precision using only fundamental QCD parameters without external parameters bringing in. Applications to the composite Higgs particle problem [8] and to the spontaneous generation of the wouldbe anomalous three-boson interaction [9, 10], to be discussed below, could be also mentioned. The analogous approach was applied to low energy gluon interaction [11]. The method and applications are also described in full in the book [12].

The main specific feature of the approach consists in formulation of compensation equations, which non-trivial solutions are connected with phenomenon of a spontaneous generation of effective interactions. Of course, there is also possibility of the trivial solution, which corresponds to absence of anything new. However a non-trivial solution exists only provided a number of conditions on parameters of a problem under a study being fulfilled. This just allows to define links between parameters, for example, mass ratios and dimensionless coupling constants. Emphasize, that such effective interactions correspond to purely non-perturbative effects. The perturbative solution is always just the trivial one.

Our aim in this work is to demonstrate principal possibility of determination of fundamental SM parameters with fine structure constant α taken as an example. The considerations are performed just in terms of spontaneously generated effective interactions. Correspondingly, we build a simple model, being guided by our previous experience in similar, but more advanced models construction. In case of success of this attempt it would be really important step, opening a way to the more sophisticated and more close to reality theorizing upon this important subject. But, in the same time, just grounding on this simplified approach we shall present some predictions, suitable for upgraded LHC experiments checking.

The present results, which are also described in [13], deals with possible effective interactions in the framework of the electro-weak gauge theory. In view of a forthcoming use in the main body of the paper, let us present few formulas, which are related to would be triple effective interaction of the electro-weak bosons [9,10]

$$-\frac{G}{3!}\,F\,\epsilon_{abc}\,W^a_{\mu\nu}\,W^b_{\nu\rho}\,W^c_{\rho\mu}\,; \tag{1}$$
$$W^a_{\mu\nu} \;=\; \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\,\epsilon_{abc}W^b_\mu W^c_\nu\,;$$

with uniquely defined form-factor $F(p_i)$, which guarantees effective interaction (1) acting in a limited region of the momentum space. The considerations were performed the framework of an approximate scheme, which accuracy was estimated to be $\simeq$ 10% [3]. Would-be existence of effective interaction (1) leads to important non-perturbative effects in the electro-weak interaction. It is usually called anomalous three-boson interaction and it is considered for long time on phenomenological grounds [14,15] and it was studied in a number of experiments. Our interaction constant G is connected with conventional definitions in the following way

$$G \;=\; -\,\frac{g\,\lambda}{M_W^2}\,; \tag{2}$$

where $g \simeq 0.65$ is the electro-weak coupling. The best limitations for parameter λ read [16]

$$\lambda_\gamma = -\,0.022 \pm 0.019\,;\quad \lambda_Z = -\,0.09 \pm 0.06\,; \tag{3}$$

where subscript denote a neutral boson being involved in the experimental definition of λ.

The conditions for existence of the non-trivial solution for the spontaneous generation of interaction (1) lead to the following set of parameters [8–10]

$$g(z_0) = 0.60366\,;\quad z_0 = 9.6175\,; \tag{4}$$
$$|\lambda| = 3.5\cdot 10^{-6}\,;\; G \;=\; 0.000352\,TeV^{-2}.$$

Here z_0 is a dimensionless parameter, which is connected with value of a boundary momentum, that is with effective cut-off Λ_0 according to the following definition

$$\frac{2\,G^2\,\Lambda_0^4}{1024\,\pi^2} = \frac{g^2\,\lambda^2\,\Lambda_0^4}{512\,\pi^2\,M_W^4} = z_0;\; \Lambda_0 = 7.914\cdot 10^5\,GeV. \tag{5}$$

Let us note, that the solution of the analogous compensation procedure in QCD correspond to $g(z_0) = 3.817$ [11], that gives satisfactory description of the low-momentum behavior of the running strong coupling, including absence of the Landau pole.

We have already mentioned that the existence of a non-trivial solution of a compensation equation impose essential restrictions on parameters of a problem. Just the example of these restrictions is the definition of coupling constant $g(z_0)$ in (4), which is really of an appropriate magnitude for the electro-weak coupling. It is advisable to consider other possibilities for spontaneous generation of effective interactions in the electro-weak theory and to find out, which restrictions on physical parameters may be imposed by an existence of non-trivial solutions. In the present work we consider possibilities of definition of links between important physical parameters, first of all with relation to the fine structure constant α.

2 Weinberg mixing angle and the fine structure constant

Let us demonstrate a simple model, which illustrates how the well-known Weinberg mixing angle could be calculated. Let us consider a possibility of a spontaneous generation of the following effective interaction of electroweak gauge bosons

$$L_{eff}^{W} = -\frac{G_2}{8} W_\mu^a W_\mu^a W_{\rho\sigma}^b W_{\rho\sigma}^b - \frac{G_3}{8} W_\mu^a W_\mu^a B_{\rho\sigma} B_{\rho\sigma} - \frac{G_4}{8} Z_\mu Z_\mu W_{\rho\sigma}^b W_{\rho\sigma}^b - \frac{G_5}{8} Z_\mu Z_\mu B_{\rho\sigma} B_{\rho\sigma}. \tag{6}$$

where we maintain the residual gauge invariance for the electromagnetic field. Here index a corresponds to charged W-s, that is it takes values 1, 2, while index b corresponds to three components of W defined by the initial formulation of the electro-weak interaction. Definition of coefficients in (6) corresponds to a convenient form for Feynman rules for corresponding vertices, *e.g.* for the first term in (6) the vertex reads

$$\imath\, \delta^{a_1}_{a_2}\, \delta^{b_1}_{b_2}\, G_2\, g_{\mu\nu}\, (g_{\rho\sigma}(p\,q) \,-\, p_\sigma\, q_\rho)\,; \tag{7}$$

where components of W^a have indices μ, ν and incoming momenta and indices (p,ρ) and (q,σ) refer to fields W^b.

Let us remind the relation, which connect fields W^0, B with physical fields of the Z boson and of the photon

$$W_\mu^0 = \cos\theta_W\, Z_\mu + \sin\theta_W\, A_\mu; \qquad B_\mu = -\,\sin\theta_W\, Z_\mu + \cos\theta_W\, A_\mu. \tag{8}$$

Thus in terms of the physical states ($W^+\, W^-\, Z\, A$) wouldbe effective interaction (6) has the following form

$$L_{eff}^{W} = -\frac{G_2}{2} W_\mu^+ W_\mu^-\, W_{\rho\sigma}^+ W_{\rho\sigma}^- - \frac{G_2}{4} W_\mu^+ W_\mu^- \Big(\cos^2\theta_W Z_{\rho\sigma} Z_{\rho\sigma} + 2\cos\theta_W \sin\theta_W Z_{\rho\sigma} A_{\rho\sigma} + \sin^2\theta_W A_{\rho\sigma} A_{\rho\sigma}\Big) - \frac{G_4}{4} Z_\mu\, Z_\mu\, W_{\rho\sigma}^+\, W_{\rho\sigma}^- \,-\, \frac{G_4}{8} Z_\mu Z_\mu \Big(\cos^2\theta_W Z_{\rho\sigma} Z_{\rho\sigma} + \sin^2\theta_W A_{\rho\sigma} A_{\rho\sigma} + 2\cos\theta_W \sin\theta_W Z_{\rho\sigma} A_{\rho\sigma}\Big) - \frac{G_3}{4} W_\mu^+ W_\mu^- \Big(\sin^2\theta_W Z_{\rho\sigma} Z_{\rho\sigma} + \cos^2\theta_W A_{\rho\sigma} A_{\rho\sigma} - 2\cos\theta_W \sin\theta_W Z_{\rho\sigma} A_{\rho\sigma}\Big) - \frac{G_5}{8} Z_\mu Z_\mu \Big(\sin^2\theta_W Z_{\rho\sigma} Z_{\rho\sigma} + \cos^2\theta_W A_{\rho\sigma}\, A_{\rho\sigma} \,- 2\,\cos\theta_W \sin\theta_W\, Z_{\rho\sigma} A_{\rho\sigma}\Big). \tag{9}$$

Interactions of type (9) were earlier introduced on phenomenological grounds in works [17,18] and are now also subjects for experimental studies including the recent study at LHC [19]. Let us consider a possibility of a spontaneous generation of interaction (6,9). In doing this we start with the standard form of the Lagrangian, which describes electro-weak gauge boson fields W^a, Z and γ and the Higgs scalar field H in the unitary gauge with the usual division into the free and the interaction parts

$$L = L_0 \,+\, L_{int}\,. \tag{10}$$

Then we perform the add-subtract procedure [1–3, 12] of expression (6)

$$L = L_0' \,+\, L_{int}'\,; \qquad L_0' = L_0 \,-\, L_{eff}^{W}\,; \tag{11}$$

$$L_{int}' \,=\, L_{int} \,+\, L_{eff}^{W}\,. \tag{12}$$

includegraphics[scale=0.6]compencZB.pdf

Figure 1: Diagram representation of set (13). Simple line represent W^a and W^b, dotted lines represent B or Z with indications in the figure. A black spot means effective interaction (9).

Now, compensation equations for wouldbe interaction (9). We are to demand, so that in the theory with Lagrangian L'_0 (11), all contributions to four-boson connected vertices, corresponding to interaction (9) are summed up to zero. That is the undesirable interaction part in the would-be free Lagrangian (11) is to be compensated. Then we are rested with interaction (9) only in the proper place (12). Emphasize, that all contributions of the SM interactions are included in L'_{int} (12). We would formulate these compensation equations using experience being acquired in the course of application of the method to the Nambu - Jona-Lazinio interaction and to the triple weak boson interaction (1). As it is demonstrated in book [12] (Section 3.3), the first approximation for the problem of spontaneous generation of the Nambu - Jona-Lazinio interaction assumes a form-factor $F(p)$, analogous to that introduced in effective interaction (1), to be unit step function $\Theta(\Lambda^2 - p^2)$ and only horizontal diagrams of the type presented in Fig. 1 are taken into account. The next approximation, described in detail in [7] and in [12] (Chapter 5) includes also vertical diagrams and form-factor $F(p)$ is uniquely defined as a solution of a set of compensation conditions. We have demonstrated, that the first approximation gives satisfactory results and the next one serves for its specification. In the present work we just use the first approximation.

So let us introduce effective cut-off Λ, which is a subject for a definition in the course of a solution of the problem and use just unit step function $\Theta(\Lambda^2 - p^2)$ for the effective form-factor.

In this way we have the following set of compensation equations, which corresponds to diagrams being presented in FIG. 1

$$\begin{aligned}
&-x_2 - 2F_W x_2^2 - (1-a^2)F_Z x_3 x_4 - a^2 F_Z x_2 x_4 = 0;\\
&-x_3 - 2\,F_W\,x_2\,x_3 - a^2 F_Z\,x_2\,x_5\,-\,(1-a^2)\,F_Z\,x_3\,x_5\,=\,0\,;\\
&-x_4 - 2F_W x_2 x_4 - a^2 F_Z x_4^2 - (1-a^2)F_Z x_3 x_4 = 0;\\
&-x_5 - 2F_W x_3 x_4 - a^2 F_Z x_4 x_5 - (1-a^2)F_Z x_5^2 = 0;
\end{aligned} \tag{13}$$

where the following notations are used

$$F_W = 1 - \frac{2M_W^2}{\Lambda^2}\Big(L_W - \frac{1}{2}\Big);\; L_W = \ln\frac{\Lambda^2 + M_W^2}{M_W^2}\;;\; F_Z = 1 - \frac{2M_Z^2}{\Lambda^2}\Big(L_Z - \frac{1}{2}\Big); \tag{14}$$

$$L_Z = \ln\frac{\Lambda^2 + M_Z^2}{M_Z^2};\quad x_i = \frac{3G_i\Lambda^2}{16\pi^2};\quad a\,=\,\cos\theta_W. \tag{15}$$

Factor 2 in equations (13) here corresponds to sum by weak isotopic index $\delta^a_a\,=\,2$, $a = 1, 2$.

We have the following solutions [2] of set (13) in addition to the evident trivial one: $x_2 = x_3 = x_4 = x_5 = 0$

$$x_3\,=\,x_5\,=\,0;\; x_2\,=\,-\,\frac{1 + a^2 F_Z x_4}{2\,F_W}; \tag{16}$$

$$x_3\,=\,x_5\,=\,0;\; x_2\,=\,-\,\frac{1}{2\,F_W};\; x_4\,=\,0; \tag{17}$$

$$x_2\,=\,x_4\,=\,0;\; x_3\,=\,\frac{a^2}{2(1-a^2)F_W}\;x_5\,=\,-\,\frac{1}{(1-a^2)F_Z};; \tag{18}$$

[2]Note, that an absence of some x_i in a solution means that the variable is arbitrary.

$$x_2 = x_4 = -\frac{1}{2\,F_W};\; x_3 = \frac{a^2}{2(1-a^2)F_W};\; x_5 = -\frac{1}{(1-a^2)F_Z}; \tag{19}$$

$$x_2 = -\frac{1}{2\,F_W};\; x_4 = 0;\; x_3 = 0;\; x_5 = 0; \tag{20}$$

$$x_2 = x_4 = 0;\; x_5 = -\frac{1}{(1-a^2)F_Z}; \tag{21}$$

$$x_2 = -\frac{1}{2\,F_W};\; x_4 = 0;\; x_3 = \frac{a^2}{2(1-a^2)F_W};\; x_5 = -\frac{1}{(1-a^2)F_Z}; \tag{22}$$

$$x_2 = -\frac{1}{2\,F_W};\; x_4 = 0;\; x_5 = 0; \tag{23}$$

$$x_2 = x_4 = -\frac{1+(1-a^2)F_Z\,x_5}{2\,F_W + a^2\,F_Z};\; x_3 = x_5. \tag{24}$$

Then, following the reasoning of the approach (see, *e.g.* [8, 12]), we assume, that the Higgs scalar corresponds to a bound state consisting of a complete set of fundamental particles. Here we study the wouldbe effective interaction (6,9) of the electroweak bosons, so we take into account just these bosons as constituents of the Higgs scalar. This assumption is carried out provided corresponding Bethe-Salpeter equations for this bound state being fulfilled. There are two equations because constituents are either $W^a\,W^a$ or $Z\,Z$. These equations are graphically presented in the two rows of FIG. 2. Calculations are performed in the unitary gauge. All power divergences in single SM terms of course properly cancel. In approximation of sufficiently large cut-off Λ these equations have the following form with notations (14,15)

$$\begin{aligned}&-3\,x_2\,(2\,F_W + a\,F_Z) - \frac{x_3(1-a^2)}{a} - \frac{3\,\alpha_{ew}}{16\,\pi}\Big[-\frac{a^2(a^6-a^4-5\,a^2+1)}{1-a^2}L_W + \\ &\frac{(1+a^2)(1-3\,a^2)}{a^2(1-a^2)}L_Z - \frac{(1-a^2-a^4)(1-a^2)}{a^2}\Big] + \frac{3\,\alpha_{ew}\,M_W^2}{32\,\pi}\Big[\frac{3\,M_H^2}{(M_H^2-M_W^2)^2}\ln\Big[\frac{M_H^2}{M_W^2}\Big] - \\ &\frac{3}{M_H^2-M_W^2} - \frac{8}{M_W^2}\Big] = \frac{1}{B_W};\end{aligned} \tag{25}$$

$$\begin{aligned}&-x_4(2F_W + aF_Z) - \frac{x_5(1-a^2)}{a} - \frac{\alpha_{ew}a^2}{4\pi} + \frac{3\alpha_{ew}M_Z^2}{32\pi a^4}\Big[\frac{3M_H^2}{(M_H^2-M_W^2)^2}\ln\Big[\frac{M_H^2}{M_Z^2}\Big] - \\ &\frac{3}{M_H^2-M_Z^2} - \frac{8}{M_Z^2}\Big] = \frac{1}{a^2 B_Z};\end{aligned} \tag{26}$$

$$\begin{aligned}&B_W = F_W + \frac{M_H^2}{2\,\Lambda^2}\Big(L_W - \frac{13}{12}\Big);\; B_Z = F_Z + \frac{M_H^2}{2\,\Lambda^2}\Big(L_Z - \frac{13}{12}\Big);\; \alpha_{ew} = \frac{\alpha_0}{1+\frac{5\alpha_0}{4\pi}\ln\frac{\Lambda^2}{M_Z^2}}; \\ &\alpha_0 = 0.0337;\; a = \cos\theta_W(\Lambda);\; 1-a^2 = \frac{\alpha\Big(1+\frac{5\alpha_0}{6\pi}\ln\frac{\Lambda^2}{M_Z^2}\Big)}{\alpha_0\Big(1-\frac{5\alpha}{6\pi}\ln\frac{\Lambda^2}{M_Z^2}\Big)}; \\ &\alpha = \frac{e^2(M_Z)}{4\,\pi} = \alpha(M_Z) = 0.007756\,.\end{aligned} \tag{27}$$

We would draw attention to definitions of the running electro-weak coupling α_{ew} and the electromagnetic one α. Here we have used the standard one-loop evolution formulas for these running couplings with number of flavors $N_f = 6$. We have also applied the well known relation $M_W = \cos\theta_W\,M_Z$.

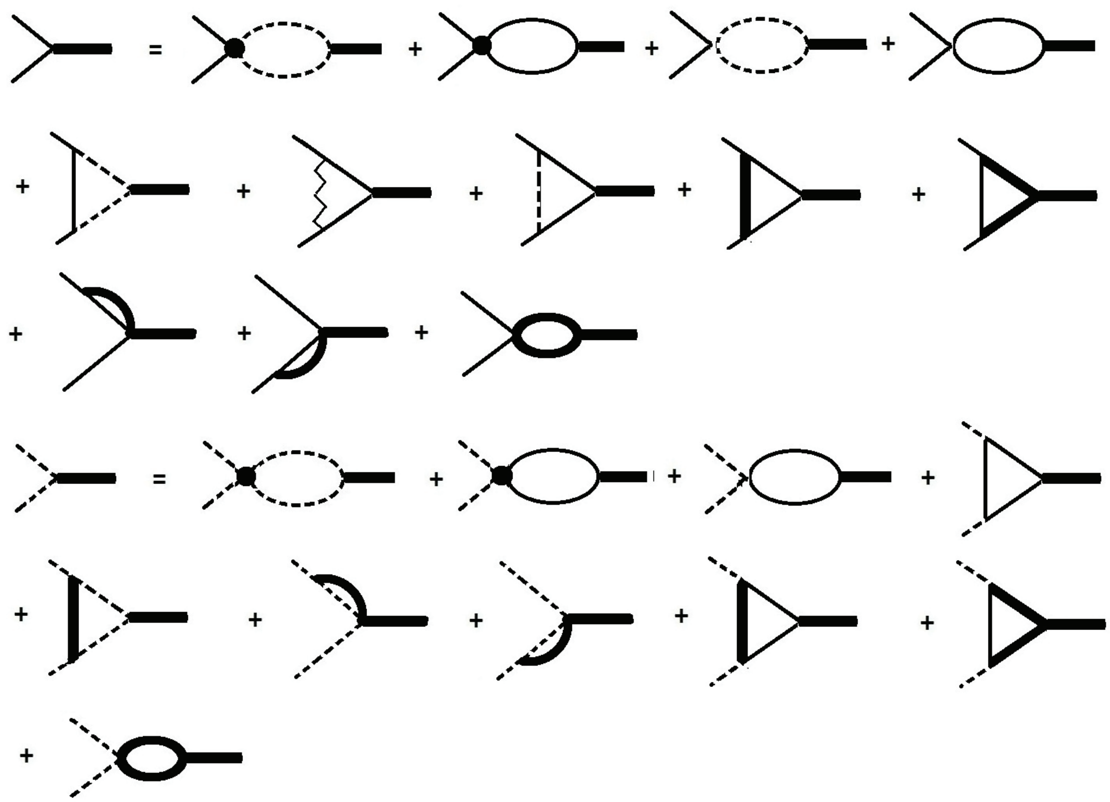

Figure 2: Diagram representation of set (25, 26) of the Bethe-Salpeter equations. Simple lines represent W-s, dotted lines represent Z, wave line represent the photon and thick lines represent Higgs scalar H. Black spots mean effective interaction (9) and simple points mean the SM electro-weak couplings in the unitary gauge.

Now we look for solutions of set (13)(four equations), (25), (26) for variables x_2, x_3, x_4, x_5, a, Λ, which give appropriate value for $\alpha(M_Z) = 0.007756$, according to relation (27). We use values for physical masses

$$M_W = 80.4GeV, \quad M_Z = 91.2GeV, \quad M_H = 125.1GeV. \tag{28}$$

We have studied solutions of the set of equations and have come to the conclusion, that only solutions (16), (21) and (24) of compensation set (13) give necessary value $\alpha(M_Z) = 0.007756$. For the first option (16) there are two solutions which satisfy our conditions. Namely, the following two ones

$$\begin{aligned} &\Lambda = 5.2262 \cdot 10^5\, GeV;\; x_2 = -\,0.32378;\; x_3 = 0;\\ &x_4 = -\,0.48652;\; x_5 = 0;\; a = 0.85114\,; \qquad (29)\\ &\Lambda = 8.6866 \cdot 10^{19} GeV;\; x_2 = -\,0.31602;\; x_3 = 0;\\ &x_4 = -\,0.71131;\; x_5 = 0;\; a = 0.71923\,; \qquad (30)\end{aligned}$$

These solutions define coupling constants of effective interaction (9) again for the two solutions respectively

$$G_2 = -6.24 \cdot 10^{-5}\, TeV^{-2};\; G_3 = 0;\; G_4 = -9.376 \cdot 10^{-5}\, TeV^{-2};\; G_5 = 0; \tag{31}$$
$$G_2 = -2.205 \cdot 10^{-33}\, TeV^{-2};\; G_3 = 0;\; G_4 = -4.962 \cdot 10^{-33}\, TeV^{-2};\; G_5 = 0. \tag{32}$$

From definition of parameters in experimental work [19]

$$L_{eff} \;=\; -\,\frac{e^2 a_0^W}{8\Lambda'^2} A_{\mu\nu} A_{\mu\nu} W^+_\rho W^-_\rho \;-\; \frac{e^2 g^2 k_0^W}{\Lambda'^2}\, A_{\mu\nu} Z_{\mu\nu} W^+_\rho W^-_\rho\;; \tag{33}$$

and from (9) we have

$$\frac{a_0^W}{\Lambda'^2} = \frac{2\,G_2}{g^2}; \quad \frac{k_0^W}{\Lambda'^2} = \frac{G_2 \cos\theta_W}{2\,g^4 \sin\theta_W}. \tag{34}$$

Results (31,32) lead to the following prediction for parameters a_0^W, k_0^W for the two solutions respectively

$$\frac{a_0^W}{\Lambda'^2} = -0.000147\, TeV^{-2}\,; \quad \frac{k_0^W}{\Lambda'^2} = -0.000142\, TeV^{-2}; \tag{35}$$
$$\frac{a_0^W}{\Lambda'^2} = -1.044 \cdot 10^{-32}\, TeV^{-2}\,; \quad \frac{k_0^W}{\Lambda'^2} = -1.13 \cdot 10^{-32}\, TeV^{-2}. \tag{36}$$

Bearing in mind relations (34) and taking from experimental work [19] the following limitations

$$-\,21\, TeV^{-2} < \frac{a_0^W}{\Lambda'^2} < 20\, TeV^{-2}; \quad -12\, TeV^{-2} < \frac{k_0^W}{\Lambda'^2} < 10\, TeV^{-2}; \tag{37}$$

we see, that predictions (35, 36) are deeply inside boundaries of limitations (37). Of course the second solution (36) gives a negligible small value, whereas the first one (35) for a possibility of its checking needs five orders of magnitude of an improvement of the precision. It seems, that such precision can be hardly achieved even at the upgraded LHC.

The second solution (21) of the set of compensation equations gives the following parameters

$$\begin{aligned} &x_2 \,=\, x_4 \,=\, 0;\; x_3 \,=\, -4.21777;\; x_5 \,=\, -\,5.95333;\; a \,=\, -\,0.87338; \qquad (38)\\ &\Lambda \,=\, 364.5845\, GeV;\; G_2 = G_4 = 0;\; G_3 = -\frac{0.00167}{GeV^2};\; G_5 = -\frac{0.00236}{GeV^2}.\end{aligned}$$

The third solution (24) of the set of compensation equations gives two solutions with the same cut-off. We have the following sets of parameters

$$\begin{aligned}
&x_2 = x_4 = -1.72596;\; x_3 = x_5 = 3.9589;\\
&a = -0.876955;\; \Lambda = 106.7934\, GeV;\\
&G_2 = G_4 = -\frac{0.007966}{GeV^2};\; G_3 = G_5 = \frac{0.018272}{GeV^2};
\end{aligned} \qquad (39)$$

$$\begin{aligned}
&x_2 = x_4 = -0.864885;\; x_3 = x_5 = -2.61273;\\
&a = 0.876955;\; \Lambda = 106.7934\, GeV;\\
&G_2 = G_4 = -\frac{0.0039918}{GeV^2};\; G_3 = G_5 = -\frac{0.012059}{GeV^2};
\end{aligned} \qquad (40)$$

Solutions (38,39,40) evidently contradict limitations (37) due to very low value for cut-off Λ and so have to be rejected.

Solution (30) with very large cut-off Λ is of considerable interest. It is remarkable, that this solution corresponds to the cut-off being of the order of magnitude of the Planck mass $M_{Pl} = 1.22 \times 10^{19}\, GeV$. Of course effective coupling constants G_i in this case are extremely small. This possibility in case of its realization may serve as an explanation of hierarchy problem [20]. Indeed, the experimental values for masses of W, Z, H and value $\alpha(M_Z)$ lead to effective cut-off being defined by the gravitational Planck mass. So the relation between the electro-weak scale and the gravity scale might become natural.

Let us pay more attention to the low cut-off case. Value of Λ (29) is close to boundary value (5) of the momentum in the problem of a spontaneous generation of anomalous triple W interaction (1)

$$\Lambda_0 = 7.91413 \cdot 10^5\, GeV\,. \qquad (41)$$

We see, that this value is of the same order of magnitude as value $5.2262 \cdot 10^5\, GeV$ in solution (29).

Now we could formulate results in a rather different manner. We have two interesting values for possible cut-off Λ. Low value (41), which follows from previous results [9,10], and the Planck mass. Let us consider set of equations (16, 25, 26) for these values of the cut-off. Earlier we have fixed actual value for electromagnetic constant $\alpha(M_Z)$ and calculated values for the cutoff (29,30). Now we fix Λ and calculate $\alpha(M_Z)$. In this way for values (41) and the Planck mass we obtain respectively

$$\alpha(M_Z)_{41} = 0.00792; \quad \alpha(M_Z)_{Pl} = 0.00790. \qquad (42)$$

Both values are almost the same and differ from actual value $\alpha(M_Z) = 0.007756$ by 2%. Thus it might be possible to interpret results (42) just as a calculation of the value of α with this precision. Note, that few *per cent* contributions are expected at the next approximation in the development by powers of α_{ew}.

Of course, there is the trivial solution of set (13): all $x_i = 0$, which gives no additional information. However we have also quite informative non-trivial solutions.

The problem of the choice of the genuine solution is undoubtedly essential. The answer is to be connected with the problem of a stability of solutions. This problem is quite difficult and needs extensive additional studies.

In the present work we try to show the way to decide if the non-trivial solution (29,31) really exists just from forthcoming experiments at the LHC.

In view of this, in the next section we consider possible experimental consequences of solutions (29,31) and (30,32), which could be studied at the upgraded LHC.

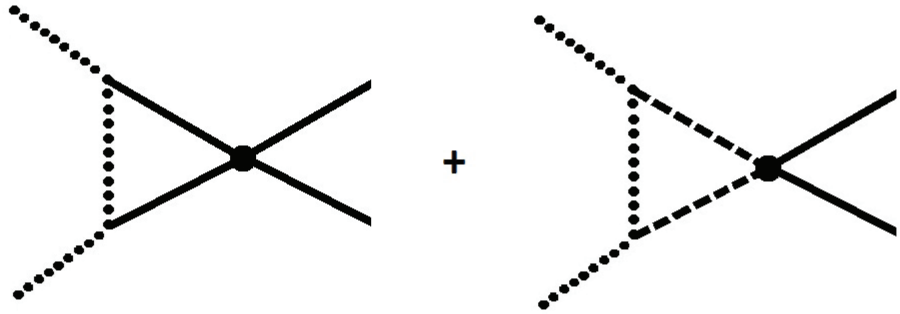

Figure 3: Diagram representation of $\bar{t}tWW$ vertex. Continuous lines represent W, dotted line represents Z and dotty lines at the left of each diagram represent the t-quarks. Notations for vertices are the same as in FIG. 2

3 Experimental implications

Effective interaction (9) directly leads to effects in reactions

$$p + p \to W^+ + W^- + W^\pm(Z, \gamma). \tag{43}$$

Unfortunately with values for effective coupling constants G_2, G_4 for solution (29,31) under consideration one could hardly hope for achieving the necessary precision even at the upgraded LHC. The more so it is right for solution (30,32).

However there is a possibility for an enhancement of the effect in processes involving t-quarks due to large value of M_t. Let us consider the wouldbe contribution of interaction (9) with parameters (29, 31) to vertex

$$\frac{G_{W\bar{t}t}}{4}\,\bar{t}t\,W^b_{\mu\nu}\,W^b_{\mu\nu}\,; \tag{44}$$

where b takes three values $1, 2, 3$. The effective coupling for this vertex is defined by diagrams presented in FIG.3. These diagrams diverge quadratically, but we have to bear in mind a presence of effective form-factor $\Theta(\Lambda^2 - p^2)$ in interaction (9). For low-cutoff solution we are to use the same cutoff Λ (29) in calculation of the diagrams and we obtain with account of definitions of x_i (15)

$$G_{W\bar{t}t} \;=\; -\frac{g^2(\Lambda)\,M_t(\Lambda)}{24\,M_W^4}\Big(2\,x_2 + a^2(\Lambda)\,x_4\Big) \;=\; 4.25\cdot 10^{-8}\,GeV^{-3}; \tag{45}$$

where we take $g(\Lambda)$ from (4) and parameters (29). For $M_t(\Lambda)$ we use the standard evolution $N_f = 6$ expression

$$M_t(\Lambda) \;=\; \frac{M_t}{\left(1 + \frac{7\alpha_s(M_t)}{4\pi}\ln\left[\frac{\Lambda^2}{M_t^2}\right]\right)^{\frac{4}{7}}}; \tag{46}$$

where $M_t = 173.2\,GeV$ is the table value for the t-quark mass [16].

For solution (30) with the cut-off being close to the Planck mass we obtain in the same way

$$G_{W\bar{t}t} \;=\; 1.506\cdot 10^{-8}\,GeV^{-3}; \tag{47}$$

Table 1: SM results for cross-sections of processes $p+p \to \bar{t}tV$ at $\sqrt{s}=8TeV$ and predictions for additional contribution due to effective interaction (44) with solutions (29) and (30) . Values of coupling $G_{W\bar{t}t}$ are indicated in subscripts.

channel	$\sigma_{SM}\, fb$, 8 TeV	$\Delta\sigma_{45}\, fb$, 8 TeV	$\Delta\sigma_{47}\, fb$, 8 TeV
$\bar{t}tW^+$	161^{+19}_{-32}	103.5 ± 20.7	13.0 ± 2.6
$\bar{t}tW^-$	71^{+11}_{-15}	28.0 ± 5.6	3.5 ± 0.7
$\bar{t}tZ$	197^{+22}_{-25}	47.2 ± 9.4	5.9 ± 1.2

Let us consider processes $p+p \to \bar{t}\, t\, W^{\pm}(Z)+X$. With values (45,47) we have additional contributions of the new effective interaction (44) to cross sections $\sigma_{\bar{t}tW}$, $\sigma_{\bar{t}tZ}$ of processes [3].

$$p+p \;\to\; \bar{t}+t+W^{\pm}+X; \tag{48}$$
$$p+p \;\to\; \bar{t}+t+Z+X; \tag{49}$$

For $\sqrt{s}=8\,TeV$ we obtain the following estimate with values of coupling $G_{W\bar{t}t}$ indicated in superscripts

$$\Delta\sigma^{45}_{\bar{t}tW^+}(8\,TeV) \;=\; 103.5\, fb. \tag{50}$$
$$\Delta\sigma^{47}_{\bar{t}tW^+}(8\,TeV) \;=\; 13.0\, fb. \tag{51}$$

For the same process with the negative W we have

$$\Delta\sigma^{45}_{\bar{t}tW^-}(8\,TeV) \;=\; 28.0\, fb. \tag{52}$$
$$\Delta\sigma^{47}_{\bar{t}tW^-}(8\,TeV) \;=\; 3.5\, fb. \tag{53}$$

For process (49) we have the following additional contributions

$$\Delta\sigma^{45}_{\bar{t}tZ}(8\,TeV) \;=\; 47.2\, fb. \tag{54}$$
$$\Delta\sigma^{47}_{\bar{t}tZ}(8\,TeV) \;=\; 5.9\, fb. \tag{55}$$

These results, as well as the subsequent ones, are obtained with the use of the CompHEP package [21].

Recent CMS result at $\sqrt{s}=8\,TeV$ [22] for these processes reads [4]

$$\begin{aligned}\sigma_{\bar{t}tW^+}(8\,TeV) \;&=\; 170^{+110}_{-100}\, fb;\\ \sigma_{\bar{t}tZ}(8\,TeV) \;&=\; 200 \pm 90\, fb;\end{aligned} \tag{56}$$

Results (56) are compatible with wouldbe additional contributions (50, 54) as well as with (51, 55) and with the Standard Model. There is no data for process (52) in [22].

However $\Delta\sigma(\bar{t}tW, Z)$ increases with the energy increasing and for the updated energy of the LHC $\sqrt{s} = 14\,TeV$ we have for the low cutoff solution

$$\begin{aligned}\Delta\sigma^{45}_{\bar{t}tW^+}(14\,TeV) \;&=\; 1257\, fb.\\ \Delta\sigma^{45}_{\bar{t}tW^-}(14\,TeV) \;&=\; 355\, fb.\\ \Delta\sigma^{45}_{\bar{t}tZ}(14\,TeV) \;&=\; 578\, fb.\end{aligned} \tag{57}$$

Table 2: SM results for cross-sections of processes $p + p \to \bar{t}tV$ at $\sqrt{s} = 14 TeV$ and predictions for additional contribution due to effective interaction (44) with solutions (29) and (30) . Values of coupling $G_{W\bar{t}t}$ are indicated in subscripts.

channel	$\sigma_{SM}\, fb$, 14 TeV	$\Delta\sigma_{45}\, fb$, 14 TeV	$\Delta\sigma_{47}\, fb$, 14 TeV
$\bar{t}tW^+$	507^{+147}_{-111}	1257 ± 251	158 ± 32
$\bar{t}tW^-$	262^{+81}_{-60}	355 ± 71	45 ± 9
$\bar{t}tZ$	760^{+74}_{-84}	578 ± 116	73 ± 15

Our predictions are to be compared with the SM calculations [24–26] in Table 2 [5].

Let us comment uncertainties of $\Delta\sigma$ being presented in Tables for the calculated contributions of new effective interactions. There are two the most important contributions to the uncertainty. The first one is the uncertainty inherent to the compensation approach. We have already mentioned in the Introduction, that accuracy of the approach, in the approximation being used, is estimated to be around 10% according to the experience of applications of the approach to several examples (see [3] and book [12]). The second contribution is provided by uncertainties of parton distribution functions, which, as a matter of fact, contribute significantly the uncertainties for σ_{SM} in the Table. With combination of these sources we approximately estimate uncertainties of our calculations of $\Delta\sigma$ to be around 20%.

We have already noted, that results for $\sqrt{s} = 8\, TeV$ do not contradict the current data (56). As for $\sqrt{s} = 14\, TeV$, we see from the Table, that the most promising process for testing of the present results at the upgraded LHC is $p + p \to \bar{t}\, t\, W^{\pm}$. Indeed, the total additional contribution to the production of the charged W with the top pair for the first solution (45) is around 1.6 pb, that more than twice exceeds the corresponding total SM value. Thus in case of a successful confirmation of the present results the cross section for this process is to be three times as much as its SM value. On the other hand such wouldbe significant effect guaranties the reliable disproof of an existence of interaction (44) with coupling (45) and thus the rejection of a realization of solution (29,31) in case of a disagreement with the prediction.

In case of absence of such significant effect, connected with low cut-off solution (29) there remains the possibility of the high cut-off solution (30). However, the same calculations give results, which hardly could be detected at the upgraded LHC. For example, additional contribution $\Delta\sigma$ for process $p + p \to \bar{t}tW^+ + X$ is now $158 \pm 32\, fb$. We include results for the second solution also in TABLE 1, where we denote different solutions in the same way as was done above. From the Table we see, that even maximal effect in the first reaction for the second solution do not exceed uncertainties of SM calculations. So the reliable study of effects of this solution presumably needs even larger energy, than it is available at the upgraded LHC.

Note, that we do not include in the Table process $p + p \to \bar{t}\, t\, \gamma$, because the effect here is significantly less pronounced. Namely, for $\sqrt{s} = 13\, TeV$ we have $\sigma_{SM} = 1.744 \pm 0.005\, pb$ [26], whereas the effect of interaction (44) with coupling (45) is calculated to be $\Delta\sigma = 0.125\, pb$. We have looked for other possible observable effects and have not succeeded in this. For example, effects in pair Higgs scalar production accompanied by W or Z are not significant for solutions (29,30).

Provided the predictions being confirmed, the first discovery of the non-perturbative effect in the electroweak interaction would be established.

The most important conclusion in this case would be the confirmation of the mechanism of definition of fundamental parameters of the Standard Model. Indeed, the solution of compensation equations, which

[3] We have got persuaded, that an interference of contributions of effective interaction (44) with the SM terms is negligible.
[4] Results for $\sqrt{s} = 7\, TeV$ see in [23].
[5] The result for $\sigma_{SM}(\bar{t}tZ)$ in the second column corresponds to $\sqrt{s} = 13\, TeV$.

leads to the results being discussed above, gives the calculation of the adequate value for electromagnetic coupling α (42), the result, which can not be obtained within any other approach as yet.

4 Conclusion

To conclude let us draw attention to the the results in view of the compensation approach to the problem of a spontaneous generation of an effective interaction. First of all, the results are obtained exclusively due to application of this approach. We would emphasize that the existence of a non-trivial solution of compensation conditions always impose strong restrictions on parameters of the problem. We see such restrictions in both problems of the spontaneous generation of the Nambu – Jona-Lazinio interaction [7] and the triple anomalous weak boson interaction [9, 10]. Here we have considered consequences of the existence of nontrivial solutions of compensation conditions for a spontaneous generation of the anomalous four-boson interaction.

The most interesting result is just relation (42). Indeed, we see, that the adequate value of fine structure constant is achieved in two cases. The first case corresponds to the electro-weak scale $\simeq 10^2\,TeV$ and the second case corresponds to the Planck mass scale. We have two phases and may assume, that these phases occur in different stages of the Universe evolution. Under some conditions there may be a phase transition between them. For example, it might be, that at the very early stage of the evolution the Planck scale solution (30) is realized. Then in the course of expanding of the Universe the phase transition occurs to the low cut-off solution (29) with the electro-weak scale and the same other parameters including the fine structure constant. In the contemporary Universe we observe just this solution. This point of view could be confirmed provided the effects presented in Table 1 would be discovered. Thus it would be possible to understand such tremendous gap between the electro-weak scale and the gravity scale. In context of the results of the present work we might hope to obtain explanation of the connection between different scales, that is one might come to the solution of the problem of the hierarchy [20].

In case of a confirmation of results under the discussion, for example, of effects (57), the following consequences might become clear.
1. The first non-perturbative effect in the electro-weak interaction would be established.
2. The efficiency of the compensation approach to description of the phenomenon of spontaneous generation of an effective interaction would be ascertained.
3. The restrictive nature of compensation conditions would be confirmed.
4. The last but not the least result consists in the successful calculation of the fine structure constant α (42), that already could be considered as a sound argument on behalf of the compensation approach.

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References

[1] N. N. Bogoliubov, *On the compensation principle and the self-consistent field method*, Soviet Phys.-Uspekhi **67** (1959) 236.

[2] N. N. Bogoliubov, *On some problems of the theory of superconductivity*, Physica Suppl. (Amsterdam) **26** (1960) 1.

[3] B. A. Arbuzov, *Spontaneous generation of effective interaction in a renormalizable quantum field theory model*, Theor. Math. Phys. **140** (2004) 1205.

[4] B. A. Arbuzov, *Spontaneous generation of the Nambu–Jona-Lasinio interaction in quantum chromodynamics with two light quarks* Phys. Atom. Nucl. **69** (2006) 1588.

[5] Y. Nambu and G. Jona-Lasinio, *Dynamical model of elementary particles based on an analogy with superconductivity. I*, Phys. Rev. **122** (1961) 345.

[6] Y. Nambu and G. Jona-Lasinio, *Dynamical model of elementary particles based on an analogy with superconductivity. II*, Phys. Rev. **124** (1961) 246.

[7] B. A. Arbuzov, M. K. Volkov and I. V. Zaitsev, *NJL model derived from QCD*, Int. J. Mod. Phys. A **21** (2006) 5721.

[8] B. A. Arbuzov and I. V. Zaitsev, *Nonperturbative solutions in the electro-weak theory with $\bar{t}t$ condensate and the t-quark mass*, Int. J. Mod. Phys. **A26** (2011) 4945.

[9] B. A. Arbuzov, *Non-perturbative solutions in the electroweak theory*, Theor. Math. Phys. **171** (2012) 448.

[10] B. A. Arbuzov and I. V. Zaitsev, *LHC would-be $\gamma\gamma$ excess as a non-perturbative effect of the electro-weak interaction*, Phys. Rev. **D85** (2012) 093001.

[11] B. A. Arbuzov and I.V. Zaitsev, *On a possibility to eliminate the Landau pole in QCD*, Int. J. Mod. Phys. **A28** (2013) 1350127.

[12] B. A. Arbuzov, *Non-perturbative Effective Interactions in the Standard Model*, De Gruyter, Berlin, 2014.

[13] B. A. Arbuzov and I.V. Zaitsev, *On a possible validation in upgraded LHC experiments of the SM fundamental parameters' calculation method based on the Bogoliubov compensation approach*, arXiv: 1505.07269 [hep-ph].

[14] K. Hagiwara, R. D. Peccei, D. Zeppenfeld and K. Hikasa, Probing of the weak boson sector in $e^+e^- \to W^+W^-$, Nucl. Phys. **B282** (1987) 253.

[15] K. Hagiwara, S. Ishihara, R. Szalapski and D. Zeppenfeld, *Low energy effects of new interactions in the electroweak boson sector*, Phys. Rev. **D48** (1993) 2182.

[16] K. A. Olive *et al.* (Particle Data Group), *Review of particle physics*, Chin. Phys. **C38** (2014) 090001.

[17] G. Belanger and F. Boudjema, *Probing quartic couplings of weak bosons through tree vectors production at 500-GeV NLC*, Phys. Lett. **B288** (1992) 201.

[18] G. Belanger *et al.*, *Bosonic quartic couplings at LEP-2*, Eur. Phys. J. **C13** (2000) 283.

[19] S. Chatrchyan *et al.* (CMS Collaboration), *Search for $WW\gamma$ and $WZ\gamma$ production and constraints on anomalous quartic gauge couplings in pp collisions at $\sqrt{s}$ = 8 TeV*, Phys. Rev. **D90** (2014) 032008.

[20] E. Gildener, *Gauge Symmatry Hierarchiies*, Phys. Rev. **D14** (1976) 1667.

[21] E. E. Boos *et al.* (CompHEP Collaboration), *CompHEP: Specialized package for automatic calculations of elementary particles' decays and collisions*, Nucl. Instr. Meth. **A534** (2004) 250.

[22] S. Chatrchyan *et al.* (CMS Collaboration), Eur. Phys. J. **C74**, 3060 (2014).

[23] S. Chatrchyan *et al.* (CMS Collaboration), *Measurement of associated production of vector bosons and top quark-antiquark pairs at sqrt(s)=7 TeV*, Phys. Rev. Lett **110** (2013) 172002.

[24] J. M. Campbell and R. K. Ellis, $t\bar{t}W^{+-}$ *production and decay at NLO*, JHEP **1207** (2012) 052.

[25] M. V. Garzelli, A. Kardos, C. G. Papadopulos and Z. Trocsanyi, $t\bar{t}W^{+-}$ *and Hadroproduction at NLO accuracy in QCD with Parton Shower and Hadronization effects*, JHEP **1211** (2012) 056.

[26] J. Alwall *et al.*, *The automated computation of tree-level and next-to-leading order differential cross-sections, and their matching to parton shower simulations*, JHEP **1407** (2014) 079.