MSSM scenarios after discovery of the Higgs boson

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Higgs boson discovery at the LHC with the mass \( m_H = 125 \text{ GeV} \), not accompanied by any signals of additional scalar resonances and superpartners of the minimal supersymmetric standard model (MSSM), indicates that the supersymmetry breaking scale is not smaller than a few TeV. Such a situation favors the MSSM scenarios close to the decoupling limit in the Higgs sector that could apparently restrict the MSSM parameter space. We analyse, whenever possible, precise restrictions imposed on the MSSM parameter space by the observed Higgs boson mass \( m_H = 125 \text{ GeV} \) and capabilities of a simplified approximations of the radiative corrections in the MSSM Higgs sector, which could reduce the multidimensional MSSM parameter space to a simple two-dimensional parameter space with a sufficient degree of precision.

1 Introduction

Higgs boson discovery at the LHC with the mass \( m_H = 125.09 \pm 0.24 \text{ GeV} \) [1] results in an important consequences for the minimal supersymmetric standard model (MSSM) two-doublet Higgs sector. The Higgs sector includes five physical states in the CP-conserving limit: two CP-even neutral Higgs bosons \( h \) and \( H \), two charged scalars \( H^\pm \) and one CP-odd neutral Higgs boson \( A \) [2]. Large radiative corrections from the third generation quarks and quark superpartners play an important role in the generation of the effective two-doublet Higgs potential at the scale below \( M_{\text{SUSY}} \). If in the absence of any corrections the tree mass of the lightest state \( m_h \) cannot exceed the \( Z^0 \)-boson mass [3] and the observables (masses and couplings) can be defined by means of the two parameters, the mixing angle \( \tan \beta = v_2/v_1 \) and the charged scalar mass \( m_{H^\pm} \), then the primary calculation [4] of radiative corrections to the effective potential changes significantly this simple picture. Multidimensional MSSM parameter space even in the simplified variant is described besides the two-dimensional set \( \tan \beta, m_{H^\pm} \) by the three additional parameters: \( \mu \) (Higgs superfield mass parameter), \( A_t = A_b \) (degenerate trilinear Higgs boson - third generation squarks mixing parameters) [3], \( M_{\text{SUSY}} \) (the quark superpartners mass scale). Radiative corrections which ‘bring up’ the lightest Higgs boson mass from the tree limit \( m_Z \) to the observable average value 125.09 GeV are defined by the terms of the order of \( \mu/M_{\text{SUSY}}, A_{t,b}/M_{\text{SUSY}} \), see explicit formulas in [5, 6], also [7].

In the framework of parametric scenarios it is natural to simplify as much as one can the parametrization of the Higgs sector in the five-dimensional parameter space \( m_{H^\pm}, \tan \beta, \mu, A_t = A_b, M_{\text{SUSY}} \) separating most essential variables which demonstrate the strongest influence on the masses and couplings of scalar particles [4]. Such parametrizations [8, 9] are based on the observation that the leading and the subleading

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\[ ^2 \text{As usual } v_{1,2} \text{ are the vacuum expectation values of the two Higgs isodoublets. Instead of } m_{H^\pm} \text{ one can use } m_A, \text{ two masses are connected by a simple formula.} \]

\[ ^3 \text{In the general case } A_t \neq A_b, \text{ for simplicity we assume } A_t = A_b. \]

\[ ^4 \text{except maybe the } gg h \text{ and } \gamma \gamma h \text{ effective couplings induced at the one-loop level.} \]
radiative corrections are most strongly dependent on the $m_h$ value in the regime which is determined by an essential difference of radiative corrections to the matrix elements of CP-even Higgs bosons \( h, H \) mixing matrix, \( \Delta M^2_{11,12} \sim 0, \Delta M^2_{22} \neq 0 \) (see Section 2.2). This circumstance allows one to reduce the number of free parameters of the effective potential from five to only two, \((m_A, \tan \beta)\), which is attractive from phenomenological point of view. At the same time it is known from numerous facts of the case \cite{10} that insignificant changes of the leading radiative corrections could result in the noticeable shifts of the Higgs boson masses and couplings\(^5\). Previous comparisons (before the Higgs boson discovery) for the three approaches FeynHiggs \cite{11}, CPsuperH \cite{12} and the present approach (realized in the CompHEP \cite{13} shell) can be found in the Appendix of \cite{5}. These comparisons have shown that careful treatment of approximations on the basis of 'leading' radiative corrections is appropriate. The question of their precision deserves a careful study.

In the effective field theory approach the MSSM with complete particle composition at the scale above \( M_{\text{SUSY}} \) is matched to an effective SM Lagrangian where the MSSM particles decouple at the scale \( m_{\text{top}} \). The SM couplings are fixed at \( M_{\text{SUSY}} \) scale by supersymmetric conditions. The third generation squark interactions (also called the threshold effects in this context) are accounted for by modification of the supersymmetric matching conditions at the scale \( M_{\text{SUSY}} \). Our analysis is based on the calculation of radiative corrections in the MSSM Higgs sector by means of the effective potential method \cite{5}, taking into account carefully the nonleading soft supersymmetry breaking \( D \)-terms, the wave-function renormalization terms and also including the QCD and weak corrections to Yukawa couplings up to two loops \cite{7,14,15}.

2 Higgs sector of the 2HDM

The most general form of 2HDM potential in the generic basis is \cite{16}

\[
U(\Phi_1, \Phi_2) = -\mu_1^2(\Phi_1^\dagger \Phi_1) - \mu_2^2(\Phi_2^\dagger \Phi_2) - \mu_{12}^2(\Phi_1^\dagger \Phi_2) - \mu_{12}^2(\Phi_2^\dagger \Phi_1) + \\
+ \lambda_1(\Phi_1^\dagger \Phi_1)^2 + \lambda_2(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \\
+ \frac{\lambda_5}{2}(\Phi_1^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + \frac{\lambda_6}{2}(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \\
+ \lambda_7(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_8(\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_1) + \lambda_9(\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_1),
\]

where \( SU(2) \)-states are

\[
\Phi_j = \begin{pmatrix} -i\omega^j_+ \\ \sqrt{2}(v_j + \eta_j + i\chi_j) \end{pmatrix}, \quad j = 1, 2.
\]

In framework of MSSM the potential (1) in the mass basis is

\[
U(h, H, A, H^\pm, G^0, G^\pm) = \frac{m_h^2}{2}h^2 + \frac{m_H^2}{2}H^2 + \frac{m_A^2}{2}A^2 + m_{H^\pm}^2 H^+ H^- + I(3) + I(4),
\]

where \( I(3) \) and \( I(4) \) are the triple and the quartic couplings of Higgs bosons, respectively.\(^5\)

\(^5\) for example, triple self-couplings in the mass basis include the terms proportional to \( m_{H,H,A}^2/v \)
Here the SU(2) states \( \eta_{1,2}, \chi_{1,2}, \omega_{1,2}^{\pm} \) and the mass eigenstates \( h, H, A, H^{\pm}, G^{0}, G^{\pm} \) can be written as \(^6\)\(^5\)\(^6\)

\[
\begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = O_x \begin{bmatrix} h \\ H \end{bmatrix}, \quad \begin{bmatrix} \chi_1 \\ \chi_2 \end{bmatrix} = O_\beta \begin{bmatrix} G^0 \\ A \end{bmatrix}, \quad \begin{bmatrix} \omega_1^+ \\ \omega_2^- \end{bmatrix} = O_\beta \begin{bmatrix} G^\pm \\ H^\pm \end{bmatrix},
\]

where the rotation matrix

\[
O_X = \begin{bmatrix} \cos X & -\sin X \\ \sin X & \cos X \end{bmatrix}, \quad X = \alpha, \beta.
\]

### 2.1 Masses and mixing angles

The masses in (3) are obtained as the eigenvalues of the mass matrix in the representation used in \(^6\)

\[
m_{1,2}^2 = \frac{1}{2} \left( trM_Y^2 \pm \sqrt{(trM_Y^2)^2 - 4 detM_Y^2} \right),
\]

or as diagonal elements of the mass matrix \( O_X^T M_Y^2 O_X \) after rotations of the SU(2) eigenstates parametrized by \( \alpha, \beta \) \(^6\), see also \(^7\), where

\[
M_Y^2 = \begin{pmatrix} M_{11}^2 & M_{12}^2 \\ M_{12}^2 & M_{22}^2 \end{pmatrix}, \quad M_{ij}^0 = \frac{\partial^2 U}{\partial Y_i \partial Y_j}, \quad Y = \eta, \chi, \omega^\pm, \quad i, j = 1, 2.
\]

For example the mixing angle \( \alpha \) and the masses of CP-even Higgs bosons have the forms

\[
m_{H, h}^2 = \frac{1}{2}(m_A^2 + m_Z^2 + \Delta M_{11}^2 + \Delta M_{22}^2 \pm \sqrt{m_A^4 + m_Z^4 - 2m_A^2m_Z^2c_\beta + C}),
\]

\[
\tan 2\alpha = \frac{2\Delta M_{12}^2 - (m_Z^2 + m_A^2)s_\beta}{(m_Z^2 - m_A^2)c_\beta + \Delta M_{11}^2 - \Delta M_{22}^2},
\]

where

\[
C = 4\Delta M_{12}^4 + (\Delta M_{11}^2 - \Delta M_{22}^2)^2 - 2(m_A^2 - m_Z^2)(\Delta M_{11}^2 - \Delta M_{22}^2)c_\beta - 4(m_A^2 + m_Z^2)\Delta M_{12}^2s_\beta.
\]

or

\[
m_H^2 = s_{a+\beta}^2m_A^2 + c_{a-\beta}^2m_Z^2 - v^2(\Delta\lambda_1s_{a+\beta}^2 + \Delta\lambda_2c_{a-\beta}^2)
\]

\[
- 2(\Delta\lambda_3 + \Delta\lambda_4)c_\beta s_\lambda s_\beta + \Re\Delta\lambda_5(s_{a+\beta}^2c_\beta^2 + c_{a-\beta}^2c_\beta^2) - 2c_{a+\beta}(\Re\Delta\lambda_6s_\lambda c_\beta - \Re\Delta\lambda_7c_\lambda s_\beta)),
\]

\[
m_H^2 = c_{a+\beta}^2m_A^2 + s_{a-\beta}^2m_Z^2 - v^2(\Delta\lambda_1c_\beta^2 + \Delta\lambda_2s_\lambda s_\beta^2)
\]

\[
+ 2(\Delta\lambda_3 + \Delta\lambda_4)c_\lambda s_\beta s_\lambda + \Re\Delta\lambda_5(c_{a+\beta}^2c_\beta^2 + s_{a-\beta}^2c_\beta^2) + 2s_{a+\beta}(\Re\Delta\lambda_6c_\lambda c_\beta + \Re\Delta\lambda_7c_\lambda s_\beta)),
\]

\[
\tan 2\alpha = \frac{s_{2\beta}(m_A^2 + m_Z^2) + v^2((\Delta\lambda_3 + \Delta\lambda_4)s_{2\beta} + 2s_{a+\beta}\Re\Delta\lambda_6 + 2s_{a+\beta}\Re\Delta\lambda_7)}{c_{2\beta}(m_A^2 - m_Z^2) + v^2((\Delta\lambda_1c_\beta^2 - \Delta\lambda_2s_\lambda^2 - \Re\Delta\lambda_3c_{2\beta} + (\Re\Delta\lambda_6 - \Re\Delta\lambda_7)s_{2\beta})}
\]

before and after rotation by angle \( \alpha \), respectively. The above mentioned representations give identical results. Numerical value of the Higgs boson mass \( m_h = 125 \) GeV allows one to simplify the equations describing the MSSM two-doublet sector. Instead of Eq.(8) which contains an inconvenient square root the compact equation for the heavy CP-even state \( H \) is valid with good numerical precision, as a rule

\[
m_H^2 = m_A^2 + m_Z^2 - \Delta M_{11}^2 + \Delta M_{22}^2.
\]
2.2 Mass matrix corrections

Radiative corrections to the mass matrix of the fields $\eta_1, \eta_2$ can be written as

$$M_{\eta}^2 = \begin{pmatrix} M_{11}^2 & M_{12}^2 \\ M_{12}^2 & M_{22}^2 \end{pmatrix} + \begin{pmatrix} \Delta M_{11}^2 & \Delta M_{12}^2 \\ \Delta M_{12}^2 & \Delta M_{22}^2 \end{pmatrix},$$

where the tree-level matrix elements are

$$M_{11}^2 = m_\lambda^2 s_\beta^2 + m_\zeta^2 c_\beta^2, \quad M_{22}^2 = m_\lambda^2 c_\beta^2 + m_\zeta^2 s_\beta^2, \quad M_{12}^2 = - s_\beta c_\beta (m_\lambda^2 + m_\zeta^2)$$

and $\Delta M_{ij}, \Delta \lambda_k (i, j = 1, 2, k = 1, \ldots, 7)$ in the general case include corrections of all orders of the perturbation theory.

It is demonstrated in [8] that at $m_h = 123 - 129$ GeV the corrections $\Delta M_{11,12}^2 \sim 0$ but $\Delta M_{22}^2 \neq 0$. In other words, all contributions coming from higher order corrections are contained in $\Delta M_{22}^2$. It is obvious from (8) that $\Delta M_{22}^2$ can be effectively accounted for using the experimentally measured value of $m_h$ [8]

$$\Delta M_{22}^2 = \frac{m_h^2 (m_A^2 + m_\zeta^2 - m_\lambda^2) - m_A^2 m_\zeta^2 c_\beta^2}{m_\zeta^2 c_\beta^2 + m_\lambda^2 s_\beta^2 - m_h^2},$$

then (8) are simplified (approximate hMSSM scenario) [8]

$$m_H^2 = \frac{(m_A^2 + m_\zeta^2 - m_\lambda^2)(m_\zeta^2 c_\beta^2 + m_\lambda^2 s_\beta^2) - m_A^2 m_\zeta^2 c_\beta^2}{m_\zeta^2 c_\beta^2 + m_\lambda^2 s_\beta^2 - m_h^2},$$

$$a = - \arctan \left( \frac{(m_\zeta^2 + m_A^2)s_\beta c_\beta}{m_\zeta^2 c_\beta^2 + m_A^2 s_\beta^2 - m_h^2} \right).$$

3 Numerical results

In this section the $\Delta M_{ij}^2$ are estimated and where it is possible the hMSSM consequences are compared with predictions of approach developed in [5, 6]. In Fig. 1 $m_h$ is plotted as a function of $A$ for three sets of supersymmetric parameters and $m_A$ equal to 300 GeV or 1 TeV. The distribution is remarkably stable for various $m_A$ (the maximal shift of value $m_h$ is about 3 GeV), so in the following we use the intermediate value $m_A = 300$ GeV. Also for numerical analysis we have taken 16 sets of supersymmetric parameters (see Table 1). For example, the set $(3; 4)$ correspond to $M_{SSVY} = 3.5$ TeV, $\tan \beta = 5$, $A = 5$ TeV and any $\mu$ from $(5; 5)$ TeV. For $\tan \beta \geq 5$ the parameter $A$ may be a constant$^8$ so that the condition $m_h = 123 - 128$ GeV is satisfied. For this type of sets the values of $\Delta M_{ij}^2$ are plotted as functions of $\mu$, for other sets the values of $\Delta M_{ij}^2$ are

$^8$Note that there exist various sets of $(A, \mu)$ for which $A$ is not a constant and $m_h = 123 - 128$ GeV, but we want to find these exceptional cases.
Figure 1: $m_h$ as function of $A$ where $M_{\text{SUSY}} = 1 \text{ TeV}$, $\tan \beta = 30$ (green line), $M_{\text{SUSY}} = 1 \text{ TeV}$, $\tan \beta = 5$ (red line), $M_{\text{SUSY}} = 3 \text{ TeV}$, $\tan \beta = 2.5$ (blue line) and $\mu = 1 \text{ TeV}$, $m_A = 300 \text{ GeV}$ (solid line) or $m_A = 1 \text{ TeV}$ (dashed line).

estimated numerically. In Fig. 2 these values are presented for parametric sets with small and large $\tan \beta$: for intermediate and high $\tan \beta$ the assumptions of hMSSM approach may be relevant, for $\tan \beta < 5$ this approach is not reliable.

In Fig. 3 $m_H$ and $\alpha$ are presented in the framework of two scenarios for three parametric sets. The agreement between hMSSM and the approach [5,6] predictions is very good for large $\tan \beta$ (the solid and dashed green lines coincide). As discussed above for $\tan \beta < 5$ the disagreement is substantial. This comes into particular prominence for $\alpha$ (the maximal shift is about 14%). The disagreement of the scenarios for $m_H$ with small $\tan \beta < 5$ is observable though it is only around 1% (blue lines in Fig. 3(left)). Note that curved part of lines for hMSSM $m_H$ is an unphysical consequence of the pole in (18).

<table>
<thead>
<tr>
<th>$\tan \beta$</th>
<th>$M_{\text{SUSY}}$ [TeV]</th>
<th>$M_{\text{SUSY}}$ [TeV]</th>
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<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
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<tr>
<td></td>
<td>3.5</td>
<td>3.1</td>
</tr>
<tr>
<td>2.5</td>
<td>2</td>
<td>4</td>
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<tr>
<td></td>
<td>6.5</td>
<td>6.5</td>
</tr>
<tr>
<td>5</td>
<td>2.2 (7; 10)</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>(−5; 5)</td>
<td>(−5; 5)</td>
</tr>
<tr>
<td>30</td>
<td>1.4 (−2; 2)</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>(−3; 3)</td>
<td>(−9; 9)</td>
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<tr>
<td></td>
<td>(−11; 11)</td>
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Table 1: Supersymmetric parametric sets where the first number is $A$ [TeV] ($A$ is such that $m_R = 123 - 128 \text{ GeV and } A_t = A_t = A$), the second number is $\mu$ [TeV] in each set, $m_A = 300 \text{ GeV}$. 
An analysis of constraints imposed by the condition $m_h = 125$ GeV on the five-dimensional MSSM parameter space is performed in the MSSM scenarios. The MSSM parameters are chosen to define in an adequate form the radiative corrections in the MSSM Higgs sector, which are evaluated in the effective potential approach. It is demonstrated that the restriction imposed by the condition $m_h = 125$ GeV is not strong, isolating rather limited regions of the five-dimensional MSSM parameter space. The evaluations are performed for the sixteen parametric sets including the small $\tan \beta$ region.

First of all for the parametric sets (three of them coincide with [8]) we find the real $\mu$ (Higgs superfield mass parameter) and $A$ (trilinear soft SUSY breaking parameter) with respect to the condition $m_h = 125$ GeV and evaluate the corrections to the Higgs bosons mixing matrix $\Delta M^2_{ij}$. For these parametric sets the corrections are different and not stable when $\tan \beta$ is small ($< 5$) and they may be in a good agreement with hMSSM assumption $\Delta M^2_{11,12} \sim 0$, $\Delta M^2_{22} \neq 0$ when $\tan \beta$ is intermediate or large. The hMSSM expressions for CP-even heavy Higgs mass $m_H$ and mixing angle $\alpha$ are compared with complete expressions where again
the dependence of hMSSM scenario validity on the tan\(\beta\) variation range is demonstrated. At the same time the appropriate tan\(\beta\) does not guarantee the validity of assumption \(\Delta M_{11,12}^2 \sim 0,\Delta M_{22}^2 \neq 0\) (as a rule the values of \(\Delta M_{ij}^2\) are of the same order). Therefore hMSSM scenario is reliable in some limited regions of the parameter space. For complete and precise analysis the full five-dimensional MSSM parameter space implementation is appropriate.

References


