

Radiative decays $V \rightarrow P\gamma^*$ in the instant form of relativistic quantum mechanics

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Calculations of form factor for the radiative decays $V \rightarrow P\gamma^*$ process are performed in the framework of an instant form of relativistic quantum mechanics. The electromagnetic current operator for this decay is constructed. The transition form factor is obtained in the so called relativistic modified impulse approximation (MIA). The current operator satisfies the conditions of Lorentz-covariance and current conservation in MIA. The results of the calculations are compared with the analogous results in the light-front dynamics and in the model of vector meson dominance

1 Introduction

Understanding the relation between the observable properties of mesons or baryons and the underlying dynamics is a serious challenge in modern hadron physics. Quantum chromodynamics is a good candidate for the role of a complete theory. Moreover, it is known that the perturbative calculation methods in QCD yield reliable results only when describing so-called "hard" processes characterized by large transferred momenta and fail when calculating characteristics determined by "soft" processes. This, for instance, pertains to describing particle electromagnetic form factors and other electromagnetic characteristics that are measured experimentally. The field theory of strong interactions involves infinitely many degrees of freedom transferred by local fields - quarks and gluons. But the success of constituent models indicates that the main characteristics of hadronic states can be described using only a finite number of these degrees of freedom, leaving other degrees of freedom frozen. One of the most reliable methods for such a description is to use the relativistic quantum mechanics (RQM) model. As in every other theoretical description of electromagnetic structures of composite systems, we have three main points in the relativistic constituent model.

1. First, we must choose the dynamics. Foundations of RQM were laid by Dirac in [1], where three main types of dynamics differing by evolution parameters were introduced. Those are the point form of dynamics, the instant form of dynamics, and the light-front dynamics, each of which can be related to a three-dimensional hypersurface in the four-dimensional space. The initial data are defined on this hypersurface, and its evolution is the main object of study. A point form hypersurface is determined by the conditions $-x^\mu x_\mu = a^2$, $t > 0$, an instant form hypersurface is determined by the condition $t = 0$, and a light-front form hypersurface is determined by the equation $x^0 + x^3 = 0$.

Numerous actual calculations based on the Dirac model currently exist. These calculations illustrate the power and flexibility of the RQM as an instrument for studying systems of strongly interacting particles in the energy range below several GeV (see [2,3,4,5,6,7,8] for a discussion and relevant references).

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2. Second, we must construct the operator of the transition current. This step is important in all schemes for studying particle structures, not only in RQM. Separating the kinematic parts and invariant parts (form factors) of the matrix elements of the current is an important point in all such methods. We call a representation of a current matrix element in terms of form factors a parameterization of the matrix element. We note that form factors, which are Lorentz-invariant functions, are customarily determined from experimental data. Therefore, we must consider a matrix element of the electroweak current that has correct transformation properties, and we must perform the procedure for relativistic parameterization of the current matrix element, i.e., we must separate relativistic-invariant form factors or, in other words, separate reduced matrix elements using the Wigner-Eckart theorem for the Poincare group [9]. An important feature of composite systems is that all these form factors, strictly speaking, are distributions or generalized functions.

3. Third, choosing a concrete approximation for calculations is important. As a rule, we use the impulse approximation in the case of two-component systems, which means that a virtual γ -quantum interacts with only one of the components, not with both of them simultaneously. But when choosing a approximation, we must always verify that it does not break the relativistic invariance. In the instant form of RQM, the impulse approximation is not relativistic invariant and depends on the choice of the coordinate system. To ensure the relativistic invariance, we construct a modified impulse approximation (MIA) in the spirit of [10]. This method uses a double-integral representation with respect to the invariant mass of the system. These integrals appear as a result of solving equations of the Muskhelishvili-Omnes type. This method was laid out in detail in [11] (see Secs. 3B and 3C), and its use in calculating structures of concrete two-quark systems can be found in [12,13,14]. In particular, the method was used to calculate the pion form factor [15], both in the asymptotic domain [16] and in the timelike domain [17]. We emphasize that the calculations in [15] also demonstrated their predictive power (see [18,19] for the details).

In this paper we are consider the description of radiative decays $V \rightarrow P\gamma^*$ in instant form of (RQM). Construction of electromagnetic current operator that satisfies the Lorentz-covariance and current conservation conditions is performed. We calculate the transition form factor and compare it with results of other approaches.

2 Parameterization an electromagnetic current matrix element nondiagonal in the angular momentum in the case of a free two-particle system with quantum numbers of pions and ρ -mesons.

The state vector of the system of two interacting particles in the RQM belongs to the direct product of two single-particle Hilbert spaces. Therefore, as a bases, we can consider the following two sets of vectors.

1. The basis of the individual momenta and spins of the particles:

$$\begin{aligned} |\vec{p}_1, m_1; \vec{p}_2, m_2\rangle &= |\vec{p}_1, m_1\rangle \otimes |\vec{p}_2, m_2\rangle, \\ \langle \vec{p}, m | \vec{p}', m'\rangle &= 2 p_0 \delta(\vec{p} - \vec{p}') \delta_{m m'}, \end{aligned} \quad (1)$$

where $\vec{p}_{1,2}$ - three-momentum, $m_{1,2}$ -spin projection, $p_0^2 - \vec{p}^2 = M^2$.

2. Basis with separated center of mass motion of two particles:

$$|\vec{P}, \sqrt{s}, J, L, S, m\rangle, \quad (2)$$

where $\vec{P} = \vec{p}_1 + \vec{p}_2$, \sqrt{s} - invariant mass of two particle system, $P^2 = s$, L - orbital momentum, S -spin momentum.

Basis (1) and (2) are related by the Clebsh-Gordon decomposition:

$$\begin{aligned}
 |\vec{P}, \sqrt{s}, J, L, S, m\rangle &= \sum_{m_1, m_2} \int \frac{d^3 \vec{p}_1}{2p_{10}} \frac{d^3 \vec{p}_2}{2p_{20}} |\vec{p}_1, m_1; \vec{p}_2, m_2\rangle \\
 &\times \langle \vec{p}_1, m_1; \vec{p}_2, m_2 | \vec{P}, \sqrt{s}, J, L, S, m \rangle,
 \end{aligned} \tag{3}$$

where

$$\begin{aligned}
 \langle \vec{p}_1, m_1; \vec{p}_2, m_2 | \vec{P}, \sqrt{s}, J, L, S, m \rangle &= \frac{2\sqrt{s}}{\sqrt{\lambda(s, M^2, M^2)}} \\
 &\times 2P_0 \delta(P - p_1 - p_2) \sum_{\tilde{m}_1, \tilde{m}_2} D_{m_1 \tilde{m}_1}^{1/2}(p_1, P) D_{m_2 \tilde{m}_2}^{1/2}(p_2, P) \\
 &\times \langle \frac{1}{2} \tilde{m}_1 \frac{1}{2} \tilde{m}_2 | S m_S \rangle Y_{L m_L}(\vartheta, \varphi) \langle S L m_S m_L | J m \rangle,
 \end{aligned} \tag{4}$$

$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$, $D^{1/2}$ - matrix of three-dimensional rotations (Wigner's D -function), $Y_{L m_L}$ - are spherical functions.

In the constituent quark model pion and ρ -meson are represented by bound states of u and \bar{d} -quarks with the quantum numbers $J = L = S = 0$ for the pion and $J = S = 1; L = 0$ for the ρ -meson. We omit zero-values quantum numbers in the corresponding state vectors. Equating the quark masses, $M_u = M_{\bar{d}} = M$, we can write the matrix element of the electromagnetic current operator for the free two-particle system with the given quantum numbers in the following form:

$$\begin{aligned}
 \langle \vec{P}, \sqrt{s} | j_\mu^0(0) | \vec{P}', \sqrt{s'}, 1, 0, 1, m' \rangle &= \int \frac{d^3 \vec{p}_1}{2p_{10}} \frac{d^3 \vec{p}_2}{2p_{20}} \frac{d^3 \vec{p}'_1}{2p'_{10}} \frac{d^3 \vec{p}'_2}{2p'_{20}} \cdot \\
 &\cdot \langle \vec{P}, \sqrt{s} | \vec{p}_1, m_1; \vec{p}_2, m_2 \rangle \langle \vec{p}_1, m_1; \vec{p}_2, m_2 | j_\mu^0(0) | \vec{p}'_1, m'_1; \vec{p}'_2, m'_2 \rangle \cdot \\
 &\cdot \langle \vec{p}'_1, m'_1; \vec{p}'_2, m'_2 | \vec{P}', \sqrt{s'}, 1, 0, 1, m' \rangle,
 \end{aligned} \tag{5}$$

where

$$\begin{aligned}
 \langle \vec{p}_1, m_1; \vec{p}_2, m_2 | j_\mu^0(0) | \vec{p}'_1, m'_1; \vec{p}'_2, m'_2 \rangle &= \langle \vec{p}_1, m_1 | j_{\mu 1}^0(0) | \vec{p}'_1, m'_1 \rangle \cdot \\
 &\cdot \delta(\vec{p}_2 - \vec{p}'_2) \delta_{m_2 m'_2} + \langle \vec{p}_2, m_2 | j_{\mu 2}^0(0) | \vec{p}'_2, m'_2 \rangle \delta(\vec{p}_1 - \vec{p}'_1) \delta_{m_1 m'_1}.
 \end{aligned} \tag{6}$$

Using nondiagonal parameterization of the matrix element [20, 21] with the zero value of the total angular momentum of the pion taken into account, we can write the matrix element of the current by following way:

$$\begin{aligned}
 \langle \vec{P}, \sqrt{s} | j_0(0) | \vec{P}', \sqrt{s'}, 1, 0, 1, m' \rangle &= \\
 &= \sum_{\tilde{m}', l', k'} D_{m', \tilde{m}'}^1(P', w) \langle 1 \tilde{m}' l' k' | 00 \rangle Y_{l' k'}(\vec{q}) G_{01}^{0, l'}(s, Q^2, s'),
 \end{aligned} \tag{7}$$

$$\begin{aligned}
 & \langle \vec{P}, \sqrt{s} | j_i^1(0) | \vec{P}', \sqrt{s'}, 1, 0, 1, m' \rangle = \\
 & = \sum_{\tilde{m}', l, k, j, n} D_{m', \tilde{m}'}^1(P', w) \langle 1 \tilde{m}' j n | 00 \rangle \langle 1 l k | j n \rangle Y_{lk}(\vec{q}) G_{01}^{1,l,j}(s, Q^2, s').
 \end{aligned} \tag{8}$$

Comparing expressions (5) and (7), (8) and integrating in the Breit reference system with $\vec{q} = (0, 0, q)$, we obtain analytic expressions for the so called free two-particle form factors. These expressions are very cumbersome, and we therefore write only one form factor, which we need in what follows, explicitly:

$$\begin{aligned}
 G_{01}^{111}(s, Q^2, s') & = \frac{\Theta(s, Q^2, s')(s + s' + Q^2)^2}{\sqrt{2}\sqrt{s - 4M^2}\sqrt{s' - 4M^2}\sqrt{4M^2 + Q^2}[\lambda(s, -Q^2, s')]^{1/2}} \cdot \\
 & \cdot \cos(\omega_1 + \omega_2) / 2 \left(\frac{s'(s' - s + 3Q^2)}{[\lambda(s, -Q^2, s')]^{1/2}} (G_M^u(Q^2) + G_M^{\bar{d}}(Q^2)) \right) + \\
 & + \sin(\omega_1 + \omega_2) / 2 \left(\frac{(s' - s - Q^2)}{(s + s' + Q^2)} \frac{\xi(s, s', Q^2)}{\sqrt{s'}} (G_M^u(Q^2) + G_M^{\bar{d}}(Q^2)) - \right. \\
 & \left. - \xi(s, s', Q^2) \frac{4M}{(s + s' + Q^2)} (G_E^u(Q^2) + G_E^{\bar{d}}(Q^2)) \right),
 \end{aligned} \tag{9}$$

where: $\Theta(s, Q^2, s') = \vartheta(s' - s_1) - \vartheta(s' - s_2)$, $\xi(s, s', Q^2) = \sqrt{ss'Q^2 - M^2\lambda(s, -Q^2, s')}$,

$$s_{1,2} = 2M^2 + \frac{1}{2M^2}(2M^2 + Q^2)(s - 2M^2) \mp \frac{1}{2M^2}\sqrt{Q^2(Q^2 + 4M^2)s(s - 4M^2)},$$

ϑ - is the step function;

$$\begin{aligned}
 \omega_1 & = \arctan \frac{\xi(s, s', Q^2)}{M[(\sqrt{s} + \sqrt{s'})^2 + Q^2] + \sqrt{ss'}(\sqrt{s} + \sqrt{s'})}, \\
 \omega_2 & = \arctan \frac{\xi(s, s', Q^2)(2M + \sqrt{s} + \sqrt{s'})}{M(s + s' + Q^2)(2M + \sqrt{s} + \sqrt{s'}) + \sqrt{ss'}(4M^2 + Q^2)}.
 \end{aligned}$$

3 The transition form factor $F_{\pi\rho}(Q^2)$

We can write the matrix element of the electromagnetic current of the transition $\rho \rightarrow \pi\gamma^*$ as (see, e.g., [22]):

$$\langle \vec{P}_\pi | j_\mu^c(0) | \vec{P}_\rho, 1, m_\rho \rangle = F_{\pi\rho}(Q^2) \varepsilon_{\mu\nu\sigma\delta} \tilde{\zeta}^\nu(m_\rho) P_\pi^\sigma P_\rho^\delta, \tag{10}$$

where \vec{P}_π and \vec{P}_ρ are the three-momentum of the pion and ρ -meson respectively, $\tilde{\zeta}^\nu(m_\rho)$ is the polarization four-vector, $\varepsilon_{\mu\nu\sigma\delta}$ is the rank-four anti-symmetric tensor, and $F_{\pi\rho}(Q^2)$ is the transition form factor measured experimentally.

For subsequent treatment of matrix element (10), we pass to the Breit reference system. In this system, the polarization vector is

$$\tilde{\zeta}^\nu(m_\rho) = -\frac{1}{\sqrt{2}}(0, 0, 1, i). \tag{11}$$

Substituting expression (11) in (10), we obtain

$$\langle \vec{P}_\pi | \tilde{j}_1^c(0) | \vec{P}_\rho, 1, \tilde{m}_\rho \rangle = -\frac{q(\tilde{P}_\pi^0 + \tilde{P}_\rho^0)}{\sqrt{2}} F_{\pi\rho}(Q^2). \quad (12)$$

We note that the components \tilde{j}_0^c , \tilde{j}_2^c and \tilde{j}_3^c of current matrix element (10) vanish. On the other hand, we can write matrix element (10) using the procedure presented in the Section 2 for parametrization of the current matrix element that is nondiagonal in the total angular momentum:

$$\begin{aligned} \langle \vec{P}_\pi | j_0^c(0) | \vec{P}_\rho, 1, m_\rho \rangle &= \\ &= \sum_{\tilde{m}_\rho, l', k'} D_{m_\rho, \tilde{m}_\rho}^1(P_\rho, w) \langle 1\tilde{m}_\rho l' k' | 00 \rangle Y_{l'k'}(\vec{q}) G_{01}^{0l'}(Q^2), \end{aligned} \quad (13)$$

$$\begin{aligned} \langle \vec{P}_\pi | j_i^c(0) | \vec{P}_\rho, 1, m_\rho \rangle &= \\ &= \sum_{\tilde{m}_\rho, l, k, j, n} D_{m_\rho, \tilde{m}_\rho}^1(P_\rho, w) \langle 1\tilde{m}_\rho j n | 00 \rangle \langle 1l k | j n \rangle Y_{lk}(\vec{q}) G_{01}^{1l, j}(Q^2). \end{aligned} \quad (14)$$

As mentioned above, for the radiative transition $\rho \rightarrow \pi\gamma^*$, we have a unique form factor expressed in terms of the first component of the electromagnetic current matrix element:

$$\langle \vec{P}_\pi | j_1^c(0) | \vec{P}_\rho, 1, \tilde{m}_\rho \rangle = -\frac{1}{\sqrt{3}} G_{01}^{111}(Q^2), \quad (15)$$

where $G_{01}^{111}(Q^2)$ is the transition form factor in RQM.

Equating expressions (12) and (15), we obtain the relation between experimentally measured form factor (10) and form factor (15) arising in our parametrization procedure:

$$F_{\pi\rho}(Q^2) = \sqrt{\frac{2}{3}} \frac{1}{q(\tilde{P}_\pi^0 + \tilde{P}_\rho^0)} G_{01}^{111}(Q^2), \quad (16)$$

where $\tilde{P}_\pi^0 = \sqrt{M_\pi^2 + \vec{q}^2}$, $\tilde{P}_\rho^0 = \sqrt{M_\rho^2 + \vec{q}^2}$. M_π , M_ρ are the masses of the pion and ρ -meson respectively.

For further calculations, we need an analytic expression for the composite system form factor $G_{01}^{111}(Q^2)$ in terms of wave functions in RQM.

4 The expression for the transition form factor in the instant form of RQM

Let us again consider matrix element (10) of the operator of the transition electromagnetic current. We assume that we have two-quark systems with quantum numbers corresponding to pions and ρ -mesons.

Because a state vector of a two-particle system in RQM belongs to the direct product of two one-particle Hilbert spaces, we can expand matrix element (10) as:

$$\begin{aligned} \langle \vec{P}_\pi | j_\mu^c(0) | \vec{P}_\rho, 1, m_\rho \rangle &= \sum \int \frac{d^3 \vec{P}}{N} \frac{d^3 \vec{P}'}{N'} d\sqrt{s} d\sqrt{s'} \langle \vec{P}_\pi | \vec{P}, \sqrt{s} \rangle \cdot \\ &\cdot \langle \vec{P}, \sqrt{s} | j_\mu^c(0) | \vec{P}', \sqrt{s'}, 1, 0, 1, m' \rangle \langle \vec{P}', \sqrt{s'}, 1, 0, 1, m' | \vec{P}_\rho, 1, m_\rho \rangle, \end{aligned} \quad (17)$$

where $\langle \vec{P}_\pi | \vec{P}, \sqrt{s} \rangle$ and $\langle \vec{P}', \sqrt{s'}, 1, 0, 1, m' | \vec{P}_\rho, 1, m_\rho \rangle$ are the wave functions in the instant form of RQM,

$$\begin{aligned} \langle \vec{P}_\pi | \vec{P}, \sqrt{s} \rangle &= N_c \delta(\vec{P} - \vec{P}_\pi) \varphi(s), \\ \langle \vec{P}', \sqrt{s'}, 1, 0, 1, m' | \vec{P}_\rho, 1, m_\rho \rangle &= N'_c \delta(\vec{P}' - \vec{P}_\rho) \varphi_1^1(s') \delta_{m' m_\rho}, \end{aligned} \quad (18)$$

$$\varphi(s) = \sqrt[4]{s} k \psi(k), \quad \varphi_1^1(s') = \sqrt[4]{s'} k' \psi_1^1(k'), \quad (19)$$

and $\psi(k)$ and $\psi_1^1(k')$ are the wave functions satisfying the normalization condition

$$\int \psi^2(k) k^2 dk = 1, \quad \int [\psi_1^1(k')]^2 k'^2 dk' = 1. \quad (20)$$

Integrating over \vec{P} and \vec{P}' using delta functions, we obtain

$$\begin{aligned} \langle \vec{P}_\pi | j_\mu^c(0) | \vec{P}_\rho, 1, m_\rho \rangle &= \int d\sqrt{s} d\sqrt{s'} \frac{N_c N'_c}{N N'} \varphi(s) \varphi_1^1(s') \cdot \\ &\cdot \langle \vec{P}_\pi, \sqrt{s} | j_\mu^c(0) | \vec{P}_\rho, \sqrt{s'}, 1, 0, 1, m_\rho \rangle. \end{aligned} \quad (21)$$

The bra and ket vectors of the matrix element in the right-hand side of equality (21) physically describe a system of two free particles and are transformed under a representation whose generators are independent of the interaction. The current operator describes the transition between two-quark systems with interaction and is transformed under another representation whose generators in RQM are interaction dependent. Therefore, we cannot apply the parametrization procedure of Sec. 2. But we can treat the right-hand side of equality (21) as a functional determined on the finite function space $\phi(s, s') = \varphi(s) \varphi_1^1(s')$ and interpret the matrix element of the electromagnetic current as a distribution, i.e., as an object that makes sense only when it is in the integrand.

The transformation properties of the left- and right-hand sides of (21) coincide. Therefore, we can represent the nondiagonal parametrization of the matrix element in the integrand in a form analogous to (13), (14):

$$\begin{aligned} \frac{N_c N'_c}{N N'} \langle \vec{P}_\pi, \sqrt{s} | j_0^c(0) | \vec{P}_\rho, \sqrt{s'}, 1, 0, 1, m_\rho \rangle &= \\ = \sum_{\vec{m}_\rho, l', k'} D_{m_\rho, \vec{m}_\rho}^1(P_\rho, w) \langle 1 \vec{m}_\rho l' k' | 00 \rangle Y_{l' k'}(\vec{q}) H_{01}^{0l'}(s, Q^2, s'), \end{aligned} \quad (22)$$

$$\begin{aligned} \frac{N_c N'_c}{N N'} \langle \vec{P}_\pi, \sqrt{s} | j_t^c(0) | \vec{P}_\rho, \sqrt{s'}, 1, 0, 1, m_\rho \rangle &= \\ = \sum_{\vec{m}_\rho, t, k, n} D_{m_\rho, \vec{m}_\rho}^1(P_\rho, w) \langle 1 \vec{m}_\rho j n | 00 \rangle \langle 1 t l k | j n \rangle Y_{l k}(\vec{q}) H_{01}^{1, l, j}(s, Q^2, s'), \end{aligned} \quad (23)$$

where $H_{01}^{01'}(s, Q^2, s')$, $H_{01}^{1,1,1}(s, Q^2, s')$ – are form factors.

Substituting representations (13), (14) and (22), (23) in (21), we obtain

$$G_{01}^{01}(Q^2) = \int d\sqrt{s}d\sqrt{s'}H_{01}^{01}(s, Q^2, s')\varphi(s)\varphi_1^1(s'), \quad (24)$$

$$G_{01}^{1,1,1}(Q^2) = \int d\sqrt{s}d\sqrt{s'}H_{01}^{1,1,1}(s, Q^2, s')\varphi(s)\varphi_1^1(s'), \quad (25)$$

We note that the explicit form of the invariant functions $H_{01}^{01}(s, Q^2, s')$, $H_{01}^{1,1,1}(s, Q^2, s')$ is unknown in general. We determine their explicit form using the MIA in [11]. This MIA consists in replacing the functions $H_{01}^{01}(s, Q^2, s')$, $H_{01}^{1,1,1}(s, Q^2, s')$ with free two-particle form factors $G_{01}^{01}(s, Q^2, s')$ and $G_{01}^{1,1,1}(s, Q^2, s')$.

$$G_{01}^{01}(Q^2) = \int d\sqrt{s}d\sqrt{s'}G_{01}^{01}(s, Q^2, s')\varphi(s)\varphi_1^1(s'), \quad (26)$$

$$G_{01}^{1,1,1}(Q^2) = \int d\sqrt{s}d\sqrt{s'}G_{01}^{1,1,1}(s, Q^2, s')\varphi(s)\varphi_1^1(s'). \quad (27)$$

Therefore, taking (16) into account, we obtain the final expression for the experimentally measurable transition form factor $F_{\pi\rho}(Q^2)$ in the MIA framework:

$$F_{\pi\rho}(Q^2) = \sqrt{\frac{2}{3}} \frac{1}{q(\bar{P}_\pi^0 + \bar{P}_\rho^0)} \int d\sqrt{s}d\sqrt{s'}G_{01}^{111}(s, Q^2, s')\varphi(s)\varphi_1^1(s'). \quad (28)$$

5 Numerical calculation of the form factor $F_{\pi\rho}(Q^2)$

When calculating the transition form factor by formulas 28) it is customary to use the harmonic oscillator ground state function (see, e.g., [22,23]),

$$\psi(k) = \frac{2}{\pi^{1/4}\beta^{3/2}} \exp\left(-\frac{k^2}{2\beta^2}\right), \quad (29)$$

where β is a parameter.

We fix this parameter based on data for mean square meson radii: $\beta = 0.278$ GeV for the pion [24] and $\beta = 0.231$ GeV for the ρ -meson [25].

To compare our results with those of other approaches, we choose the Sax form factors in the form

$$G_E(Q^2) = e_q f_q(Q^2), \quad G_M(Q^2) = (e_q + \kappa_q) f_q(Q^2), \quad f_q(Q^2) = \frac{1}{1 + \langle r_q^2 \rangle Q^2 / 6}, \quad (30)$$

where e_q is the quark charge, κ_q is the anomalous magnetic moment of a quark expressed in the natural units, and $\langle r_q^2 \rangle$ is the mean square radius of a quark.

Let us briefly discuss the choice of parameter values for our numerical calculations. We use the value $M = 0.22$ GeV for masses of constituent u - and \bar{d} quarks. This value is commonly used in modern relativistic calculations (see, e.g., [22,23]). We set the value of the mean square radius of a constituent quark to be $\langle r_q^2 \rangle = 0.3/M^2$. This value was obtained in the model with spontaneous chiral symmetry breaking [22], [26]. We set the sum of anomalous magnetic moments of constituent quarks in the formula for the transition form factor to be $\kappa_u + \kappa_{\bar{d}} = 0.09$ in accordance with the Gerasimov sum rules [27].

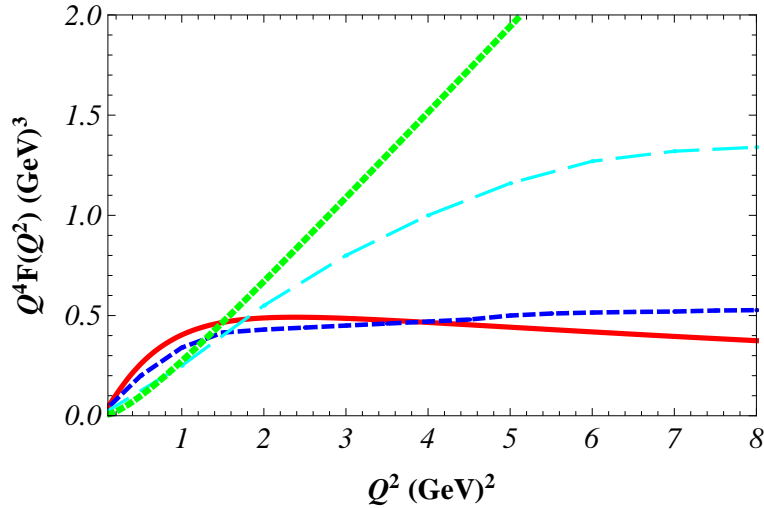


Figure 1: The transition form factor of the process $\rho \rightarrow \pi\gamma^*$ calculated in various approaches: the solid line is the result of calculating by formula (28), short-dashed line is the result of calculating in the framework of the light-front dynamics in [22], long-dashed line is the result of calculating in the framework of the light-front dynamics in [23], and the dotted line is the result of calculating in the vector meson dominance model (see, e.g., [23]).

6 Conclusion

In the framework of instant form of relativistic quantum mechanics the calculation of the transition form factor for the radiative decay $\rho \rightarrow \pi\gamma^*$ is performed. For the calculation the original method of the parametrization of the matrix element of the transition current is used. In the modified relativistic impulse approximation we obtained analytic expression for the transition form factor. The transition form factor is written via the two-quark wave functions and the so-called free two-particle form factor which enters in the matrix element of the transition current between two systems of free quarks with the quantum numbers of pion and ρ -meson. In contrast to the standard impulse approximation, the modified relativistic impulse approximation does not break the Lorentz covariance and the current conservation law. The numerical calculation results in the framework of our approach are close to the results obtained in the light-front dynamics and agree with the vector meson dominance model.

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