Two-Higgs-doublet model in terms of observable quantities and problems of renormalization

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We found a minimal and a comprehensive set of directly measurable quantities defining the most general two-Higgs-doublet model (2HDM), we call these quantities *observables*. The potential parameters of the model are expressed explicitly via these observables (plus nonphysical parameters which are similar to gauge parameters). The model with arbitrary values of these observables can, in principle, be realized (up to general enough limitations). Our results open the door for the study of Higgs models in terms of measurable quantities only. The experimental limitations can be implemented here directly, without complex, often model-dependent, analysis of the Lagrangian coefficients.

The principal opportunity to determine all parameters of the 2HDM from the (future) data meets strong practical limitation. It is the problem for a very long time.

1 Introduction

The essential part of this report is based on results [1].

The recent discovery of a Higgs-like particle with $M \approx 125$ GeV at the LHC [2] hints that the spontaneous electroweak symmetry breaking is most probably realized by the Higgs mechanism. The minimal realization of the Higgs mechanism introduces a single scalar isodoublet ϕ with the Higgs potential $V_H = -m^2(\phi^\dagger\phi)/2 + \lambda(\phi^\dagger\phi)^2/2$. This model is usually called "the Standard Model" (SM). The experimental results favor the realization of that minimal scenario [3] (SM-like scenario [4], or SM alignment limit [5]). Nevertheless, many variants of extended Higgs models are not ruled out.

The two Higgs doublet model (2HDM) presents the simplest extension of the standard Higgs mechanism [6]. This name unites a group of models in which the standard Higgs doublet is supplemented by an extra hypercharge-one doublet. It offers a number of phenomenological scenarios with different physical content realized in different regions of the model parameter space (see e.g. in [7]). After electroweak symmetry breaking the 2HDM contains three neutral Higgs bosons $h_a \equiv h_{1,2,3}$ and charged Higgs boson $H^\pm$ with masses M_a, $M_\pm$ respectively.

In the SM, parameters of the Higgs potential can be treated as measurable quantities. These are the mass of the Higgs boson M_h and the Higgs self-coupling parameter $\lambda = M_h^2/v^2$, where $v = 246$ GeV is the vacuum expectation value of the Higgs field. Physical problems in this model can be equally discussed in terms of parameters of the potential or in terms of these observables.

The 2HDM contains two fields with identical quantum numbers. Therefore, its description in terms of original fields or in terms of their linear superpositions are equivalent. This freedom makes clear that the study in terms of the Lagrangian may be likened to discussion of electrodynamical effects in a certain gauge defined by some particular gauge-fixing conditions. The discussion of the 2HDM in terms of only well-measurable quantities seems preferable. Here we present solution of this problem [1].

• The 2HDM describes a system of two scalar isospinor fields ϕ_1, ϕ_2 with hypercharge $Y = 1$. The most

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general form of the 2HDM potential is

$$\begin{aligned} V \; &= \frac{\lambda_1}{2}\left(\phi_1^\dagger\phi_1\right)^2 + \frac{\lambda_2}{2}\left(\phi_2^\dagger\phi_2\right)^2 + \lambda_3\left(\phi_1^\dagger\phi_1\right)\left(\phi_2^\dagger\phi_2\right) + \lambda_4\left(\phi_1^\dagger\phi_2\right)\left(\phi_2^\dagger\phi_1\right) \\ &+ \left[\frac{\lambda_5}{2}\left(\phi_1^\dagger\phi_2\right)^2 + \lambda_6\left(\phi_1^\dagger\phi_1\right)\left(\phi_1^\dagger\phi_2\right) + \lambda_7\left(\phi_2^\dagger\phi_2\right)\left(\phi_1^\dagger\phi_2\right) + \text{h.c.}\right] \\ &- \frac{m_{11}^2}{2}\left(\phi_1^\dagger\phi_1\right) - \frac{m_{22}^2}{2}\left(\phi_2^\dagger\phi_2\right) - \left[\frac{m_{12}^2}{2}\left(\phi_1^\dagger\phi_2\right) + \text{h.c.}\right] \end{aligned} \tag{1}$$

Its coefficients are restricted by the requirement that the potential be positive at large quasiclassical values of ϕ_i (*positivity constraints*).

• The model contains two doublets of fields with identical quantum numbers. Therefore, it can be described either in terms of the original fields ϕ_1, ϕ_2, which enter (1), or in terms of fields ϕ_1', ϕ_2', which are obtained from ϕ_k by a global unitary *reparameterization* (RPA) transformation $\hat{\mathcal{F}}$ of the form

$$\begin{pmatrix}\phi_1'\\ \phi_2'\end{pmatrix} = \hat{\mathcal{F}}_{gen}(\theta,\tau,\rho)\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}, \quad \hat{\mathcal{F}}_{gen} = e^{-i\rho_0}\begin{pmatrix}\cos\theta\, e^{i\rho/2} & \sin\theta\, e^{i(\tau-\rho/2)}\\ -\sin\theta\, e^{-i(\tau-\rho/2)} & \cos\theta\, e^{-i\rho/2}\end{pmatrix}. \tag{2}$$

This transformation induces a transformation of the parameters of the Lagrangian $\lambda_i \to \lambda_i'$ in such a way that the new Lagrangian, written in fields ϕ_i', describes the same physical content. We refer to these different choices as different RPa bases.

Transformation (2) is parameterized by angles θ, ρ, τ and ρ_0. The parameter ρ_0 describes an overall phase transformation of the fields, and since it does not affect the parameters of the potential, we do not consider this degree of freedom.

In the potential (1), parameters λ_{1-4}, m_{11}^2 and m_{22}^2 are real while λ_{5-7}, m_{12}^2 are generally complex. So it takes 14 real quantities to fully define the scalar part of the 2HDM. Since the three remaining parameters of the RPa transformation cannot influence description of physical phenomena, the actual number of physically relevant parameters of the potential is $14 - 3 = 11$.

• Extrema of the potential satisfy the stationarity equations $\partial V/\partial\phi_i|_{\phi_1=\langle\phi_1\rangle,\phi_2=\langle\phi_2\rangle} = 0$ $(i = 1, 2)$. The most general solution that describes the $SU(2)\times U(1)_Y \to U(1)_{EM}$ symmetry breaking is expressed via two positive numbers v_i and the relative phase factor $e^{i\xi}$ as:

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_1\end{pmatrix}, \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_2 e^{i\xi}\end{pmatrix}, \quad v_1 = v\cos\beta, \quad v_2 = v\sin\beta, \quad v = \sqrt{v_1^2+v_2^2}. \tag{3}$$

The ground state of potential (the vacuum) is the extremum with the lowest energy, and its vacuum expectation value (v.e.v.) is $v = 246$ GeV.
The fields ϕ_i are then decomposed into their v.e.v.'s and the quantized component fields, their linear combinations describe Goldstone modes $G^\pm$, G^0, charged Higgses $H^\pm$ with mass $M_\pm$ and neutral Higgses $h_{1,2,3}$ with masses $M_{1,2,3}$

• **Relative couplings.** We use the relative couplings for each neutral Higgs boson h_a:

$$\chi_a^P = \frac{g_a^P}{g_{\text{SM}}^P}, \quad \chi_a^\pm = \frac{g(H^+H^-h_a)}{2M_\pm^2/v}, \qquad \chi_a^{H^+W^-} = \frac{g(H^+W^-h_a)}{M_W/v}. \tag{4}$$

The quantities χ_a^P are the ratios of the couplings of h_a with the fundamental particles $P = V\,(W,Z),\; q = t,b,...,\; \ell = \tau,...$ to the corresponding couplings for the would be SM Higgs boson with $M_h = M_a$. The other relative couplings describe interaction of h_a with charged Higgs boson $H_b^\pm$. The quantity $\chi_a^{\pm b}$ describes interaction $H_b^+H_b^-h_a$, the quantity $\chi_a^{H_b^+W^-}$ describes off-diagonal interaction $H_b^\pm W^\mp h_a$. (Below we omit the adjective "relative".)

The neutrals h_a generally have no definite CP parity. Couplings χ_a^V and $\chi_a^{\pm b}$ are real due to Hermiticity of Lagrangian, while other couplings are generally complex. The $Re(\chi_a^f)$ and $Im(\chi_a^f)$ are responsible for the interaction of fermion f with CP-even and CP-odd components of h_a respectively.

2 Higgs basis. Basic equations

Any RPa basis can be used for solving physical problems. Some of them are more suitable than others when solving specific problems. In particular, when the system possesses an additional symmetry, the preferable RPa basis is the one in which this symmetry is made obvious.

We find it useful here to analyze the model with known vacuum (the ground state of the potential) using the basis with $v_2 = 0$. This basis is called *the Higgs (or Georgi) basis* [8]. This basis is obtained from any given basis with known v.e.v.'s by transformation (2) with

$$\begin{pmatrix}\Phi_1\\ \Phi_2\end{pmatrix} = \hat{\mathcal{F}}_{HB}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}, \quad \hat{\mathcal{F}}_{HB} = \hat{\mathcal{F}}_{gen}(\theta=\beta, \tau=\rho-\xi). \tag{5}$$

The phase factor $e^{\pm i\rho/2}$ represents the remaining rephasing (RPh) freedom in the choice of the Higgs basis that is, independence of the physical picture from the choice of relative phase ϕ_i, the RPh phase.

Vise versa, any form of the potential can be obtained from the Higgs basis form with the transformation, $\hat{\mathcal{F}}_{HB}^{-1} = \hat{\mathcal{F}}_{gen}(\theta = -\beta, \tau = \rho + \xi)$ with $\rho \to -\rho$, $\rho_0 \to -\rho_0$. Again, we do not fix in this definition the RPh phase ρ and the irrelevant parameter ρ_0.

The potential obtained has the same form as (1). To distinguish its parameters in the Higgs basis from a generic basis, we use the capital letters Λ, Φ for parameters and fields. Using the extremum conditions, one can rewrite the potential in the simple form [9]

$$\begin{aligned} V_{HB} = M_{\pm}^2\,(\Phi_2^\dagger\Phi_2) + \frac{\Lambda_1}{2}\left(\Phi_1^\dagger\Phi_1 - \frac{v^2}{2}\right)^2 + \frac{\Lambda_2}{2}\,(\Phi_2^\dagger\Phi_2)^2 + \Lambda_3\left(\Phi_1^\dagger\Phi_1 - \frac{v^2}{2}\right)(\Phi_2^\dagger\Phi_2) + \Lambda_4\,(\Phi_1^\dagger\Phi_2)\,(\Phi_2^\dagger\Phi_1) \\ + \left[\frac{\Lambda_5}{2}\,(\Phi_1^\dagger\Phi_2)^2 + \Lambda_6\left(\Phi_1^\dagger\Phi_1 - \frac{v^2}{2}\right)(\Phi_1^\dagger\Phi_2) + \Lambda_7\,(\Phi_2^\dagger\Phi_2)\,(\Phi_1^\dagger\Phi_2) + \text{h.c.}\right]. \end{aligned} \tag{6}$$

In the Higgs basis the decomposition of fields around v.e.v. has form

$$\Phi_1 = \begin{pmatrix} G^+ \\ \dfrac{v + \eta_1 + iG^0}{\sqrt{2}} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} H^+ \\ \dfrac{\eta_2 + i\eta_3}{\sqrt{2}} \end{pmatrix}. \tag{7}$$

To arrive at the description in terms of physically observable fields, one should start by substituting these expressions into the potential (6). Also, by choosing the unitarity gauge for the gauge fields, we omit the Goldstone modes G^a from now on.

As a result, the potential (6) takes the form in which coefficients are expressed via parameters of (6) (here and below, the usual convention of summation over repeated indices is adopted):

$$\begin{aligned} V = M_{\pm}^2\,H^+H^- + \frac{M_{ij}}{2}\,\eta_i\eta_j + vT_i\,H^+H^-\eta_i + vT_{ijk}\,\eta_i\eta_j\eta_k \\ + CH^+H^-H^+H^- + \frac{B_{ij}}{2}\,H^+H^-\eta_i\eta_j + Q_{ijkl}\eta_i\eta_j\eta_k\eta_l. \end{aligned} \tag{8}$$

3 Quadratic terms of potential. First subset of observables

In Eq. (8), the coefficients M_{ij} form the neutral scalar *mass matrix* (here $N = M_{\pm}^2/v^2 + \Lambda_4$):

$$M_{ij} = v^2\begin{pmatrix} \Lambda_1 & Re\,\Lambda_6 & -Im\,\Lambda_6 \\ Re\,\Lambda_6 & \dfrac{N + Re\Lambda_5}{2} & -Im\,\Lambda_5/2 \\ -Im\,\Lambda_6 & -Im\,\Lambda_5/2 & \dfrac{N - Re\Lambda_5}{2} \end{pmatrix}. \tag{9}$$

The physical neutral Higgs states h_a are such superpositions of fields η_i that diagonalize this matrix:

$$h_a = R_a^i\eta_i\,, \quad \eta_i = R_i^a h_a\,; \qquad M_{ij}\eta_i\eta_j/2 = \textstyle\sum_a M_a^2 h_a^2/2, \quad M_{ij} = R_i^a R_j^a M_a^2. \tag{10}$$

The mixing matrix R_i^a is a real-valued orthogonal matrix determined by the parameters of the mass matrix. It can be parameterized with three Euler angles. One of them is responsible for rephasing transformation of fields, i.e. it is irrelevant. The overall sign of this matrix is insignificant, we fix $R_1^1 > 0$.

The trace of the mass matrix is invariant under transformations (10). Therefore we obtain *a sum rule* $v^2(\Lambda_1 + \Lambda_4) = \sum_a M_a^2 - M_\pm^2$.

One of the advantages of the Higgs basis as compared to other RPa bases is the fact that elements of the rotation matrix are directly related to the couplings (4), which are, in principle, measurable:

$$\chi_a^V = R_1^a, \quad \chi_a^{H^+W^-} \equiv \left(\chi_a^{H^-W^+}\right)^* = R_2^a + iR_3^a. \tag{11}$$

It can be seen easily after writing the kinetic term of the Higgs Lagrangian with definitions (7) and (10). The absolute values of the real quantities χ_a^V are directly measurable in the decays $h_a \to WW$ (or W-fusion process), etc.

The phases of quantities $\chi_a^{H^+W^-}$, i.e. the ratios R_3^a/R_2^a, cannot be fixed because of the rephasing freedom of potential in the Higgs basis . Their relative phases for different h_a are determined unambiguously. We fix the RPh basis by the condition $R_3^2 = 0$.

The orthogonality of the mixing matrix means that its elements obey a set of relations $\sum\limits_i R_i^a R_i^b = \delta_{ab}$, $\sum\limits_a R_i^a R_j^a = \delta_{ij}$, which can be rewritten as very useful **Sum Rules**:

$$\sum_a (\chi_a^V)^2 = 1\,, \qquad |\chi_a^V|^2 + |\chi_a^{H^+W^-}|^2 = 1\,. \tag{12}$$

This orthogonality allows to express all elements R_i^a via couplings of different Higgs neutrals h_a to gauge bosons χ_a^V (ρ is arbitrary phase, determined RPh freedom):

$$R_a^i = \begin{pmatrix} \chi_1^V & \chi_2^V & \chi_3^V \\ \dfrac{-\chi_1^V\chi_2^V}{\sqrt{1-(\chi_2^V)^2}} & \sqrt{1-(\chi_2^V)^2} & \dfrac{-\chi_2^V\chi_3^V}{\sqrt{1-(\chi_2^V)^2}} \\ \dfrac{\chi_3^V}{\sqrt{1-(\chi_2^V)^2}} & 0 & \dfrac{-\chi_1^V}{\sqrt{1-(\chi_2^V)^2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\rho & \sin\rho \\ 0 & -\sin\rho & \cos\rho \end{pmatrix} \tag{13}$$

with limitation, given by the first sum rule (12).

Finally, one can read (9) as expressions of some Λ's via elements of the mass matrix and then, with the aid of (10), express them via the masses of Higgs bosons and their couplings to gauge bosons:

$$\begin{array}{ll} v^2\Lambda_1 = \sum\limits_a (\chi_a^V)^2 M_a^2; & v^2\Lambda_4 = \sum\limits_a M_a^2 - M_\pm^2 - v^2\Lambda_1; \\ v^2\Lambda_5^* = \sum\limits_a (\chi_a^{H^+W^-})^2 M_a^2; & v^2\Lambda_6^* = \sum\limits_a \chi_a^V \chi_a^{H^+W^-} M_a^2. \end{array} \tag{14}$$

These equations describe parameters Λ_1, Λ_4, Λ_5, Λ_6 of Lagrangian via observables of the first subset, i.e. **masses of all Higgs bosons $M_{1,2,3}$, $M_\pm$, vacuum expectation value of Higgs field $v = 246$ GeV and the couplings χ_a^V of any two (of three) chosen neutrals to the gauge bosons** – 7 quantities. Below we assume that h_1 is the discovered Higgs boson with $M_1 \approx 125$ GeV. The final equations also contain the couplings $\chi_a^{H^+W^-}$, expressed via χ_a^V with the aid of Eq. (13). This subset determines explicitly all quadratic (mass) terms of potential (6).

4 Other terms of potential. Second subset of observables

The Higgs boson masses and couplings to the gauge bosons do not depend on Λ_2, Λ_3, Λ_7. In turn, these parameters are necessary to determine triple and quartic Higgs boson vertices. The triple and quartic Higgs vertices of the potential (6) can be determined completely only if one supplements the parameters of the first subset with additional information. In turn, to form the second subset, one needs to use triple and quartic Higgs self-interactions. For this goal we use **three triple couplings $H^+H^-h_a$ (quantities $\chi_a^\pm$) and one quartic coupling $g(H^+H^-H^+H^-)$** – 4 quantities. The analysis is simple but cumbersome.

The parameters of the first subset plus three couplings $\chi_a^\pm$ determine all triple Higgs couplings. The coefficients Λ_3, Λ_7 of the Lagrangian are expressed simply via these three couplings and observables of the first subset.

$$\Lambda_3 = (2M_\pm^2/v^2)\sum_a \chi_a^V \chi_a^\pm; \qquad \Lambda_7^* = (2M_\pm^2/v^2)\sum_a \chi_a^{H^-W^+}\chi_a^\pm\,. \tag{15}$$

The description of quartic interactions of Higgs particles demands adding one more observable

$$\Lambda_2 = 2g(H^+H^-H^+H^-)\,. \tag{16}$$

• Certainly, the second subset of observables can be constructed with other triple and quartic couplings.

Using processes involving charged Higgses looks preferable for two reasons. First, with charged Higgses, this procedure requires the fewest calculations, improving accuracy and reducing uncertainties. Second, the amplitudes of the processes $e^+e^- \to H^+H^-h_a$, $\gamma\gamma \to H^+H^-h_a$, $e^+e^- \to H^+H^-H^+H^-$, $\gamma\gamma \to H^+H^-H^+H^-$ at ILC/CLIC [10] are directly proportional to the corresponding couplings, without any nonrelevant diagrams interfering.

• The obtained equations for parameters of the Lagrangian in the Higgs basis contain one irrelevant parameter: the RPh phase ρ related to a rephasing freedom in the Higgs basis . In order to switch to another RPa basis, which could be more useful for some special reasons, one should use two parameters $\tan\beta$ and ξ, which are determined by the RPa basis choice. Once these parameters are determined from problem-specific conditions, the transition to this RPa basis is performed with the aid of the back rotation $\hat{\mathcal{F}}_{HB}^{-1}$ (5). The final equations for parameters λ_i, m_{ij}^2 are constructed from measurable quantities discussed above and RPa basis-choice parameters β, ρ, ξ.

5 Notes about renormalization scheme

The standard calculation of the radiative corrections (RC) in the model is based on the parameters of Lagrangian which are RPa dependent. This RPa ambiguity can be removed, for example, by using the renormalization procedure fixing parameters of the basic set. In the modern approach the calculation of any physical effect should be supplemented by calculation of renormalized values of masses and other parameters of basic set which should be taken into account in the data analysis. The development of such scheme looks important problem.

6 Discussion

• We have found the minimal complete set of measurable quantities (named *observables*) that determines all parameters of the 2HDM Lagrangian – *the basic set of observables*.

• The observables of the basic set are measurable quantities, independent of each other. The models with arbitrary values of these observable parameters can, in principle, be realized, provided that the positivity constraints are satisfied and the couplings χ_a^V are not too large, in order not to violate the sum rule (12). In some special variants of the 2HDM, additional relations between these parameters may appear (for example, in the CP conserving case $\chi_3^V = \chi_3^\pm = 0$).

Our results open the door for the study of Higgs models in terms of measurable quantities alone. It allows to remove from the data analysis the widely spread intermediate stages with complex, often model-dependent, analysis of coefficients of the Lagrangian.

• **Possible strong interaction in the Higgs sector**. The fact that free parameters of the potential naturally fall into three very distinct categories offers a new opportunity that was absent in the SM. Before the Higgs discovery, the large coupling constant λ was, in principle, possible within the SM. In this case, the Higgs boson would be very heavy and wide, and it could not be seen as a separate particle. Instead, its dynamics would be governed by the strong interaction in the Higgs sector, which would manifest itself in the form of resonances in the W_LW_L, W_LZ_L, Z_LZ_L scattering in the $1-2$ TeV energy range. In the SM this opportunity is closed by the discovery of the Higgs boson with $M \approx 125$ GeV.

Our analysis shows that, within the 2HDM, the reasonably low values of all Higgs masses are well compatible with large Λ_3, $|\Lambda_7|$, Λ_2, i.e. with the strong interaction in the Higgs sector. A signal of this feature can be observed in the multi-Higgs final states or (for Λ_3, $|\Lambda_7|$) in the anomalously large two-photon width of some neutral Higgs boson. Moreover, this strong interaction can coexist even with moderate values of triple Higgs couplings as it could be driven exclusively by the large value of a single parameter Λ_2.

• The principal possibility to determine all parameters of the 2HDM from the (future) data meets strong practical limitations (which can be hidden in other approaches). This will remain a problem for a very long time.

Indeed, the modern data on the Higgs boson couplings, the analysis of many particular models (see, e.g., [11]), and the using of sum rules (12) allow us to conclude that the discovery of new Higgs bosons $h_{2,3}$, $H^{\pm}$ is a difficult problem for the LHC and e^+e^- colliders [12].

If these $h_{2,3}$ are discovered, the inaccuracies in the measuring of their masses and couplings are not expected to be small.

The measuring of triple and quartic interactions of Higgs bosons looks more difficult problem. So it is natural to expect that these measurements will be made later and with bigger inaccuracy.

The general limitations for the model, similar to the positivity constraint, contain parameters of the first and second subsets simultaneously. Thus, there are few chances that such restrictions can be verified in the near future.

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