Investigation of $\pi^+\pi^-$, $K^+\pi^-$ and π^+K^- atoms for pion-pion and pion-kaon scattering length measurements

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Theory, using Low Energy QCD, calculated with high precision the $\pi\pi$ and πK scattering length. Lifetime of $\pi^+\pi^-$ atoms are related with a difference of pion-pion s-wave scattering lengths with isospin 0 and 2: $|a_0 - a_2|$. Energy splitting between p-states (long-lived atoms) and s-states of $\pi^+\pi^-$ atoms allows to investigate another combination: $2a_0 + a_2$. Lifetime of πK atoms gives information about difference of pion-kaon s-wave scattering lengths with isospin 1/2 and 3/2: $|a_{1/2} - a_{3/2}|$. Experimental measurements of pion-kaon lifetimes and observation of long-lived $\pi^+\pi^-$ atoms at experiment DIRAC are presented.

1 Introduction

Chiral Perturbation Theory (ChPT) describes QCD processes at low energies. ChPT in 2-loop approximation and Roy equation predicts s-wave $\pi\pi$ scattering lengths with isosprin 0 and 2 to be [1]:

$$a_0 = 0.220 \pm 2.3\%,\ a_2 = -0.0444 \pm 2.3\%,\ a_0 - a_2 = 0.265 \pm 1.5\%\,. \tag{1}$$

Also there are predictions for s-wave πK scattering with isosprin 1/2 and 3/2, which have done by ChPT in 1-loop approximation [2,3]:

$$a_{1/2} = 0.19 \pm 0.02, a_{3/2} = -0.05 \pm 0.02, a_{1/2} - a_{3/2} = 0.23 \pm 0.01\,. \tag{2}$$

ChPT with $L^{(2)}, L^{(4)}, L^{(6)}$ in 2-loop approximation predicts s-wave scattering length difference to be [4]: $a_{1/2} - a_{3/2} = 0.267$. Another prediction for scattering length difference have been obtained, using Roy-Steiner equations [5]:

$$a_{1/2} - a_{3/2} = 0.269 \pm 0.015\,. \tag{3}$$

The πK scattering has also been studied extensively in the framework of lattice QCD. Predictions for πK scattering length $a_{1/2} = 0.183 \pm 0.039, a_{3/2} = -0.0602 \pm 0.0040$ [6] and their combination a_0^- have been obtained [7]:

$$a_0^- = \frac{1}{3}(a_{1/2} - a_{3/2}) = 0.0811 \pm 0.0143\,. \tag{4}$$

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The measurement of the s-wave $\pi\pi$ scattering lengths would test our understanding of the chiral $SU(2)_L \times SU(2)_R$ symmetry breaking (u,d quarks) and measurement of the s-wave πK scattering lengths would test our understanding of the chiral $SU(3)_L \times SU(3)_R$ symmetry breaking of QCD (u, d and s quarks).

Experimental data on the πK low-energy phases are absent.

2 Method of $\pi^+\pi^-$ and πK atom observation and investigation

The $\pi^+\pi^-$-atom ($A_{2\pi}$) is a hydrogen-like atom consisting of π^+ and π^- mesons. The $A_{2\pi}$-atom lifetime (ground state 1S), $\tau = 1/\Gamma$ is dominated by the annihilation process into $\pi^0\pi^0$. There is a relation [8, 9] between the width of $A_{2\pi}$ decay $\Gamma_{1s,2\pi^0}$ and $\pi\pi$ scattering lengths for isospin 0 and 2: $\Gamma_{1s,2\pi^0} = R \cdot |a_0 - a_2|^2$ ($\delta R \approx 1.2\%$). Taking into account (Eq. 1), lifetime is predicted to be: $\tau = (2.9 \pm 0.1) \times 10^{-15}s$. Therefore a measurement of $A_{2\pi}$ lifetime allows to measure a value for difference of s-wave $\pi\pi$ scattering lengths: $|a_0 - a_2|$ [10].

In order to get values of a_0 and a_2 separately from $\pi^+\pi^-$ data [10], one may exploit the fact that the energy splitting between the levels ns and np, $\Delta E^{ns-np} = E_{ns} - E_{np}$, depends on another combination of the scattering lengths: $2a_0 + a_2$ [11]. Detailed analysis of strong and electromagnetic interactions on the $A_{2\pi}$ energy structure has been performed in [12]. Term ΔE^{str}_{nl} takes into account effects from strong interaction and contributes up to 80% (-0.47 ± 0.01 eV) of the full energy shift. Other contributions from finite-size effect, self-energy corrections, vacuum polarization and relativistic insertions have been calculated with high precision. This fact provides a high sensitivity of a ΔE^{ns-np} measurement to the value of the term: $2a_0 + a_2$. Thus it allows to obtain a value for the new combination of s-wave $\pi\pi$ scattering lengths.

The πK-atom ($A_{\pi K}$) is a hydrogen-like atom consisting of K^+ (K^-) and π^- (π^+) mesons. Lifetime of this atom is dominated by the annihilation process into $\pi^0 K^0$ and is related with a difference of s-wave πK scattering lengths for isospin 1/2 and 3/2 [12]: $\Gamma_{1S,\pi^0 K^0} = 8\alpha^3\mu^2 p^*(a_0^-)^2(1+\delta_K)$. Here α is the fine structure constant, μ is the reduced mass of the $\pi^\pm K^\mp$ system, p^* is the outgoing π^0 momentum in the πK atom system, and δ_K accounts for corrections, due to isospin breaking, at order α and quark mass difference $(m_u - m_d)$. With prediction of scattering length difference from Eq. (3), lifetime of $A_{\pi K}$ in ground state is estimated to be: $\tau = (3.5 \pm 0.4) \times 10^{-15}$.

A method of investigation for $\pi^+\pi^-$, πK and other atoms, consisting from two oppositely charged mesons, has been proposed in [10]. Pairs of $\pi^+\pi^-$ or K^+ (K^-) and π^- (π^+) mesons are producing in proton-target interactions. Pairs, which are generated from fragmentation and strong decay (“short-lived” sources), are affected by Coulomb interaction in the final state. Some of them form Coulomb bound states — atoms, other are generated as free pairs (“Coulomb pairs”). Number of produced atoms (N_A) is proportional to a number of “Coulomb pairs” (N_C) with low relative momentum Q in a pair C.M. system: $N_A = K \cdot N_C$. The coefficient K is calculated with an accuracy better than 1%.

If at least one meson is generated from long-lived sources (electromagnetically or weakly decaying mesons or baryons: $\eta, \eta', K^0_s, \ldots$), then such pairs are not affected by interaction in the final states (“non-Coulomb pairs”).

After production, $A_{2\pi}$ and $A_{\pi K}$ travel through the target and could to annihilate into $\pi^0\pi^0$ ($\pi^0 K^0$), or to be ionised due to interaction with the target matter, producing specific “atomic pairs”. These pairs have small relative momentum ($Q < 3$ MeV/c) and a number of such pairs n_A could be measured experimentally. Ratio of “atomic pair” number to a number of atom produced is a breakup probability: $P_{\rm br}(\tau) = n_A/N_A = n_A/(K \cdot N_C)$ [13, 14]. In Fig. 1 dependence of $A_{\pi K}$ breakup probability is shown for two nickel target are

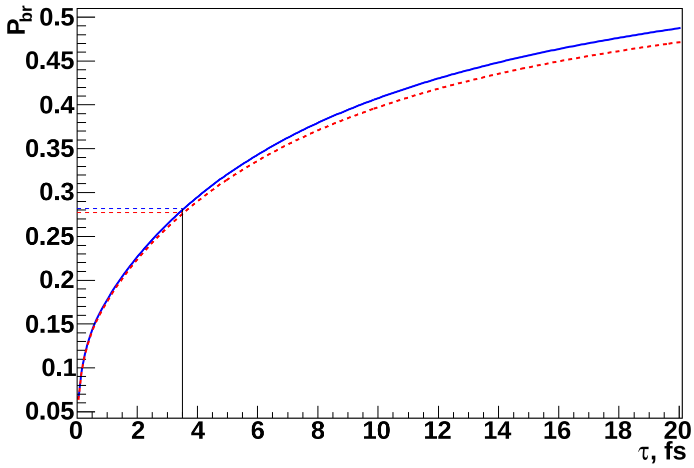

Figure 1: Dependence of the breakup probability $P_{\rm br}$ on $A_{\pi K}$ lifetime for 108μm (solid blue line) and 98μm (dashed red line) nickel targets, and an example how lifetime could be obtained from experimentally measured breakup probability.

used in experiment DIRAC for pair laboratory momentum range $4.8 \div 7.6$ GeV/c. Value is averaged, using experimentally measured spectrum of atoms.

To observe atoms in states with orbital momentum $l > 0$ this approach has been modified. In inclusive processes $A_{2\pi}$ are produced in s-states distributed over the principal quantum number n according to n^{-3}. When moving inside the target, relativistic $A_{2\pi}$ interact with the electric field of target atoms, and some of them (N_A^L) leave the target with an orbital quantum number $l > 0$ ($A_{2\pi}^L$). For these states the decays are suppressed, due to 0 value of wave function for small distances, where strong interaction acts. Therefore, the decay mechanism of such excited states is the radiative deexcitation to an ns state, annihilating subsequently into two π^0. Thus, the $A_{2\pi}^L$ decay probability is given by the shortest radiative lifetime, the $2p$ lifetime $\tau_{2p} = 1.17 \cdot 10^{-11}$ s. For an average $A_{2\pi}$ momentum of 4.5 GeV/c ($\gamma \simeq 16$), the decay lengths are 5.7 cm ($2p$), 19 cm ($3p$) and 43 cm ($4p$). Using a $\sim 100\mu$m thick Be target and inserting a $\sim 2\mu$m thick Pt foil downstream of this target [15], a large fraction of the long-lived atoms $A_{2\pi}^L$, generated in Be, reaches the Pt foil and breaks up, thus providing an extra number n_A^L of atomic pairs (see Fig. 2).

3 DIRAC setup

DIRAC setup was created to detect $\pi^+\pi^-$ with small relative momenta [16]. In 2004-2006 it has been modified in order to detect both $\pi^+\pi^-$ and πK pairs [17]. In 2011-2012 years Pt foil and additional permanent magnet between Be target and Pt foil (see Fig. 2) have been added for searching long-lived $A_{2\pi}$ [18].

4 Investigation of π^+K^- and $K^+\pi^-$ atoms

Experimental distribution of π^+K^- and $K^+\pi^-$ have been analysed and total number of πK "atomic pairs" is found to be [17]: $n_A^{K^+\pi^-} + n_A^{\pi^+K^-} = 178 \pm 49$. Effect-to-error ratio (3.6) is not sufficient for observation,

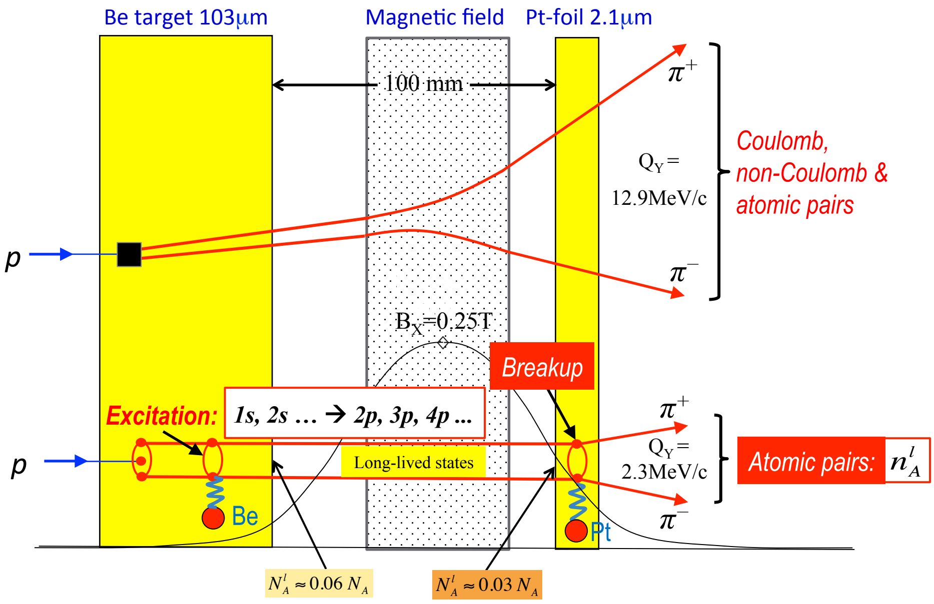

Figure 2: Method to observe long-lived $A^L_{2\pi}$ by means of a breakup foil (Pt).

but these statistic allow to make estimation of $A_{\pi K}$ atom lifetime:

$$\tau = (2.5^{+3.0}_{-1.8}|_{stat}\ ^{+0.3}_{-0.1}|_{syst})\ fs = (2.5^{+3.0}_{-1.8}|_{tot})\ fs\,. \tag{5}$$

Lifetime estimation (Eq. 5) provides the first model-independent estimation of s-wave πK scattering length combination (Eq. 4):

$$|a_0^-|M_\pi = 0.107^{+0.093}_{-0.035} = 0.11^{+0.09}_{-0.04} \tag{6}$$

5 Observation of long-lived $\pi^+\pi^-$ atoms

The event reconstruction has been performed by means of the DIRAC $\pi^+\pi^-$ analysis software already used for the analysis of the 2001-2003 data [19] and 2008-2010 [17]. One modification of procedure has been induced by presence of additional permanent magnet between Be target and Pt foil. Horizontal field of this magnet shifts Q_Y projection of relative momentum Q by 12.9 MeV/c for pairs generated in Be target and by 2.3 MeV/c for pairs generated in Pt foil (Fig. 2). Therefore new definition of transverse component of relative momentum has been used: $Q'_T = \sqrt{Q_X^2 + (Q_Y - 2.3\,MeV/c)^2}$. To improve background conditions the procedure selects events with small values Q'_T. One-dimensional (over $|Q_L|$) and 2-dimensional ($|Q_L|, Q'_T$) distributions of experimental data have been analysed, using simulated distributions of "atomic pairs" from ionisation of $A^L_{2\pi}$ in Pt foil, "Coulomb" and "non-Coulomb" pairs, generated in Be target. Simulation takes into account initial distributions of pairs over pair laboratory momentum and relative momentum Q and its projections, multiples scattering in a target, Pt foil, the setup detectors and partitions, detector resolution and tracking efficiency.

Results of fit procedure for 1- and 2-dimensional distributions are collected in Table 1. It is shown that for different criteria on Q'_T amount of background events differs up to 17 times, but a total number of "atomic pairs", obtained as detected number of "atomic pairs" n^L_A divided by efficiency of selection criterion, coincide within statistics. It demonstrates stability of results.

Table 1: Analysis of data collected in 2012 for different Q'_T cuts. The detected numbers n^L_A of atomic pairs and the corresponding total numbers $n^{L,\ tot}_A$ (via selection efficiency) are presented together with the background contribution (Coulomb, non-Coulomb and accidental pairs) and the fit quality χ^2/n (n = degrees of freedom). Errors are only statistical.

Q'_T cut (MeV/c)	n^L_A	$n^{L,\ tot}_A$	Background	χ^2/n
Fit over $\lvert Q_L\rvert, Q'_T$				
2.0	436 ± 57	488 ± 64	16790	138/140
Fit over $\lvert Q_L\rvert$				
0.5	152 ± 29	467 ± 88	971	29/27
1.0	349 ± 53	489 ± 75	3692	19/27
1.5	386 ± 78	454 ± 91	9302	22/27
2.0	442 ± 105	495 ± 117	16774	22/27

Systematic effects have been analysed and systematic error is found to be 22 [20].

From the 2-dimensional analysis the evaluated number of atomic pairs is $n^L_A = 436 \pm 61$ or 7.1 standard deviations, taking into account statistical as well as systematic errors.

6 Summary

The analysis of πK pairs statistic, collected from 2008 to 2010, allows to evaluate the number of atomic πK pairs (178 ± 49) as well as the number of produced πK atoms (653 ± 42) and thus the breakup (ionisation) probability.

Value of πK atom lifetime has been extracted to be $\tau = (2.5^{+3.0}_{-1.8})\, fs$. It provides a measurement of the S-wave isospin-odd πK scattering length: $|a^-_0| = (0.11^{+0.09}_{-0.04}) \cdot M^{-1}_\pi$.

Analysis of data collected in 2012 with Be-Pt target allows to make observation of "atomic pairs" from $\pi^+\pi^-$ atoms in long-lived states: $n^L_A = 436 \pm 61$. It provides possibility to plan experiments for measurement of "Lamb shift like" effect in $\pi^+\pi^-$ system.

Acknowledgments

We are grateful to CERN for continuous support and the PS team for the excellent performance of the accelerator. This work was funded by CERN, INFN (Italy), INCITE and MICINN (Spain), IFIN-HH (Romania), the Ministry of Education and Science and RFBR grant 01-02-17756-a (Russia), the Grant-in-Aid from JSPS and Sentanken-grant from Kyoto Sangyo University (Japan).

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