Similarity and Some Differences in Radion and Higgs Boson Production and Decay Processes Involving Off-Shell Fermions

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The Randall-Sundrum model as one of the possible extensions of the Standard Model (SM) predicts the existence of the scalar radion which mass is somewhat smaller than that of all the Kaluza-Klein modes of the SM particles. The radion interacts with the trace of the energy-momentum tensor of the SM and the fermion part of the interaction Lagrangian looks similar to that for the Higgs boson of the SM except for some additional terms. The single radion and associated Higgs boson - radion production processes are considered and it is shown that these new terms do not contribute to the radion production and decay processes making them similar to the same processes with the Higgs boson.

1 Introduction

One of the possible extensions of the SM is the stabilized Randall-Sundrum model [1], [2], [3], [4],. In the framework of this model one extra dimension is added and a system of two branes interacting with the gravitation and a real scalar field in 5-dimensional space-time is considered. The metric of the background solution is not flat and contains an exponential factor whereby the hierarchy problem is solved. The SM fields are localized on one of the branes and the distance between the branes is fixed by the additional stabilizing scalar field. The stabilizing scalar field and gravitation propagate in the whole 5-dimensional space-time. The lowest Kaluza-Klein mode of the 5-dimensional scalar field appearing from the fluctuations of the metric component corresponding to the extra dimension is called the radion and it can be potentially observed at the modern accelerators [1] - [7]. The radion couples to the trace of the energy-momentum tensor of SM, so the interaction Lagrangian has the following form $L = -1/\Lambda_r \left(r(x) T^{\mu}_{\mu} \right)$, where Λ_r is a dimensional scale parameter, $r(x)$ corresponds to the radion field and T^{μ}_{μ} is the trace of the SM energy-momentum tensor with anomaly terms included which can be written as follows

$$\begin{aligned} T^{\mu}_{\mu} = {} & \frac{\beta(g_s)}{2g_s} G^{ab}_{\rho\sigma} G^{\rho\sigma}_{ab} + \frac{\beta(e)}{2e} F_{\rho\sigma} F^{\rho\sigma} + \sum_f \left[\frac{3i}{2} \left(\left(D_\mu \overline{f} \right) \gamma^\mu f - \overline{f} \gamma^\mu \left(D_\mu f \right) \right) + 4 m_f \overline{f} f \right] \\ & - \left(\partial_\mu h \right) \left(\partial^\mu h \right) + 2 m_h^2 h^2 \left(1 + \frac{h}{2v} \right)^2 - \left(2 m_W^2 W^+_\mu W^{\mu -} + m_Z^2 Z_\mu Z^\mu \right) \left(1 + \frac{h}{v} \right)^2 \end{aligned} \quad (1)$$

where $\beta(g_s)$, $\beta(e)$ are the QCD and QED β-functions respectively, D_μ are the SM covariant derivatives and the summation here is taken over all SM fermions.

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In the case of on-shell fermions the fermion part of the Lagrangian (1) is the same as for the Higgs boson (with the replacement $\Lambda_r \rightarrow v$), but for the off-shell fermions there appear some additional terms which may contribute to the radion production and decay processes making them different from the same processes with the Higgs boson. Nevertheless there exist some processes involving off-shell fermions which are similar to the processes with the Higgs boson. In the current work we consider single radion and Higgs boson production in association with gauge bosons and associated Higgs boson - radion production processes. For the both cases the similarity in the radion and the Higgs boson production is demonstrated. The investigation of double Higgs boson production is an important task for the Higgs potential profile experimental measurements. This problem is rather tricky even in the high luminosity mode of the LHC, being one of the key arguments for the ILC creation. If, however, one of the multidimensional "brane world" scenarios occurs in nature, the presence of the radion can further complicate the problem of the Higgs potential research due to the similarity of the Higgs boson and the radion properties.

2 Cancellations of additional to the Higgs-like contributions in the single radion production processes

It can be shown [8] that all the additional contributions as compared to the Higgs boson case are canceled out in the sum of amplitudes corresponding to the single radion production processes. This property follows from the structure of any massive fermion current emitting a radion and an arbitrary number of any SM gauge bosons. One can observe it rewriting the fermion-radion vertex in terms of the inverse function to the propagator $S^{-1}(p) = \frac{p^2 - m_f^2}{\not{p} + m_f}$, so the contributions to the single radion production with one gauge boson emission take the following form

$$M_{1\mu} = -i\,C\,\overline{u}_{out}(p_{out})\,\Gamma_\mu\,S(k_1)\,[\frac{3}{2}\,(S^{-1}(k_1)\,+\,S^{-1}(p_{in})) - m_f]\,u_{in}(p_{in}) \tag{2}$$

$$M_{2\mu} = -i\,C\,\overline{u}_{out}(p_{out})\,[\frac{3}{2}\,(S^{-1}(p_{out})\,+\,S^{-1}(q_1)) - m_f]\,S(q_1)\,\Gamma_\mu\,u_{in}(p_{in}) \tag{3}$$

$$M_{3\mu} = +i\,3\,C\,\overline{u}_{out}(p_{out})\,\Gamma_\mu\,u_{in}(p_{in}) \tag{4}$$

where the constant C involves all vertex coefficients, is the Lorentz part of the vertices. Applying the Dirac equation for the external fermions and adding together (2), (3) and (4) one can see that there remains the only term proportional to the fermion mass, i.e. the Higgs-like contribution only. In the case of N gauge bosons the sum of all diagrams can be expressed as a sum of $2N+1$ terms

$$M_{N vector\,bosons} \,=\, M_0 \,+\, \sum_{l=1}^{N}(M_l \,+\, M'_l), \tag{5}$$

where M_i stands for the amplitudes containing the fermion-fermion-radion vertex and M'_i - for that with the boson-fermion-fermion-radion vertex Fig.1.

In the same manner as in previous case one can simplify the amplitudes and get the relations

$$M_l = M_l^H - \frac{1}{2}M'_l - \frac{1}{2}M'_{l+1}, \qquad M_0 = M_0^H - \frac{1}{2}M'_1, \qquad M_N = M_N^H - \frac{1}{2}M'_N, \tag{6}$$

where M_0^H, M_N^H, M_N^H stand for the Higgs-like contributions proportional to the fermion masses. With consideration of this, it is easy to show that the sum of all the contributions leads to only the Higgs-like terms and all the other parts are cancelled out $\sum_{l=0}^{N} M_l + \sum_{l=1}^{N} M'_1 = \sum_{l=0}^{N} M_l^H$.

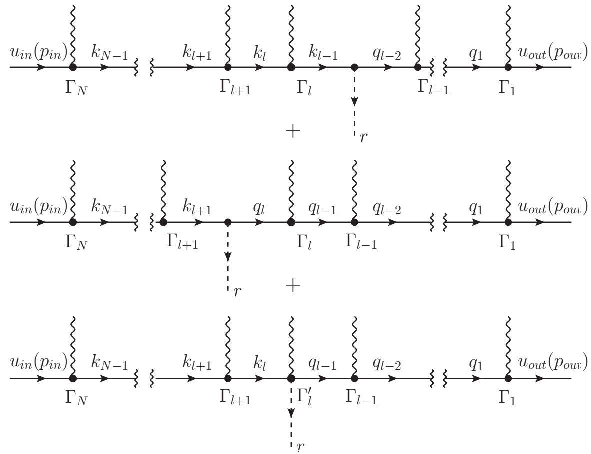

Figure 1: The minimal set of the diagrams to demonstrate the cancelation of additional to the Higgs-like contributions

The cancelation property remains valid for the loop case as well. The generalization to the loop case can be done by replacing the boson lines with virtual propagators and external spinors with additional fermion propagators in cases of boson and fermion loops respectively.

3 Associated Higgs boson - radion production in fermion collisions

Let us first consider an associated Higgs boson - radion production in fermion collisions (Fig.2).

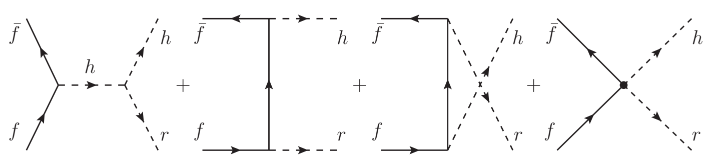

Figure 2: Feynman diagrams contributing to the associated Higgs boson - radion production in fermion collisions

Corresponding amplitudes being simplified in the same manner as in previous cases have the following form

$$M_1 = \overline{v}^r(p_1)\,\frac{-i\,m_f}{v}\,u^s(p_2)\,\frac{i}{k_h^2 - m_h^2}\,\frac{-i}{\Lambda_r}\,(-2k_h\,p_h + 4m_h^2)\,r(p_r)\,h(p_h) \tag{7}$$

$$M_2 = -i\,\overline{v}^r(p_1)\,\frac{1}{\Lambda_r}\,\frac{m_f}{v}\,\left\{-\frac{3}{2} + m_f\,\frac{\not{k}' + m_f}{k'^2 - m_f^2}\right\}\,r(p_r)\,u^s(p_2)\,h(p_h) \tag{8}$$

$$M_3 = -i\,\overline{v}^r(p_1)\,\frac{1}{\Lambda_r}\,\frac{m_f}{v}\,\left\{-\frac{3}{2} + m_f\,\frac{\not{k} + m_f}{k^2 - m_f^2}\right\}\,r(p_r)\,u^s(p_2)\,h(p_h) \tag{9}$$

$$M_4 = -i\,\overline{v}^r(p_1)\,\frac{1}{\Lambda_r}\,\frac{m_f}{v}\,\{4\}\,r(p_r)\,u^s(p_2)\,h(p_h) \tag{10}$$

In order to get (7) in more comprehensible for summing up form one can modify it using the simple kinematical identities:

$$p_h = k_h - p_r \;\Rightarrow\; 2k_h\, p_h = 2(p_h + p_r)\, p_h = (p_h + p_r)^2 + p_h^2 - p_r^2 = k_h^2 + p_h^2 - p_r^2 = k_h^2 + m_h^2 - m_r^2 \qquad (11)$$

So (7) can be rewritten as follows

$$M_1 = -i\,\frac{m_f}{\Lambda_r\, v}\,\overline{v}^r(p_1)\, r(p_r)\, h(p_h)\left\{-1 + \frac{m_r^2 + 2m_h^2}{k_h^2 - m_h^2}\right\} u^s(p_2) \qquad (12)$$

Now it is easy to put (8), (9), (10) and (12) together and write down the sum of the amplitudes which yields

$$M_{rh} = -i\,\frac{m_f}{\Lambda_r\, v}\, r(p_r)\, h(p_h)\,\overline{v}^r(p_1)\left\{m_f\,\frac{\not k + m_f}{k^2 - m_f^2} + m_f\,\frac{\not k' + m_f}{k'^2 - m_f^2} - \frac{3}{2} - \frac{3}{2} + 4 - 1 + \frac{m_r^2 + 2m_h^2}{k_h^2 - m_h^2}\right\} u^s(p_2) \qquad (13)$$

Comparing this with the result for the same process with double Higgs boson production

$$M_{hh} = -i\,\frac{m_f}{v^2}\, h(p_{h1})\, h(p_{h2})\,\overline{v}^r(p_1)\left\{m_f\,\frac{\not k + m_f}{k^2 - m_f^2} + m_f\,\frac{\not k' + m_f}{k'^2 - m_f^2} + \frac{3m_h^2}{k_h^2 - m_h^2}\right\} u^s(p_2) \qquad (14)$$

one can see the similarity (with the replacements $m_r \to m_h$ and $\Lambda_r \to v$).

4 Associated Higgs boson - radion production in gluon collisions

As another example let us compare two processes involving the gluons: an associated Higgs boson - radion production ($gg \to rh$) and double Higgs boson production ($gg \to hh$), the corresponding diagrams are shown in Fig.3 and Fig.4 respectively.

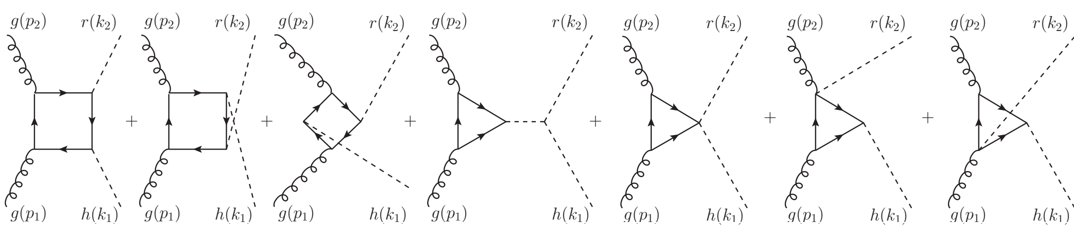

Figure 3: Feynman diagrams contributing to the associated Higgs boson - radion production ($gg \to rh$)

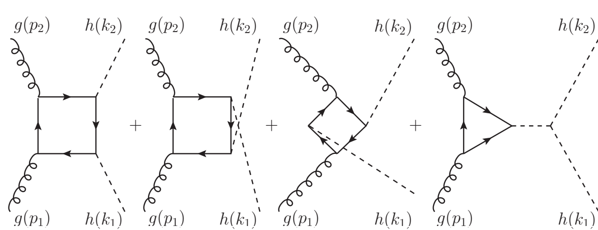

Figure 4: Feynman diagrams contributing to the double Higgs boson production ($gg \to hh$)

One can notice that all the amplitudes have the following similar structure

$$M_i = \frac{g_c^2}{v^2}\,\epsilon(p_1)_\mu\,\epsilon(p_2)_\nu\, h(k_1)\, h(k_2)\int \frac{d^d l}{(2\pi)^d}\, X_i^{\mu\nu}(p_1, p_2, k_1, k_2) \quad \text{for } gg \to hh,\; i = \overline{1,4}, \qquad (15)$$

$$M_i = \frac{g_c^2}{v\,\Lambda_r}\,\epsilon(p_1)_\mu\,\epsilon(p_2)_\nu\,h(k_1)\,r(k_2)\int\frac{d^d l}{(2\pi)^d}\,X_i^{\mu\nu}(p_1,p_2,k_1,k_2)\quad \text{for}\, gg\to rh,\ i=\overline{1,7}, \tag{16}$$

where
$$X_1^{\mu\nu}\equiv Sp[\gamma^\mu\,S_1\,\gamma^\nu\,S_2\,\Gamma_{2,3}\,S_3\,(-m)\,S_4],\qquad X_2^{\mu\nu}\equiv Sp[\gamma^\mu\,S_1\,\gamma^\nu\,S_2\,(-m)\,S_5\Gamma_{5,4}\,S_4],$$
$$X_3^{\mu\nu}\equiv Sp[\gamma^\mu\,S_1\,(-m)\,S_6\gamma^\nu\,S_5\,\Gamma_{5,4}\,S_4],\qquad X_4^{\mu\nu}\equiv Sp[\gamma^\mu\,S_1\,\gamma^\nu\,S_2\,(-m)\,S_4]\,D\,\Gamma',$$
$$X_5^{\mu\nu}\equiv Sp[\gamma^\mu\,S_1\,\gamma^\nu\,S_2\,(+4m)\,S_4],\qquad X_6^{\mu\nu}\equiv Sp[\gamma^\mu\,S_1\,(-3\gamma^\nu)\,S_3\,(-m)\,S_4],$$
$$X_7^{\mu\nu}\equiv Sp[(-3\gamma^\mu)\,S_1\,\gamma^\nu)\,S_2\,(-m)\,S_5],$$

where $S_j^{-1}=(\not{l}-\not{q}_j)-m$, $D^{-1}=(k_1+k_2)^2-m^2$, $q_1=0$, $q_2=-p_2$, $q_3=-p_2+k_2$, $q_4=-p_2+k_2+k_1$, $q_5=-p_2+k_1$, $q_6=k_1$, and Γ_{jl} takes the following forms $\Gamma_{jl}=\begin{cases}-m,\\ \frac{3}{2}S_j^{-1}+\frac{3}{2}S_l^{-1}-m.\end{cases}$

After simple transformations one can get

$$X_1^{\mu\nu}=m^2\,Sp[\gamma^\mu\,S_1\,\gamma^\nu\,S_2\,S_3\,S_4]-\tfrac{3}{8}X_5^{\mu\nu}-\tfrac{1}{2}X_6^{\mu\nu},\quad X_2^{\mu\nu}=m^2\,Sp[\gamma^\mu\,S_1\,\gamma^\nu\,S_2\,S_5\,S_4]-\tfrac{3}{8}X_5^{\mu\nu}-\tfrac{1}{2}X_7^{\mu\nu},$$
$$X_3^{\mu\nu}=m^2\,Sp[\gamma^\mu\,S_1\,S_6\,\gamma^\nu\,S_5\,S_4]-\tfrac{1}{2}X_6^{\mu\nu}-\tfrac{1}{2}X_7^{\mu\nu}, \tag{17}$$

So the sum comes up to

$$X_1^{\mu\nu}+X_2^{\mu\nu}+X_3^{\mu\nu}+X_5^{\mu\nu}+X_6^{\mu\nu}+X_7^{\mu\nu}=$$
$$=\tfrac{1}{4}X_5^{\mu\nu}+m^2\,Sp[\gamma^\mu\,S_1\,\gamma^\nu\,S_2\,S_3\,S_4]+m^2\,Sp[\gamma^\mu\,S_1\,\gamma^\nu\,S_2\,S_5\,S_4]+m^2\,Sp[\gamma^\mu\,S_1\,S_6\,\gamma^\nu\,S_5\,S_4]. \tag{18}$$

Next one can rewrite the h^3-vertex in the following way

$$\Gamma'=2\left\{(k_1+k_2)_\mu\,k_1^\mu-2m_h^2\right\}=(k_1+k_2)^2+k_1^2-k_2^2-4m_h^2=[(k_1+k_2)^2-m_h^2]-m_r^2-2m_h^2 \tag{19}$$

and multiply it by the inverse propagator: $D\,\Gamma'=1-\frac{m_r^2+2m_h^2}{(k_1+k_2)^2-m_h^2}$.

As a result one has the expression for $X_4^{\mu\nu}$:
$$X_4^{\mu\nu}=-\frac{1}{4}X_5^{\mu\nu}+m\,Sp[\gamma^\mu\,S_1\,\gamma^\nu\,S_2\,S_4]\,\frac{m_r^2+2m_h^2}{(k_1+k_2)^2-m_h^2}. \tag{20}$$

Finally the sum of all $X_i^{\mu\nu}$ for the $gg\to rh$ process yields

$$\sum_{i=1}^{7}X_i^{\mu\nu}=\quad m^2\,Sp[\gamma^\mu\,S_1\,\gamma^\nu\,S_2\,S_3\,S_4]+m^2\,Sp[\gamma^\mu\,S_1\,\gamma^\nu\,S_2\,S_5\,S_4]+$$
$$m^2\,Sp[\gamma^\mu\,S_1\,S_6\,\gamma^\nu\,S_5\,S_4]+m\,Sp[\gamma^\mu\,S_1\,\gamma^\nu\,S_2\,S_4]\,\frac{m_r^2+2m_h^2}{(k_1+k_2)^2-m_h^2}. \tag{21}$$

The same expression for the $gg\to hh$ process differs only by a factor after the trace

$$m^2\,Sp[...]+m^2\,Sp[...]+m^2\,Sp[...]+m\,Sp[...]\,\frac{3m_h^2}{(k_1+k_2)^2-m_h^2}. \tag{22}$$

Thus we get that these two processes coincide up to constants (masses, vacuum expectation values) in the case of the anomalies being ignored.

It is interesting to notice that the same result can follow from a model without the radion but with a modified factor in the Higgs backreaction:

$$L = -\frac{1}{2}\left(\partial_\mu h\right)\left(\partial^\mu h\right) - \frac{1}{2}m_h^2 h^2 - \frac{\xi}{2}\frac{m_h^2}{v}h^3 - \frac{1}{8}\frac{m_h^2}{v^2}h^4, \quad \text{where} \quad \xi = \begin{cases} 1 \text{ - for the SM,} \\ 1 + \frac{m_r^2 - m_h^2}{2m_h^2} \end{cases} \tag{23}$$

In the case of $m_r = m_h$ this model can not be distinguished from the SM. However this statement is valid only for the processes in the first order of the radion interaction constant, in more complicated cases the difference between the radion and the Higgs boson can be more significant and go beyond the simple replacement of the Higgs potential parameters.

5 Conclusions

In the current work we have shown several examples of the radion - Higgs boson similarity in single and associated production processes. First, the general idea of the cancellation of additional to the Higgs-like contributions in the single radion production processes was demonstrated. Next the associated Higgs boson - radion production was considered on two examples - in fermion and gluon collisions. In both cases the radion Higgs boson similarity was shown. For such processes with one radion in association with Higgs boson the same result can be obtained from a model with modified Higgs potential parameters and without the radion, thought it is not the same for the processes with two and more radions or with virtual radion decaying into two radions. The radion pair production is not considered in the current work being a model dependent and a complicated study due to adding the terms of higher orders to the Lagrangian.

Acknowledgments The work was supported by grant 14-12-00363 of Russian Science Foundation. The authors are grateful to V. Bunichev, M. Smolyakov, and I. Volobuev for useful discussions and critical remarks.

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